Modelling with Minsky

*The free online companion to **The New Economics: A Manifesto***

First Release: October 3rd 2021

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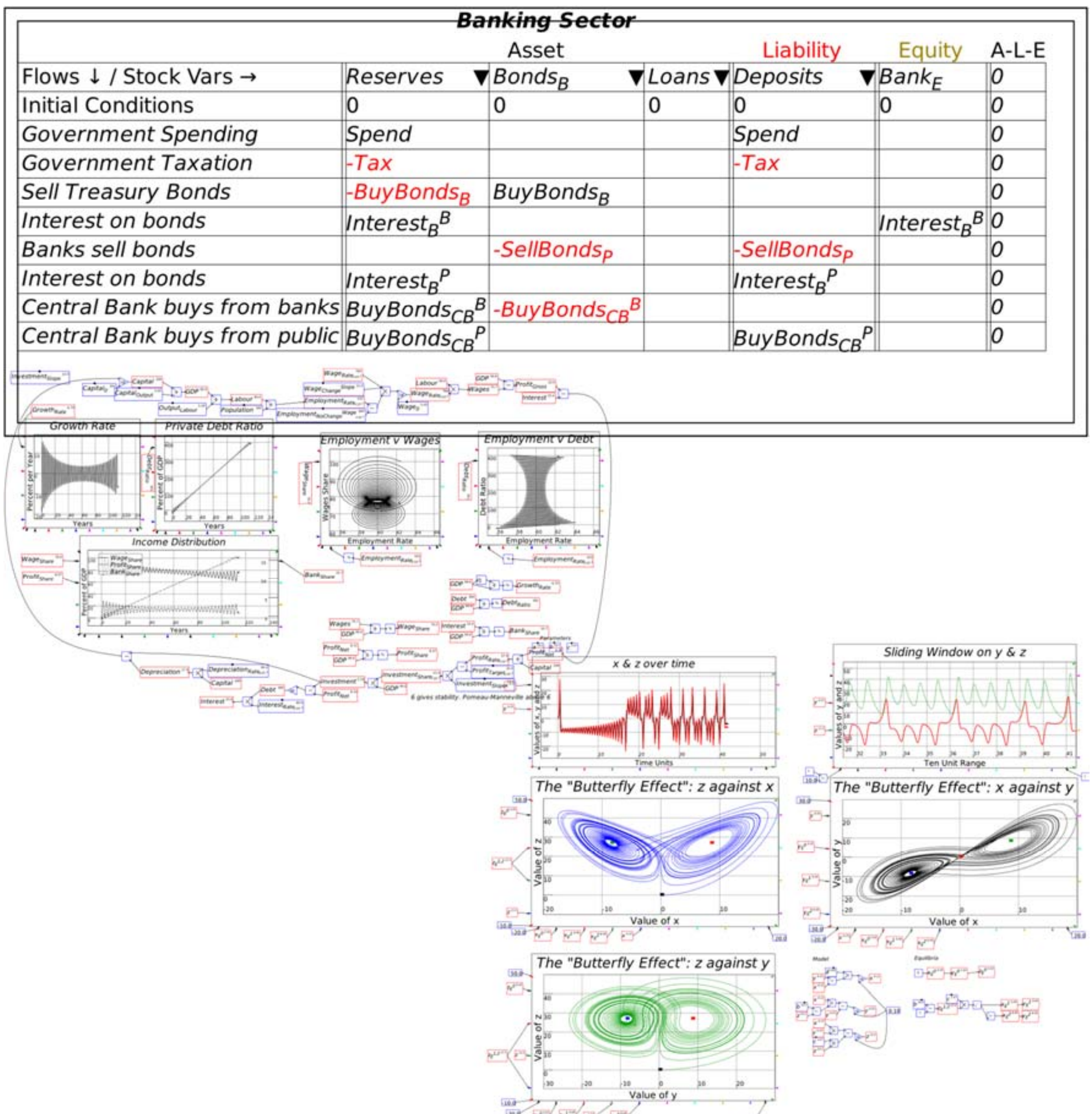

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*$Bonds_B$*	*Loans*	*Deposits*	*$Bank_E$*	*0*
Initial Conditions	0	0	0	0	0	*0*
Government Spending	*Spend*			*Spend*		*0*
Government Taxation	*-Tax*			*-Tax*		*0*
Sell Treasury Bonds	*$-BuyBonds_B$*	*$BuyBonds_B$*				*0*
Interest on bonds	*$Interest_B^B$*				*$Interest_B^B$*	*0*
Banks sell bonds		*$-SellBonds_P$*		*$-SellBonds_P$*		*0*
Interest on bonds	*$Interest_B^P$*			*$Interest_B^P$*		*0*
Central Bank buys from banks	*$BuyBonds_{CB}^B$*	*$-BuyBonds_{CB}^B$*				*0*
Central Bank buys from public	*$BuyBonds_{CB}^P$*			*$BuyBonds_{CB}^P$*		*0*

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1 Preface: A Manual with Attitude

Technically, this is a free companion book to *The New Economics: A Manifesto* (Keen 2021)—hereinafter referred to as *Manifesto*—with two main functions:

- To explain the Minsky models in *Manifesto*; and, therefore by necessity, to also be
- A Manual for the Open Source system dynamics program Minsky, which you can download from https://sourceforge.net/projects/minsky/.

In practice, it has developed a 3rd function: it's somewhere for me to rant about economic issues that I didn't cover in Manifesto, for reasons of lack of space. The main rants are:

- How Bill Phillips (of the Phillips Curve) was a far greater economist—and person—than the mainstream economics caricature of the Phillips Curve have made him out to be;
- That economists should stop modelling in "discrete time" or periods, and start modelling in continuous time, or differential equations; and
- That "Loanable Funds", Paul Krugman's preferred model of banking, is utterly misleading about the role of banks, debt and money in macroeconomics.

2 Introduction

If you're reading this book, you are also, I hope, reading *The New Economics: A Manifesto* (Keen 2021). The purpose of this book is to provide the in-depth explanation for the models used in that book, all of which were designed in *Minsky*.

Minsky is a *system dynamics* program. System dynamics programs in general demonstrate the behaviour of a system of equations that—hopefully—mimic a real-world system which changes over time. There are many other system dynamics programs out there—Stella, IThink, Vensim, Modelica, AnyLogic, Matlab's Simulink, Wolfram's System Modeler, Insight Maker, to mention but a few. What distinguishes Minsky from the pack are its "Godley Tables", which are designed to make it easy to model the dynamics of monetary systems. System dynamics itself can be challenging, and involves many concepts that may be foreign to you when you first use such a program. But Godley Tables are easy: if you have used a spreadsheet, you can design a model of a monetary system using Godley Tables.

Minsky is also Open Source, which means that (a) it is free and (b) its source code is available for anyone for anyone to inspect and, if they wish, modify.

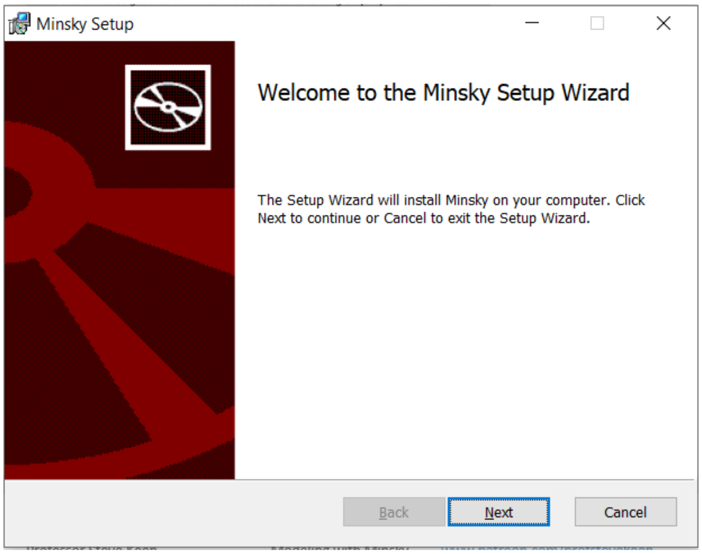
To use *Minsky*, you first have to download it from one of its online repositories. The main site is SourceForge (https://sourceforge.net/projects/minsky/), and it is also available for more technically savvy users at https://github.com/highperformancecoder/minsky. There are compiled versions for both Windows (the default download from the Download button on SourceForge) and Macs (accessed via the page https://sourceforge.net/projects/minsky/files/). Linux is also supported through distros for some popular versions of Linux.

In this manual I will exclusively use the Windows version of Minsky, which, at the time of publication of this book (online only and free), is version 2.24. This version has vast number of improvements over the previous release, thanks to a £200,000 grant from the Friends Provident Foundation of the UK. We will use the remainder of those funds to develop one more release, which will focus on improvements to Minsky's plots and data display options. If you want to test those out, download *MinskyBeta* from https://sourceforge.net/projects/minsky/files/beta%20builds/. Both release and beta versions of Minsky can be installed at the same time.

3 Installation

To install Minsky on a Windows PC, double click on the MSI ("MicroSoft Installer") file that you should have downloaded from SourceForge. This will bring up the dialog box shown in Figure 1.

Figure 1: Installer dialog box for Minsky



Click on "Next" and you will see the license agreement dialog box:

Figure 2

Click on the "I accept the terms in the License Agreement" checkbox (this is a standard Open-Source license, involving no user fees) and the *Next* button will become available. Click on it, and the install destination dialog box appears.

Figure 3

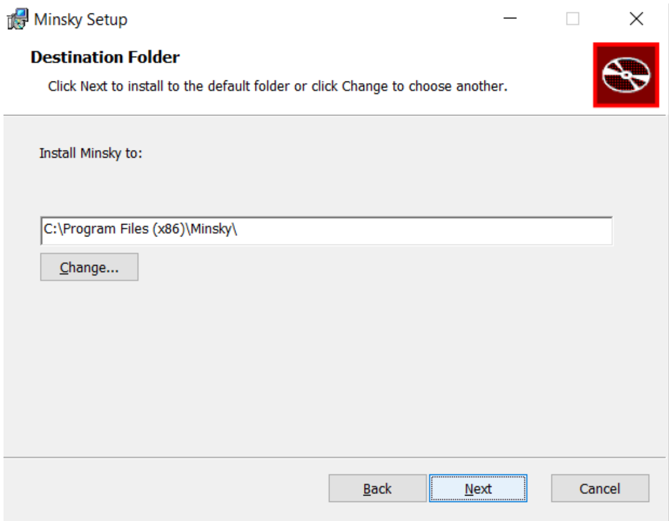
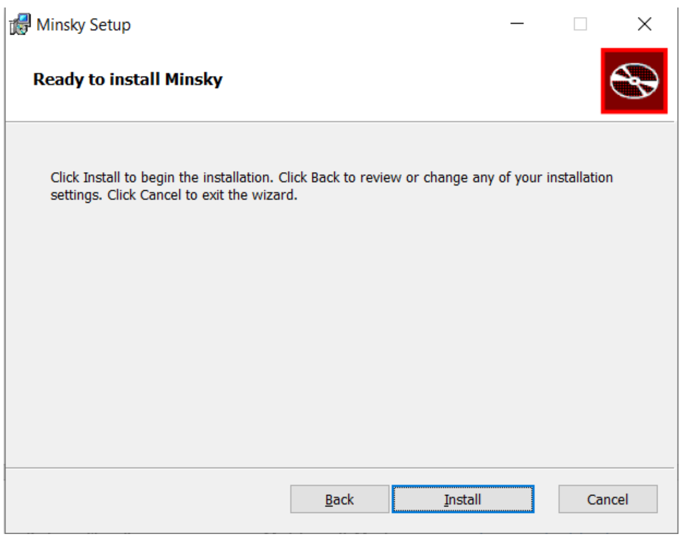
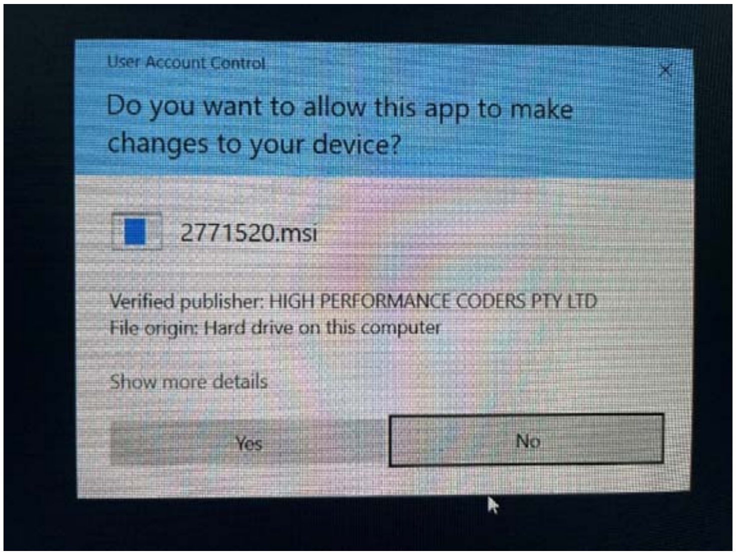


Figure 4



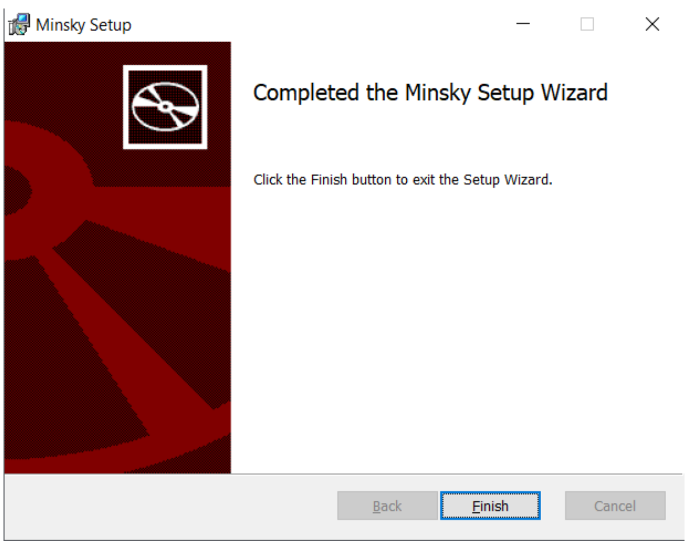
Click on *Install*, and after a short delay, you screen should go blank, except for the form shown in Figure 5. Click on "Yes", and the installation will commence.

Figure 5



When it finishes, you have one more dialog box to contend with—see Figure 6.

Figure 6



You are now ready to use *Minsky*. Press the Windows key on your keyboard to bring up the main Windows menu (or the equivalent on a Mac or Linux box), choose *Minsky*, and you're ready to delve into the world of system dynamics and monetary modelling.

I urge you to not just read this book, but also to build the models in it yourself *as you read it*. Ideally, you would be doing this in a workshop with a tutor guiding the process—something I used to do with my students at Kingston University. Especially now in the age of Covid, this isn't possible—so it's up to you to follow the instructions in this book, and then replicate them in your own *Minsky* models on your computer. You will doubtless make mistakes. But you will learn from mistakes and ultimately learn how to use Minsky to learn economics, and to create models of your own, for your own purposes. This can range from just the fun of being able to simulate chaotic systems, to building a robust, large scale model of a national economy.

You will also almost certainly encounter bugs, ranging from minor annoyances—such as, at present, text boxes for plots running outside the plot boundaries—to fatal crashes, where the program hangs and suddenly you find yourself staring at the desktop. There will, hopefully, be very few of the latter—the funding that we received from Friends Provident Foundation in 2018 has allowed us to dramatically improve the program's stability, as well as to add numerous features. But they will happen nonetheless: bugs are a given in any computer software.

If you find a bug, please report it to the beta-testers list that you can find at https://sourceforge.net/p/minsky/mailman/. The user groups that are exist there are:

- minsky-betatesters: Subscribe | Archive | Search — A list for people who'd like to test beta versions of Minsky
- minsky-developer: Subscribe | Archive | Search
- minsky-users: Subscribe | Archive | Search — For topics related to general usage of Minsky

If you plan on being an active user of Minsky, please sign up to at least *minsky-users* and *minsky-betatesters*. In the former you can get feedback and advice from other users; in the latter, you can report bugs (or feature requests) that will enable us to improve *Minsky* over time.

Finally, consider signing up to Minsky's page on Patreon, https://www.patreon.com/HPCODER. The minimum signup amount is US$1 per month (plus sales or value added taxes, which vary from country to country). Ideally, this would provide sufficient funds to enable Minsky's programmer Dr Russell Standish to work full-time maintaining and extending Minsky. At present (October 2021), it raises a small amount—$480 a month from 82 patrons. This at least covers Russell's costs in producing compiled versions, which take about 4 hours to generate, but it's a long way short of enabling him to work on Minsky itself.

4 If you don't want to model: "*Minsky* for Dummies"

This is a very technical book, for the simple reason that it explains how to build economic models using a mathematical program. But a lot can be done with *Minsky* without doing mathematical modelling, because many of the issues in dispute these days in economic theory and policy come down to "How are you going to pay for it?"

To really answer that question, you have to understand the dynamics of our monetary system—and that means you have to understand double-entry bookkeeping, because that's the way banks both make and keep track of financial transactions. *Minsky* was built to do that, with its unique feature of "Godley Tables". You can use the Godley Tables alone to answer many of the questions that dominate political debate today:

- Is there a "magic money tree"?
- Do banks create money?
- What are Reserves used for?
- What do taxes do?

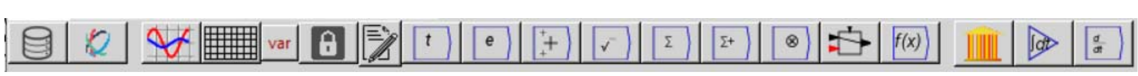
And so on. In this chapter I'll show how to pose and answer questions like these using *Minsky*, without having to write a single equation. Instead, I'll just use the unique feature of *Minsky*, it's *Godley Table* (see Figure 7).

Figure 7: The "Godley Table" icon

You can place a Godley Table on *Minsky*'s design canvas in two ways:

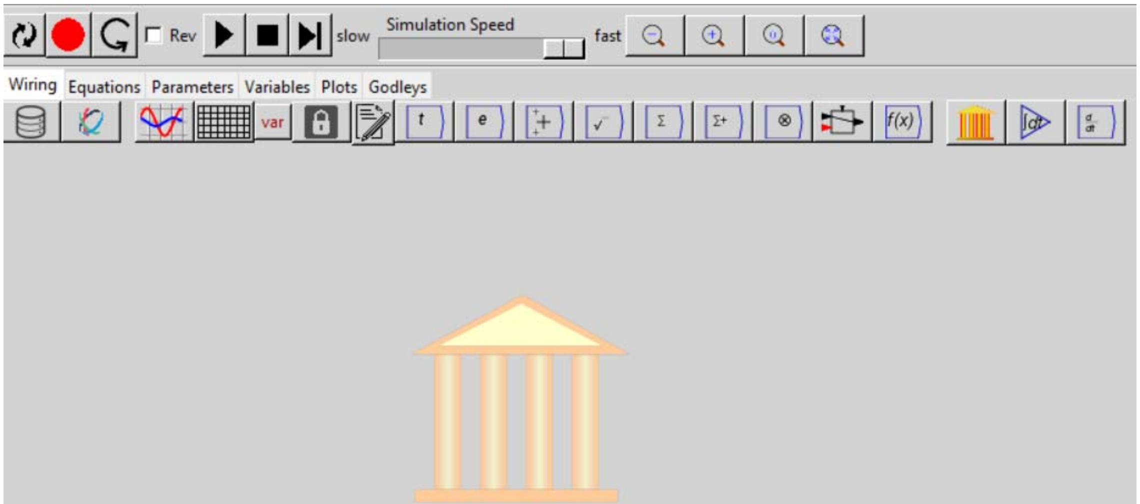
- By choosing "Insert/Godley Table" from the Insert menu; or
- By clicking once on the Godley Table icon on the toolbar (see Figure 8), and then clicking again to place the icon somewhere on the canvas.
    - You may be used to using "click and drag" to insert objects in other programs, like Paint, This doesn't work in *Minsky*—instead you click once to attach an object to the cursor, and then a second time to place the object somewhere on the canvas.
    - At some stage in the future we might support both "click, then click and place" and "click and drag", but for the moment, we only support the first method.

Figure 8: Minsky's Toolbar, with the Godley Table icon the 3rd from the right



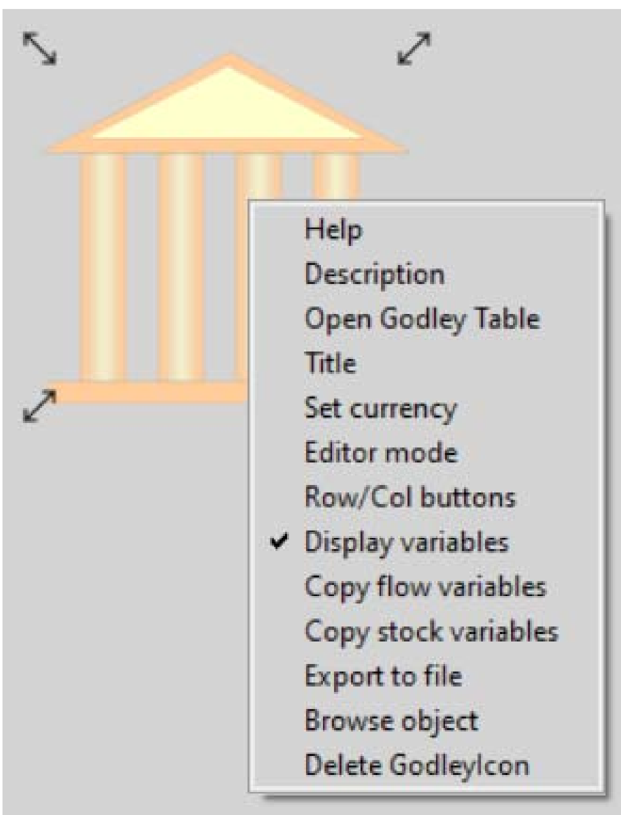
Once you've inserted the Table on the canvas somewhere, it will look like Figure 9.

Figure 9: Minsky's canvas with a single Godley Table



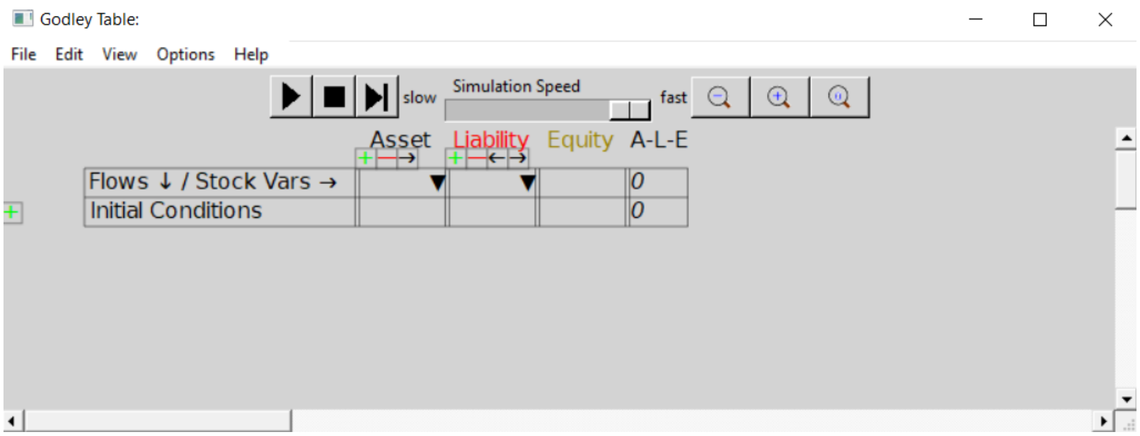
To use the Godley Table, either double-click on the icon, or click on your right-mouse-button and choose “Open Godley Table” from the menu—see Figure 10.

Figure 10: The right-click (context-sensitive) menu for a Godley Table



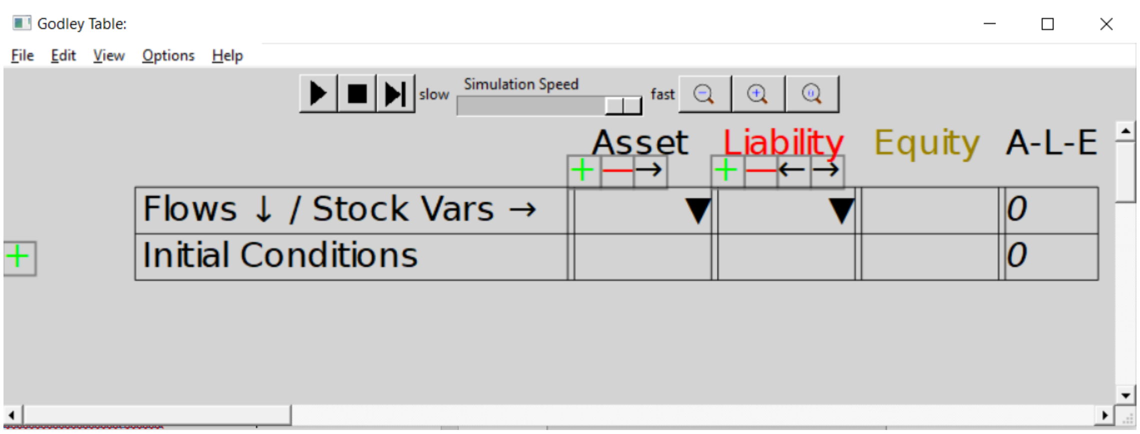
That will bring up a new window for editing the Godley Table—see Figure 11. This is a free-standing *Windows* window, so you can switch between it and the canvas using *Windows* commands (Alt-Tab is the keyboard shortcut to move between windows), and you can have multiple Godley Table windows open at once, as well as the window for the main canvas.

Figure 11: A Godley Table open for editing in its own window



The top row of the Table has tools for running a model, and for zooming in and out on the Table itself. The most useful tools at this stage are the magnifying glasses, which let you zoom out, zoom in, or set the size of the Table to its default. Figure 12 shows the same blank table as in Figure 11 after six clicks on the zoom in tool.

Figure 12: A magnified view of a Godley Table



The next row shows that all accounts in a Godley Table have to be classified either as an Asset (a claim that you have on someone else), a Liability (a claim that someone else has on you), or Equity (the gap between Assets and Liabilities).

The Table starts with room for just one Asset, one Liability, and one Equity column, but of course a significant model is going to have more than one of each. That's what the +—←→ symbols on the next row are for: the + adds a new column, the — deletes an existing column, and the ←→ symbols move a column to the left or right. Now let's build a simple model that, without the need for any equations, will show how a modern monetary system works.

4.1 Fiat Money

To use the table, you first have to name the Assets, Liabilities and Equity columns on the table. That's where the next row—which starts with "Flows ↓/Stock Vars →"—comes in. If you click in one of the cells on that row and start typing, you are providing a name for one of the stocks in the Table (ignore the upside-down triangle there for now—we'll come to that soon). In Figure 13, I've clicked in the

Asset cell and I've started to type the word "Reserves". Once I press the Enter key (or click outside the cell using the mouse), I've defined the stock "Reserves".

Figure 13: Naming a stock in a Godley Table

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Reser	▼		*0*
Initial Conditions	0			*0*

Click in the cell below Liability and enter "Deposits" (without the inverted commas of course!), and in the cell below Equity, type Bank_E. The underscore tells Minsky to subscript the next character, so when you press Enter, or click outside the cell, the program will display $Bank_E$ in that cell (the subscript stands for "Equity"). When you're finished, you'll have the basic elements of the simplest possible model of banking—in fact, one that's too simple, because it doesn't include the key thing that defines a bank, its capacity to make loans.

Figure 14: A basic Godley Table with 3 stocks: Reserves, Deposits and $Bank_E$

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*▼	*Deposits*▼	*$Bank_E$*	*0*
Initial Conditions	0	0	0	*0*

We'll add that by using the + key below the *Asset* heading. That creates an additional blank cell next to Reserves. If you type "Loans" into this cell, you have Figure 15: the starting point for understanding our monetary system: a banking sector with the Assets of Reserves and Loans, the Liability of Deposits, and the difference between them, the banking sector's equity $Bank_E$—which must be positive, since one rule of banking is that a bank must have more Assets than Liabilities.

Figure 15: The minimum stocks to show the credit and fiat money roles of banks

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*▼	*Loans*▼	*Deposits*▼	*$Bank_E$*	*0*
Initial Conditions	0	0	0	0	*0*

If we were going to build a simulation model, then the next row would be critical: this shows the initial amounts in the various accounts. But since this chapter is about using *Minsky* without building a simulation, we'll skip over it and instead click on the + key at the beginning of the row. This adds a row for recording a financial transaction. Let's start with government taxation, using the name "Tax" as a placeholder for the flow of money out of Deposits. If you type "$-Tax$" into the cell beneath Deposits, you've recorded what taxation does—it takes money out of the bank accounts of the public. At this stage, *Minsky* lets you know that your entry isn't complete, because you have only one entry for Tax, when every financial operation requires two entries per row.

Figure 16: Taxation entered as a deduction from Deposits

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*▼	*Loans*▼	*Deposits*▼	*$Bank_E$*	*0*
Initial Conditions	0	0	0	0	*0*
Government taxation			*-Tax*		*Tax*

To complete this row, you need to add another entry so that the "Fundamental Law of Accounting"—that $Assets - Liabilities - Equity = 0$—is enforced. It should be obvious that the correct thing to do is to add another "$-Tax$" to the Reserves column as well: it doesn't make sense to do insert in the Loans column (which we'll model in the next section), or to Bank Equity. Therefore taxation reduces not only Deposits, but also Reserves (Figure 17).

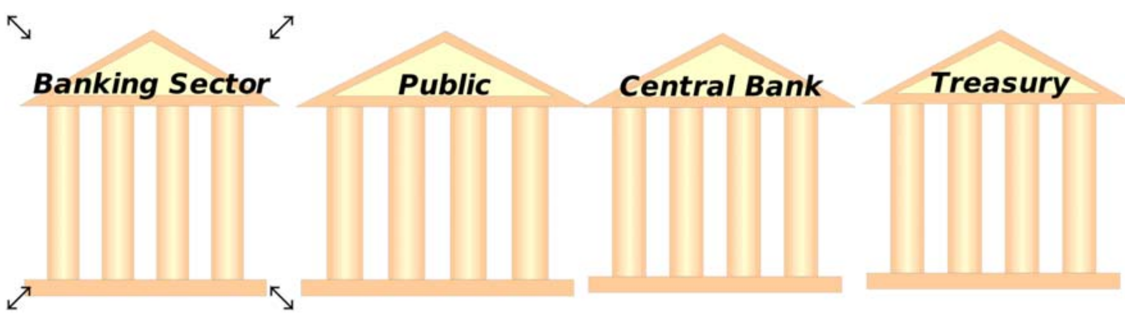
Figure 17: A fully entered double-entry bookkeeping record of taxation

		Asset		Liability	Equity	A-L-E
	Flows ↓ / Stock Vars →	Reserves▼	Loans▼	Deposits▼	$Bank_E$	0
+	Initial Conditions	0	0	0	0	0
+—↓	Government taxation	-Tax		-Tax		0

At this stage, we're simply seeing the financial system from the point of view of the banking sector. A complete model involves seeing it from all perspectives, including here the public (where Deposits, which are a Liability of the banking sector, are an Asset of the public), the Central Bank (since Reserves, which are an Asset of the banking sector, are a Liability of the Central Bank), and the Treasury (which is the originator of the taxation operation).

To do this, we need to add an additional three Godley Tables—one each for the Public, the Central Bank, and the Treasury. We should also name this initial Godley Table "Banking Sector" (using the Title option on the right-mouse menu, or on the Edit menu for the Godley Table itself).

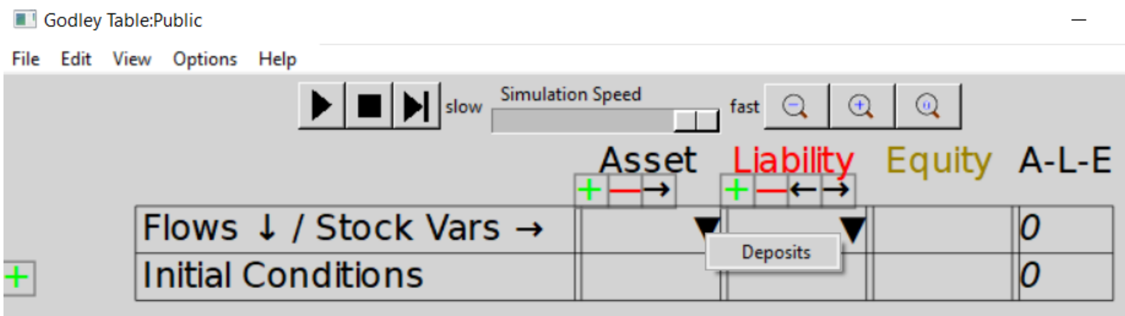
Figure 18: The model with 3 more blank Godley Tables



To populate these tables, we make use of one feature I haven't yet explained, the upside-down triangle or wedge ▼, in the cells for naming stocks. If you click on one of these wedges, Minsky returns a list of all the Liabilities (or Assets) that haven't already been recorded as an Asset (or Liability) for some other entity in the model.

Open up the Godley Table for the Public and click on the wedge in the Asset cell, and one entry will appear in a drop-down menu: Deposits—see Figure 19.

Figure 19: Using the Assets and Liabilities Wedge



Click on Deposits to choose it, and *Minsky* will show Deposits as an Asset of the Public, and auto-populate the column with all the operations that have been entered on the Banking sector table that affect Deposits—so far, this is only the negative entry for taxation. This gives us Figure 20. Notice that the A-L-E column has the entry $-Tax$ in it, showing that the matching double-entry for this table hasn't yet been entered.

Figure 20: Deposits as an Asset for the Public

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Deposits			0
Initial Conditions	0			0
Government Taxation	-Tax			-Tax

The only sensible option here is that taxation reduces the equity position of the non-bank public sector.[1] Name the Equity cell $Public_E$, add the entry "$-Tax$" on the Government Taxation row, and this operation is now shown from the public's perspective: taxation takes money out of the public's bank accounts, and reduces its equity. This is a fundamental proposition in MMT—Modern Monetary Theory—and it's obviously true, when you see the world through the double-entry bookkeeping eyes of *Minsky*.

Figure 21: Taxation shown from the public's point of view

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Deposits		$Public_E$	0
Initial Conditions	0		0	0
Government Taxation	-Tax		-Tax	0

Next, let's add the public's liability in this model—loans from the banking sector. If you click on the wedge under Liabilities, the drop-down menu will reveal two choices: Loans and Reserves. Click on Loans, and you'll get Figure 22. We'll add flows to the Loans column in the next section—in this one we're focusing on government operations.

[1] Obviously the financial sector gets taxed as well (though it's surely better at evading taxes in practice).

Figure 22: Auto-populating the Public's Liabilities

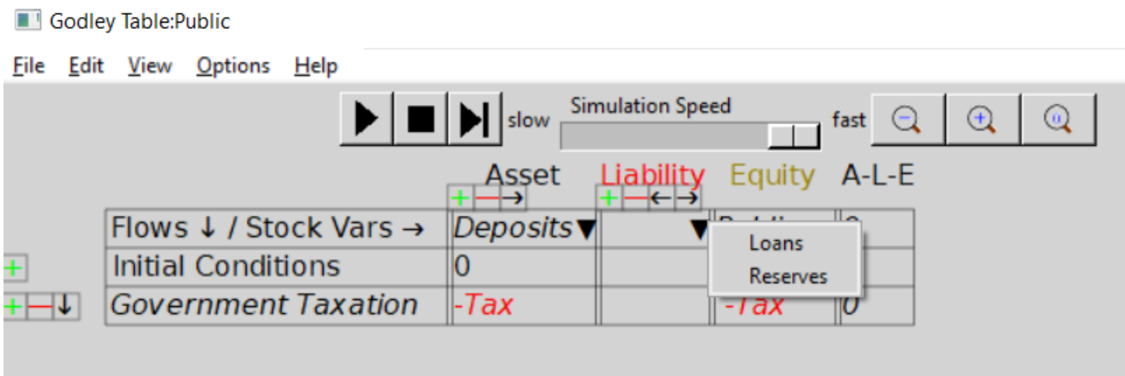


Figure 23: The Public's Godley Table completed

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Deposits	Loans	$Public_E$	0
Initial Conditions	0	0	0	0
Government Taxation	-Tax		-Tax	0

That's taken care of Deposits, which is shown as a Liability of the Banking Sector and an Asset of the Public. Now we have to do the same for Reserves. These, as is well known, are a Liability of the Central Bank: in effect, Reserves are the accounts of private banks at the Central Bank. So open the Central Bank's Godley Table, click on the wedge in the Liabilities cell, and choose "Reserves". That generates Figure 24.

Figure 24: The Central Bank Godley Table with Reserves entered

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves		0
Initial Conditions		0		0
Government Taxation		-Tax		Tax

As with the earlier exercise with the Public's table, we have just a single entry for Tax: there's nowhere obvious to record it a second time, since it's not the Central Bank that does the taxing, but the Treasury. So the sensible thing to do here is to add an additional Liability for the Central Bank, the deposit account of the Treasury—which I simply call Treasury (see Figure 25). I've also named the Equity column for the Central Bank CB_E.

Figure 25: Treasury account added to Central Bank Godley Table

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves	Treasury	CB_E	0
Initial Conditions		0	0	0	0
Government Taxation		-Tax	Tax		0

This now gives us a Liability for the Central Bank—the Treasury's account—which is an Asset for the final entity in this model, the Treasury itself. Bring up the Treasury's Godley Table, click on the wedge for Assets, choose "Treasury", and you'll have Figure 26.

Figure 26: The Treasury's Godley Table on initial entry of its Asset, the Treasury account at the Central Bank

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*			*0*
Initial Conditions	0			*0*
Government Taxation	*Tax*			*Tax*

To complete the model at this stage, you need to enter the balancing entry for Tax—and the obvious place for it to go is in the Equity column for Treasury: taxation increases the Equity of the Treasury (Figure 27). This is the obverse side of the MMT point that the Public's surplus is the Government's deficit: taxation subtracts from the Equity of the public and increases the equity of the government.

Figure 27: Treasury Equity shown

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*		$Treasury_E$	*0*
Initial Conditions	0		0	*0*
Government Taxation	*Tax*		*Tax*	*0*

This is also the point at which a genuine Fiat money system differs from a commodity-backed system—a "Gold Standard", for example—or from one like the Eurozone, where national treasuries cannot produce the currency they spend. In such systems, Tax would add to the Treasury's stock of Gold (or Euros), while government spending—which I'll introduce shortly—would run that stock down.

We now have a complete model of the impact of taxation in a Fiat money sysyem, in that every Asset is shown as another entity's Liability, and all flows are recorded four times: twice in each table they appear in, and once each as affecting an Asset and a Liability. Via double-entry bookkeeping, this gives us eight entries for the one operation.

To see this whole system, click on the Tab labelled "Godleys", and you'll see all the Godley Tables at once.[2] They'll be a jumble when you first click on the tab, but you can easily move them around to produce an arrangement like Figure 28.

[2] On Minsky's design canvas, with "Display Variables" turned off, you model consists of just the 4 banking icons shown in Figure 18.

Figure 28: The complete basic model, with all four Godley Tables

Public

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Deposits*	*Loans*	*Public$_E$*	*0*
Initial Conditions	0	0	0	*0*
Government Taxation	*-Tax*		*-Tax*	*0*

Banking Sector

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Loans*	*Deposits*	*Bank$_E$*	*0*
Initial Conditions	0	0	0	0	*0*
Government Taxation	*-Tax*		*-Tax*		*0*

Central Bank

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		*Reserves*	*Treasury*	*CB$_E$*	*0*
Initial Conditions		0	0	0	*0*
Government Taxation		*-Tax*	*Tax*		*0*

Treasury

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*		*Treasury$_E$*	*0*
Initial Conditions	0		0	*0*
Government Taxation	*Tax*		*Tax*	*0*

We can now add government spending to the model, and it's effectively the opposite of Tax: government spending increases the public's equity and reduces the government's. You can start anywhere you like in the system—from the Public's Godley Table, or the Treasury's, Central Bank or the Banking Sector—and Minsky will point out where the matching entries are needed. I started with the Banking Sector's view in Figure 29:

Figure 29: Adding government spending into the model

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Loans*	*Deposits*	*Bank$_E$*	*0*
Initial Conditions	0	0	0	0	*0*
Government Taxation	*-Tax*		*-Tax*		*0*
Government Spending	*Spend*		*Spend*		*0*

A minute or so later, I had the picture shown in Figure 30.

Figure 30: The complete model with government spending as well as taxation

Public

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Deposits*▼	*Loans*▼	*Public$_E$*	*0*
Initial Conditions	0	0	0	*0*
Government Taxation	*-Tax*		*-Tax*	*0*
Government Spending	*Spend*		*Spend*	*0*

Banking Sector

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*▼	*Loans*▼	*Deposits*▼	*Bank$_E$*	*0*
Initial Conditions	0	0	0	0	*0*
Government Taxation	*-Tax*		*-Tax*		*0*
Government Spending	*Spend*		*Spend*		*0*

Central Bank

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	▼	*Reserves*▼	*Treasury*▼	*CB$_E$*	*0*
Initial Conditions		0	0	0	*0*
Government Taxation		*-Tax*	*Tax*		*0*
Government Spending		*Spend*	*-Spend*		*0*

Treasury

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*▼	▼	*Treasury$_E$*	*0*
Initial Conditions	0		0	*0*
Government Taxation	*Tax*		*Tax*	*0*
Government Spending	*-Spend*		*-Spend*	*0*

This lets us see the key points of Modern Monetary Theory—not because I've been explaining the theory itself, but because the theory is fundamentally based on an accurate portrayal of the accounting. A government deficit creates net financial assets for the public, and simultaneously creates negative net financial assets for the government: the government deficit ***is*** the private sector surplus.

To complete the picture of a modern fiat money system, we need to include bond sales by the Treasury to the Banking Sector in the initial auction, sales by the Banking Sector to non-banks (which can include other financial institutions, such as Pension Funds), and purchases by the Central Bank of bonds from both the Banking Sector and the Public.

4.2 Bond Sales

In the model to date, if the Treasury spends more than it taxes, the Treasury's account at the Central Bank will go into overdraft—notice in Figure 30 that the only flow entries in the Treasury's account at the Central Bank are $-Spend$ and $+Tax$. So if $Spend > Tax$, then over time this account will turn negative.

For an ordinary customer of an ordinary bank, that's a serious problem. A negative deposit account might not be approved in the first place—so that any intended transaction which sends an ordinary depositor's account into negative territory would be rejected for insufficient funds. And if an overdraft is approved by the bank, it attracts a punitive interest rate, normally higher than the interest rate on loans themselves.

What's the situation for the Treasury and the Central Bank? In a country which issues its own currency, *the Treasury is the effective owner of the Central Bank*. Though specific laws can change the situation, technically, the Central Bank is obliged to let the Treasury do what it wants, even if that means a negative balance on the Treasury's account at the Central Bank. So it would be quite possible, in an accounting sense, for the government to simply operate with an overdraft account at the Central Bank: it doesn't have to sell bonds at all.

However, most countries have passed laws forbidding the Treasury from operating in overdraft mode, except in exceptional circumstances like the pandemic, where the Bank of England initially allowed Treasury to operate its overdraft account. Some countries also require the Central Bank to charge the Treasury interest on either overdrafts or loans. But even in countries which do that, the interest is returned to the Treasury as part of the operating profits of the Central Bank. This is why *there is a "magic money tree"*: a currency-issuing country can create money by running a deficit, and it does not have to borrow from either private banks or the public to finance that deficit.

So what do bond sales in fact do? Let's add them to the model and find out. This requires one more Asset column for the Banking Sector, which is the sector that initially purchases Treasury Bonds. I've named the Asset for Banks Bonds_B, to indicate that these are Bonds owned by the banks—rather than, say, the Central Bank or the Public—and labelled the transaction BuyBonds_B in Figure 31.

Figure 31: Banks buy Bonds from the Treasury

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Bonds_B*	*Loans*	*Deposits*	*Bank_E*	*0*
Initial Conditions	0	0	0	0	0	*0*
Government Spending	*Spend*			*Spend*		*0*
Government Taxation	*-Tax*			*-Tax*		*0*
Sell Treasury Bonds		*BuyBonds_B*				*BuyBonds_B*

That's showing the increase in the Banking Sector's Assets from buying the bonds, but how do they finance the purchase?: where is the second entry required by double-entry bookkeeping to show the purchase? The only viable option is that the funds used to purchase the bonds come from Reserves—and these Reserves were created by the deficit: the excess of Spend over Tax. So as well as creating money for the private sector, the deficit creates excess Reserves, which the Banking Sector uses to buy the bonds. So long as the value of bonds sold be Treasury is equal to or less than the deficit, the Banking Sector has the Reserves needed to buy them: see Figure 32

Figure 32: Bond purchase balanced by showing bonds are bought using Reserves created by the deficit

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Bonds_B*	*Loans*	*Deposits*	*Bank_E*	*0*
Initial Conditions	0	0	0	0	0	*0*
Government Spending	*Spend*			*Spend*		*0*
Government Taxation	*-Tax*			*-Tax*		*0*
Sell Treasury Bonds	*$-\text{BuyBonds}_B$*	*BuyBonds_B*				*0*

This completes the Banking Sector's Godley Table, but leaves the Central Bank's incomplete—see Figure 33.

Figure 33: The Central Bank's Godley Table after the Banking Sector's Table has been completed

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves	Treasury	CB_E	0
Initial Conditions		0	0	0	0
Government Taxation		-Tax	Tax		0
Government Spending		Spend	-Spend		0
Sell Treasury Bonds		-$BuyBonds_B$			$BuyBonds_B$

The obvious way to complete the Central Bank's Table is that the proceeds from the sale of Bonds tops up the Treasury account: see Figure 34.

Figure 34: The Central Bank's Table with the sale of Bonds fully recorded

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves	Treasury	CB_E	0
Initial Conditions		0	0	0	0
Government Taxation		-Tax	Tax		0
Government Spending		Spend	-Spend		0
Sell Treasury Bonds		-$BuyBonds_B$	$BuyBonds_B$		0

This shows the real purpose of bond sales, from the Government's point of view: they enable the Treasury's account at the Central Bank to avoid going into overdraft. If the revenue bond sales ($BuyBonds_B$ here) equals $Spend - Tax$, then there's no change to the balance in the Treasury account from running a deficit.

What bonds certainly are not is borrowing money from the banks in the way that individuals do when they take out a mortgage. When you take out a mortgage, it's because you haven't got the money needed to do what you want to do—buy a house. If you don't get the mortgage, you can't afford to buy the house.

But the government has already created the money it needs to do whatever its proposed activities are by running the deficit itself. Secondly, the Reserves that are used to buy the bonds were created by the government running a deficit. If the deficit didn't exist, then (at least initially—there's a change coming when we consider interest payments on bonds) the banks wouldn't have the funds needed to buy the bonds.

The final step in recording the impact of the bond sales is to add $Bonds_B$ as a Liability of the Treasury. Open the Treasury's Godley Table and it will look like Figure 35. Click on the wedge below Liability, and the drop down will show $Bonds_B$ as an Asset (for the Banking Sector) that hasn't yet been recorded as some other entity's Liability.

Figure 35: Treasury Godley Table before $Bonds_B$ is recorded as a liability

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Treasury		$Treasury_E$	0
Initial Conditions	0		0	0
Government Taxation	Tax		Tax	0
Government Spending	-Spend		-Spend	0
Sell Treasury Bonds	$BuyBonds_B$			$BuyBonds_B$

Select $Bonds_B$ and *Minsky* automatically balances the table: see Figure 36.

Figure 36

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Treasury	$Bonds_B$	$Treasury_E$	0
Initial Conditions	0	0	0	0
Government Taxation	Tax		Tax	0
Government Spending	-Spend		-Spend	0
Sell Treasury Bonds	$BuyBonds_B$	$BuyBonds_B$		0

To complete modeling bond sales to banks, we need to include the payment of interest on those bonds. In Figure 37 I've labelled this $Interest_B^B$—the subscript to indicate that it's interest on bonds, the superscript to indicate that it's paid to the banks, to distinguish it from interest paid to the public when, at a later stage, banks sell some of these bonds to the public. A superscript is entered into Minsky using the ^ character, which is normally on the 6 key on your keyboard (So the string you type into the cell is Interest_B^B).

Figure 37: Payment of interest to banks on Treasury Bonds

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves	$Bonds_B$	Loans	Deposits	$Bank_E$	0
Initial Conditions	0	0	0	0	0	0
Government Spending	Spend			Spend		0
Government Taxation	-Tax			-Tax		0
Sell Treasury Bonds	$-BuyBonds_B$	$BuyBonds_B$				0
Interest on bonds	$Interest_B^B$				$Interest_B^B$	0

I've already made the obvious deduction that this interest payment increases the equity of the banking system—which is one obvious reason that, when the Treasury offers to sell bonds to the banking sector, the offer is always taken up. To do otherwise is to turn down an offer to turn a non-tradeable, non-income-earning asset—Reserves—into a tradeable, income-earning asset—Bonds. To complete the model at this stage, we now need to add this flow to the Central Bank's and the Treasury's Godley Tables. When you open up the Central Bank's Godley Table, it will look like Figure 38: the addition to Reserves is already shown, but the second balancing entry is still needed.

Figure 38

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves	Treasury	CB_E	0
Initial Conditions		0	0	0	0
Government Taxation		-Tax	Tax		0
Government Spending		Spend	-Spend		0
Sell Treasury Bonds		$-BuyBonds_B$	$BuyBonds_B$		0
Interest on bonds		$Interest_B^B$			$-Interest_B^B$

The obvious thing is that the interest payments come out of the Treasury's account. Make the entry $-\text{Interest}_B^B$ in the Treasury column, and you have Figure 39.

Figure 39

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →		Reserves	Treasury	CB_E	0
Initial Conditions		0	0	0	0
Government Taxation		-Tax	Tax		0
Government Spending		Spend	-Spend		0
Sell Treasury Bonds		$-BuyBonds_B$	$BuyBonds_B$		0
Interest on bonds		$Interest_B^B$	$-Interest_B^B$		0

This change in turn cascades into the Treasury's Godley Table now—see Figure 40.

Figure 40

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Treasury	$Bonds_B$	$Treasury_E$	0
Initial Conditions	0	0	0	0
Government Taxation	Tax		Tax	0
Government Spending	-Spend		-Spend	0
Sell Treasury Bonds	$BuyBonds_B$	$BuyBonds_B$		0
Interest on bonds	$-Interest_B^B$			$-Interest_B^B$

The obvious closure of this entry is that paying interest reduces the Treasury's equity—and by precisely as much as it increases the equity of the banking sector. So just as a deficit creates net financial assets for the non-bank public (by crediting their deposit accounts with more money from government spending than is removed by taxation), the interest payments create net financial assets for the banking sector—see Figure 41.

Figure 41

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*	*$Bonds_B$*	*$Treasury_E$*	*0*
Initial Conditions	0	0	0	*0*
Government Taxation	*Tax*		*Tax*	*0*
Government Spending	*-Spend*		*-Spend*	*0*
Sell Treasury Bonds	*$BuyBonds_B$*	*$BuyBonds_B$*		*0*
Interest on bonds	*$-Interest_B^B$*		*$-Interest_B^B$*	*0*

So how does the Treasury pay the interest? In practice, there could be many methods. What I'll model here is the most sensible for a currency-issuing government: it borrows from the Central Bank.[3] If you've followed me this far, you should be familiar with the steps needed to show this: we add an Asset to the Central Bank's Godley Table—$Loans_{CB}^T$ (which uses another of Minsky's formatting tricks: encase the characters CB in a pair of curly brackets—Loans_(ECB)—and Minsky subscripts the two characters together), and use Lend_(ECB)^T to show the actual loans. Figure 42 shows the entries on the Central Bank's table (with the actual entry of the text string into the Treasury column, before *Minsky* formats it).

Figure 42

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	*$Loans_{CB}^T$*	*Reserves*	*Treasury*	*CB_E*	*0*
Initial Conditions	0	0	0	0	*0*
Government Taxation		*-Tax*	*Tax*		*0*
Government Spending		*Spend*	*-Spend*		*0*
Sell Treasury Bonds		*$-BuyBonds_B$*	*$BuyBonds_B$*		*0*
Interest on bonds		*$Interest_B^B$*	*$-Interest_B^B$*		*0*
Treasury borrows from CB	*$Lend_{CB}^T$*		Lend_{CB}^T		*0*

If the loan from the Central Bank to Treasury equals the interest payments on the bonds, then the Treasury's account at the Central Bank can avoid going into overdraft. It doesn't change the aggregate picture: the Treasury's negative equity from the deficit creates positive equity for the non-bank public, while interest payments on bonds creates negative equity for the Treasury and identical positive equity for the banking sector.

Even without attempting to build a mathematical model, this exercise in laying out the structure of the financial system eradicates a lot of popular myths in mainstream economic thinking:

- A deficit doesn't take money from the public—in the sense of the government borrowing from the public to finance its deficit—but actually puts money into the hands of the public;
- The deficit creates Reserves for the banking sector, and those Reserves are what banks later use to buy government bonds;

[3] In practice, this is forbidden in most countries by legislation that prevents the Treasury borrowing directly from the Central Bank. However the same outcome can be achieved in a two-step process: the Treasury sells bonds to the private banks to the value of the interest on outstanding bonds, and the Central Bank then purchases these bonds from the private banks on the secondary market.

- The deficit creates net equity for the non-bank public, while interest on government bonds creates net equity for the banking sector.

This symmetry—that a deficit for the government means a surplus of precisely the same magnitude for the non-government sectors—is apparent in Figure 43. The sum of the non-bank Public's and the banking sector's net equity position is $Spend + \text{Interest}_B^B - Tax$; this is the opposite of the Government's net equity $Tax - Spend - \text{Interest}_B^B$.

Figure 43: Full system with bond sales to banks

Public

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Deposits▼	Loans▼	$Public_E$	0
Initial Conditions	0	0	0	0
Government Taxation	-Tax		-Tax	0
Government Spending	Spend		Spend	0

Banking Sector

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves ▼	$Bonds_B$ ▼	Loans▼	Deposits▼	$Bank_E$	0
Initial Conditions	0	0	0	0	0	0
Government Spending	Spend			Spend		0
Government Taxation	-Tax			-Tax		0
Sell Treasury Bonds	$-BuyBonds_B$	$BuyBonds_B$				0
Interest on bonds	$Interest_B^B$				$Interest_B^B$	0

Central Bank

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	$Loans_{CB}^T$▼	Reserves ▼	Treasury ▼	CB_E	0
Initial Conditions	0	0	0	0	0
Government Taxation		-Tax	Tax		0
Government Spending		Spend	-Spend		0
Sell Treasury Bonds		$-BuyBonds_B$	$BuyBonds_B$		0
Interest on bonds		$Interest_B^B$	$-Interest_B^B$		0
Treasury borrows from CB	$Lend_{CB}^T$		$Lend_{CB}^T$		0

Treasury

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	Treasury ▼	$Bonds_B$ ▼	$Loans_{CB}^T$▼	$Treasury_E$	0
Initial Conditions	0	0	0	0	0
Government Taxation	Tax			Tax	0
Government Spending	-Spend			-Spend	0
Sell Treasury Bonds	$BuyBonds_B$	$BuyBonds_B$			0
Interest on bonds	$-Interest_B^B$			$-Interest_B^B$	0
Treasury borrows from CB	$Lend_{CB}^T$		$Lend_{CB}^T$		0

The final two steps to show to cover the fundamentals of fiat money are sales of bonds by the banking sector to the public, and purchases of bonds by the Central Bank from both banks and the public. Figure 44 shows the full system—which, if you want to learn how to use Minsky, you should try to complete for yourself. It needed:

- Two additional stock variables—$Bonds_{CB}$ for bonds owned by the Central Bank, and $Bonds_{P}$ for bonds owned by the public;
- The relevant flows for these stocks: sales of bonds by the Banking Sector to the Public, $SellBonds_{P}$; purchases of bonds by the Central Bank from the Banking Sector, $BuyBonds_{CB}^{B}$; and purchases of bonds by the Central Bank from the Public, $BuyBonds_{CB}^{P}$.

As with the previous stages of this exercise, several insights can be gleaned from these Tables that contradict widespread beliefs about government money creation. One of these is even something that I used to believe—that money is only created to the extent that the Central Bank buys government bonds. But in fact, Central Bank purchases of Treasuries are irrelevant to money creation, and indirectly slightly reduce the amount of money created.

Figure 44: Full MMT system with bond transactions between Treasury, Banks, Central Bank and the Public

Public

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Deposits*	*$Bonds_P$*	*Loans*	*$Public_E$*	*0*
Initial Conditions	0	0	0	0	*0*
Government Taxation	*-Tax*			*-Tax*	*0*
Government Spending	*Spend*			*Spend*	*0*
Banks sell bonds	*$-SellBonds_P$*	*$SellBonds_P$*			*0*
Interest on bonds	*$Interest_B^P$*			*$Interest_B^P$*	*0*
Central Bank buys from public	*$BuyBonds_{CB}^P$*	*$-BuyBonds_{CB}^P$*			*0*
Central Bank buys from banks					*0*

Banking Sector

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*$Bonds_B$*	*Loans*	*Deposits*	*$Bank_E$*	*0*
Initial Conditions	0	0	0	0	0	*0*
Government Spending	*Spend*			*Spend*		*0*
Government Taxation	*-Tax*			*-Tax*		*0*
Sell Treasury Bonds	*$-BuyBonds_B$*	*$BuyBonds_B$*				*0*
Interest on bonds	*$Interest_B^B$*				*$Interest_B^B$*	*0*
Banks sell bonds		*$-SellBonds_P$*		*$-SellBonds_P$*		*0*
Interest on bonds	*$Interest_B^P$*			*$Interest_B^P$*		*0*
Central Bank buys from banks	*$BuyBonds_{CB}^B$*	*$-BuyBonds_{CB}^B$*				*0*
Central Bank buys from public	*$BuyBonds_{CB}^P$*			*$BuyBonds_{CB}^P$*		*0*

Central Bank

	Asset		Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	*$Loans_{CB}^T$*	*$Bonds_{CB}$*	*Reserves*	*Treasury*	*CB_E*	*0*
Initial Conditions	0	0	0	0	0	*0*
Government Taxation			*-Tax*	*Tax*		*0*
Government Spending			*Spend*	*-Spend*		*0*
Sell Treasury Bonds			*$-BuyBonds_B$*	*$BuyBonds_B$*		*0*
Interest on bonds			*$Interest_B^B$*	*$-Interest_B^B$*		*0*
Treasury borrows from CB	*$Lend_{CB}^T$*			*$Lend_{CB}^T$*		*0*
Central Bank buys from banks		*$BuyBonds_{CB}^B$*	*$BuyBonds_{CB}^B$*			*0*
Central Bank buys from public		*$BuyBonds_{CB}^P$*	*$BuyBonds_{CB}^P$*			*0*
			$Interest_B^P$	*$-Interest_B^P$*		*0*

Treasury

	Asset	Liability				Equity	A-L-E
Flows ↓ / Stock Vars →	*Treasury*	*$Bonds_B$*	*$Bonds_P$*	*$Bonds_{CB}$*	*$Loans_{CB}^T$*	*$Treasury_E$*	*0*
Initial Conditions	0	0	0	0	0	0	*0*
Government Taxation	*Tax*					*Tax*	*0*
Government Spending	*-Spend*					*-Spend*	*0*
Sell Treasury Bonds	*$BuyBonds_B$*	*$BuyBonds_B$*					*0*
Interest on bonds	*$-Interest_B^B$*					*$-Interest_B^B$*	*0*
Treasury borrows from CB	*$Lend_{CB}^T$*				*$Lend_{CB}^T$*		*0*
Banks sell bonds		*$-SellBonds_P$*	*$SellBonds_P$*				*0*
Central Bank buys from banks		*$-BuyBonds_{CB}^B$*		*$BuyBonds_{CB}^B$*			*0*
Central Bank buys from public			*$-BuyBonds_{CB}^P$*	*$BuyBonds_{CB}^P$*			*0*
	$-Interest_B^P$					*$-Interest_B^P$*	*0*

The reason why Central Bank bond purchases from the banking sector don't affect the amount of money created by a deficit is apparent in the second table in Figure 44: the purchase reduces the monetary value of the bonds held by banks, and replaces them by an equivalent value of Reserves. The Banks would hope to make a trading profit out of this sale,[4] but the sale itself simply swaps one Asset for the Banks (Bonds) with another Asset (Reserves). In practice, this reduces the process of the Treasury selling bonds to the banks in the first place: it replaces Bonds with Reserves. It is therefore irrelevant to money creation, because since the level of Assets remain constant, so too do Liabilities and Bank Equity.

This is an important general point that will recur frequently in this book, and when building models using Godley Tables: *for money to be created, an operation must affect both the Asset and the Liability/Equity sides of the Banking Sector's ledger*. Central Bank bond purchases from the Banking Sector only affect the Asset side, and therefore are irrelevant to money creation. The only effect they do have is to reduce money creation slightly, because the Treasury will no longer pay interest to the Banks on these bonds.

On the other hand, Central Bank purchases of Bonds from the public do create money: the sale of the Bonds credits both the public's deposit accounts at banks, and the reserve accounts of the banks.

Conversely, the sale of bonds by the Banking Sector to the non-bank Public destroys money: the Public's deposit accounts fall and their holdings of Bonds rise. But even in this case, the money being destroyed was initially created by the deficit itself: only if all the bonds initially purchased by the banks from the Treasury at the bond auction were sold to the public would the actual creation of money by the deficit fall to zero.

That covers government money creation. Now let's turn to private money creation by the Banking Sector.

4.3 Credit Money

I'm now assuming that you have some fluency with Godley Tables—*you have been following my explanation by reproducing these tables in Minsky yourself, haven't you?*—and so I'll just cut to the chase, and enter the three necessary operations for private money creation in one go: new loans by banks, paying interest on loans, and loan repayment by bank customers. Use the words "Lend", "Interest", and "Repay" for these flows, and make the entries so that the checksum column $A-L-E$ always sums to zero (Figure 45).

[4] This could be added to the model with another row showing the trading profit (or loss)—which would be the difference between the sale price and the purchase price, multiplied by the number of bonds sold. This would add to Reserves and Bank Equity, in which case it is a mechanism for money creation.

Figure 45: The basic operations of fiat and credit money from the Banking Sector's point of view

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves	$Bonds_B$	Loans	Deposits	$Bank_E$	0
Initial Conditions	0	0	0	0	0	0
Government Spending	Spend			Spend		0
Government Taxation	-Tax			-Tax		0
Sell Treasury Bonds	$-BuyBonds_B$	$BuyBonds_B$				0
Interest on bonds	$Interest_B^B$				$Interest_B^B$	0
Banks sell bonds		$-SellBonds_P$		$-SellBonds_P$		0
Interest on bonds	$Interest_B^P$			$Interest_B^P$		0
Central Bank buys from banks	$BuyBonds_{CB}^B$	$-BuyBonds_{CB}^B$				0
Central Bank buys from public	$BuyBonds_{CB}^P$			$BuyBonds_{CB}^P$		0
Bank lending			Lend	Lend		0
Interest payments				-Interest	Interest	0
Loan repayment			-Repay	-Repay		0

As an aside, if you have a background in accounting, you may prefer to see Minsky's operations using DR and CR rather than plus and minus entries. You can engage that from the Options menu on the Table: choose "DR/CR Style". Then the model in Figure 45 will look like Figure 46 (I prefer the plus/minus approach, so that's what I'll stick with from now on).

Figure 46:Figure 45 in DR/DR style

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves	$Bonds_B$	Loans	Deposits	$Bank_E$	0
Initial Conditions	0	0	0	0	0	0
Government Spending	DR Spend			CR Spend		0
Government Taxation	CR Tax			DR Tax		0
Sell Treasury Bonds	CR $BuyBonds_B$	DR $BuyBonds_B$				0
Interest on bonds	DR $Interest_B^B$				CR $Interest_B^B$	0
Banks sell bonds		CR $SellBonds_P$		DR $SellBonds_P$		0
Interest on bonds	DR $Interest_B^P$			CR $Interest_B^P$		0
Central Bank buys from banks	DR $BuyBonds_{CB}^B$	CR $BuyBonds_{CB}^B$				0
Central Bank buys from public	DR $BuyBonds_{CB}^P$			CR $BuyBonds_{CB}^P$		0
Bank lending			DR Lend	CR Lend		0
Interest payments				DR Interest	CR Interest	0
Loan repayment			CR Repay	DR Repay		0

Following the general principle noted just above, that to create money, an operation must add to both the Asset and Liability/Equity sides of the banking sector's balance sheet, it should be obvious that lending creates money while repayment destroys it.[5] This simple fact is ignored by the mainstream model of lending, known as Loanable Funds, which treats banks as "financial intermediaries" that take in deposits from one set of customers ("Patient people", to use Paul Krugman's non-pejorative[6] term) and then lends them out to other people ("Impatient people" in Krugman's lexicon).

I'll spend a lot of time on the macroeconomic impacts of private money creation in Chapter 8. For now, without writing a single equation, we've come up with a picture of the monetary aspects of a mixed fiat-credit money system that contradict the conventional wisdoms promulgated by economics textbooks and mouthed by politicians.

[5] This last point is something that I myself didn't accept before I designed Minsky: though both Minsky himself and Graziani made this claim, it sounded crazy to me that banks would let money be destroyed after they had created it, and I wrote the money chapter of the 2nd edition of *Debunking Economics* to be agnostic on this point. But in fact Minsky and Graziani were correct whereas I was wrong, and one of the major reasons I'm writing a third (and final!) edition of *Debunking Economics* is to correct this mistake.

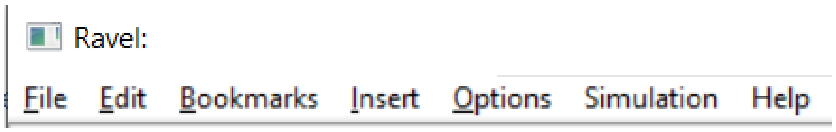
[6] Irony alert.

If you've followed the argument here to date—especially if you've done so by reproducing the model in *Minsky* for yourself—then you're well on the way to understanding the monetary dynamics of capitalism. I'll repeat a lot of the points here in subsequent chapters, but with the addition of defining a mathematical model rather than stopping at laying out the balance sheets.

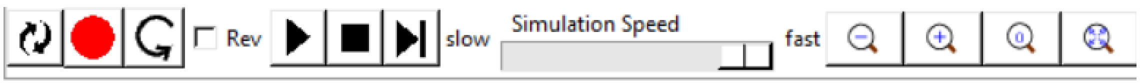
5 The User Interface

Minsky's interface has five main components:

- The menu bar, with options File/Edit/Bookmarks/Insert/Options/Simulation/Help;



- The simulation control toolbar with tools to reset a simulation, run it, stop it, step through it, change the speed of the simulation, reverse its direction (simulate backwards in time rather than forwards), zoom out/in/reset/full scale, record the construction of a model, and replay its construction;

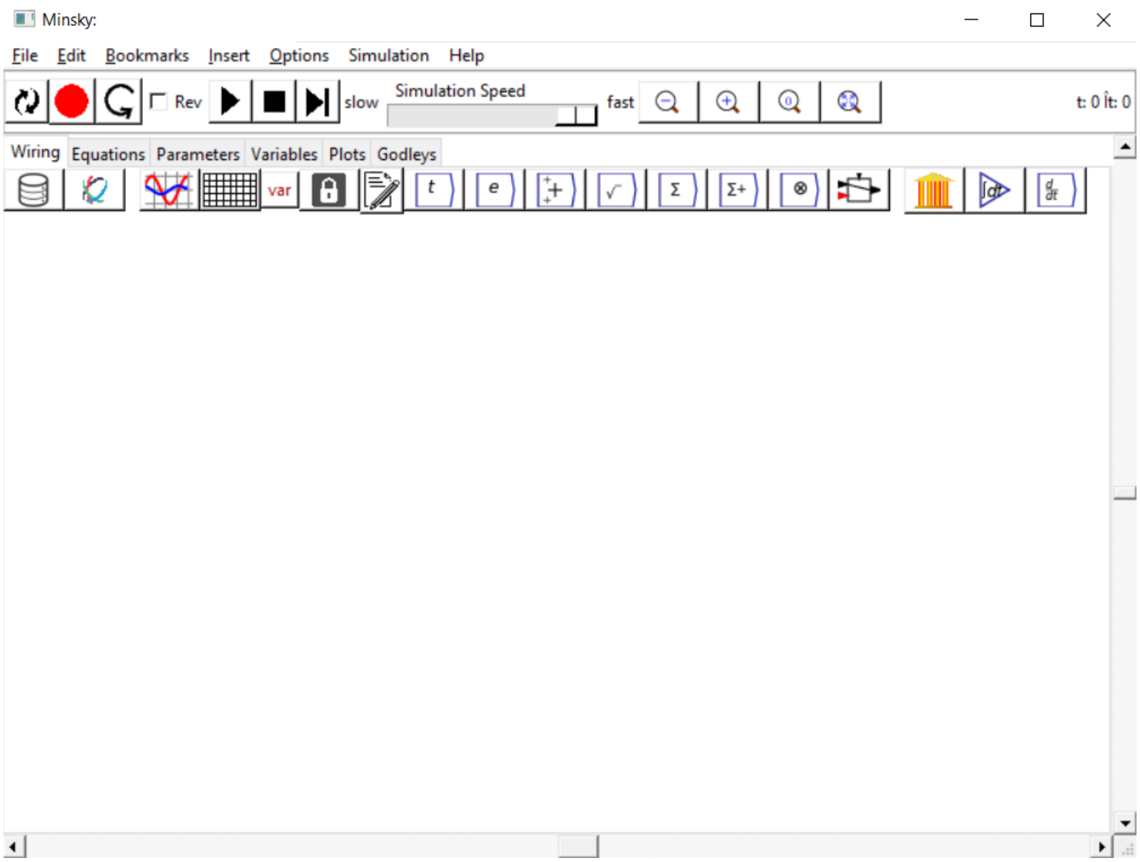


- Tabs for various aspects of the user interface. The main tab is *Wiring*, where you lay out your model using the visual design elements in Minsky; *Equations* shows the equations generated by your model; *Parameters* shows the names and values for model parameters; *Variables* lists the definition of the variables in a model; *Plots* shows selected graphs from a simulation on a separate canvas; and *Godleys* shows the double-entry bookkeeping tables used to build the financial aspects of any model you construct;

- The Toolbar for designing a model. From left to right, the tools: import data; attach data to a Ravel (a commercial extension to Minsky detailed in the final chapter of this book); insert a plot; insert a spreadsheet; from a drop-down menu, insert either a variable, a parameter, or a constant; lock an operation (so that the locked variable doesn't change when the model is altered); insert a text note; and insert a time widget. The next six icons activate a series of drop-down menus to insert mathematical operators on the canvas. Finally there is a logical switch operator, the Godley Table icon, integral block icon, and differential operator;

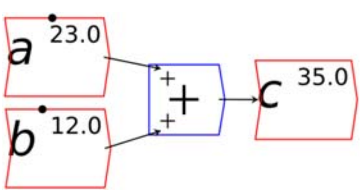
- And finally, the design canvas, where the contents depend on which Tab is active—see Figure 47. The main Wiring tab presents you with a design surface that is 100,000 by 100,000 pixels large—in terms of modern computer screens, that's equivalent to an array of 4K monitors (each with resolutions of 4,000 pixels horizontally and 2,000 pixels vertically) 25 monitors wide and 50 monitors deep. You are unlikely to design a model that uses even 1% of that design space, but the room is there if needed to build truly gargantuan models.

Figure 47: Minsky's interface, open on the "Wiring" Tab.



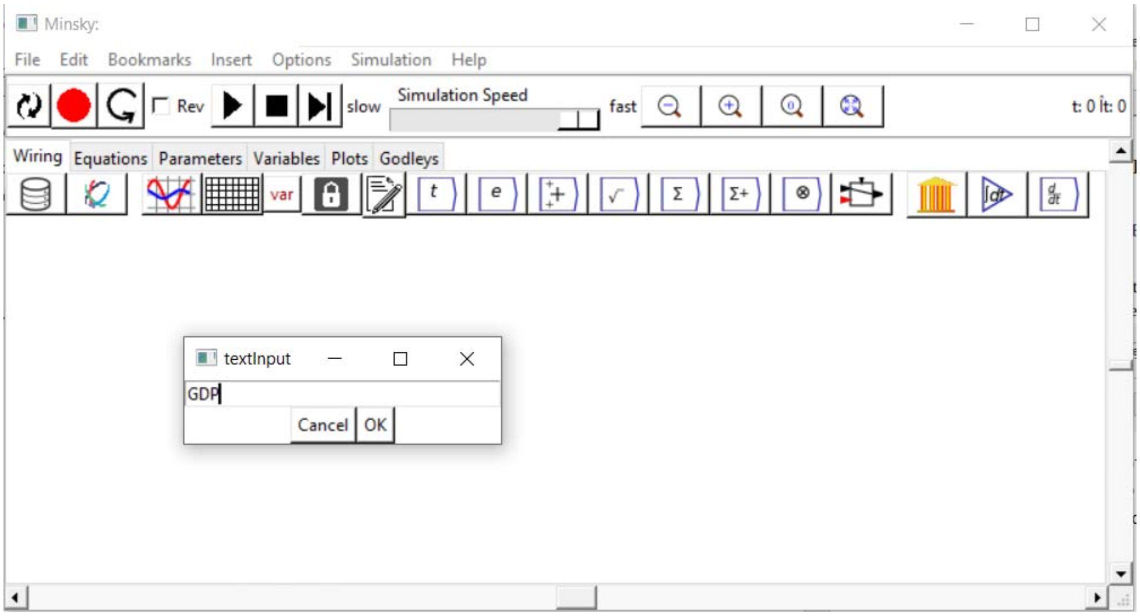
You will spend most of your time on the *Wiring* Tab when designing a *Minsky* model. As is standard in system dynamics programs, you create equations using wires that connect one or more entities to each other. A simple equation like, for example, $a+b=c$, looks like this in Minsky:

Figure 48: A simple equation in Minsky



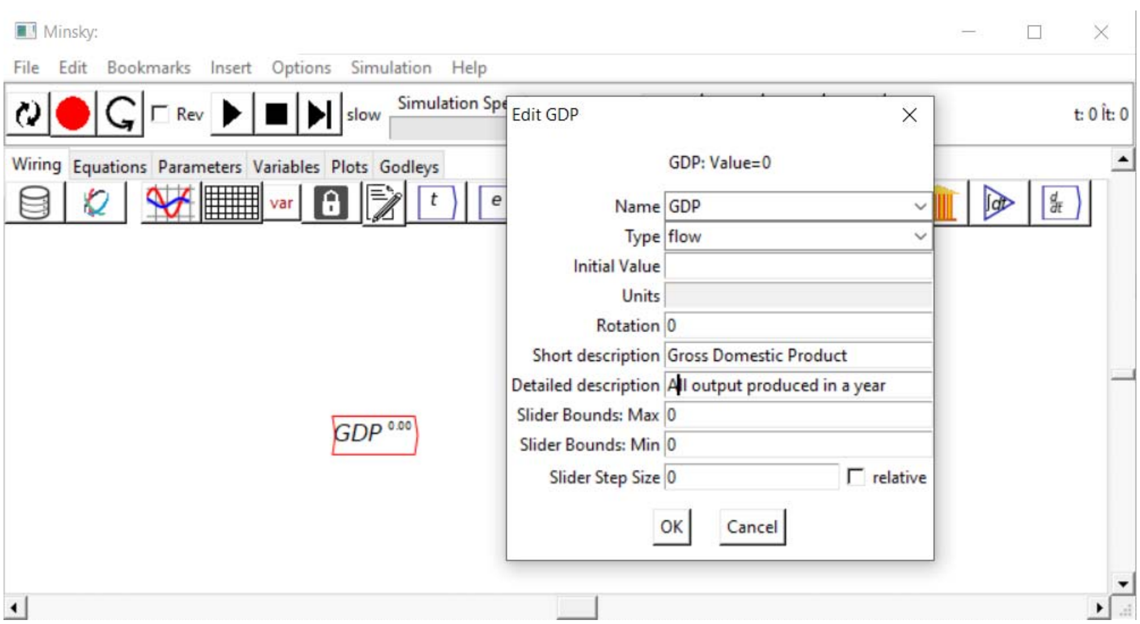
We have endeavoured to make entering equations as easy as possible, so you can just type anywhere on the canvas to add a variable to your model. For example, if you wish to define GDP, you can simply start typing “GDP” on the canvas. When you hit the “G” key, the “textInput” dialog box will pop up, where you can complete typing the expression: see Figure 49.

Figure 49



When you press the Enter key, or click on "OK", the variable GDP will be entered on the canvas, and the Edit dialog box will pop up where, if you wish, you can give it an initial value, specify its units, give it a short description, etc.—see Figure 50.

Figure 50

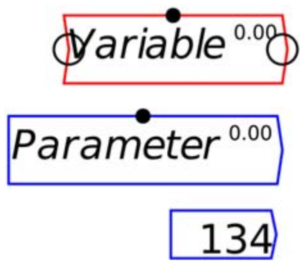


You can also change its type, from "flow" to "parameter", "constant", "integral" or "stock" (we'll meet the latter two types in the next chapter). Parameters differ from flow variables by (a) having a different colour (blue rather than red) and (b) having only an output, whereas flow variables have both an input and an output.

You can see the input and output ports if you put your mouse pointer above an object on the canvas. These are circles on the right and left ends of a Variable, and the right end only of Parameters and

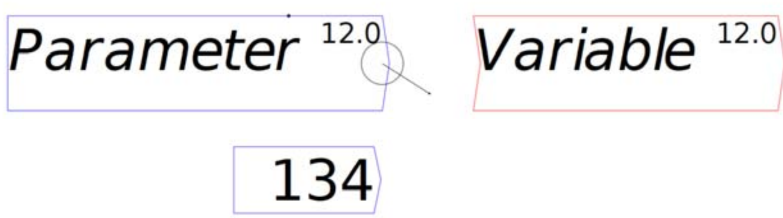
Constants—see Figure 51, where my mouse pointer was hovering over *Variable*, so that both its input and output ports are visible.

Figure 51: Variables, parameters, and constants



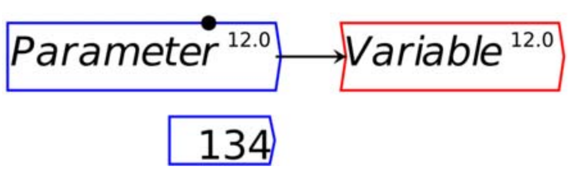
If you click anywhere apart from inside one of these circles, then you can drag the entity to somewhere else on the canvas. If you click inside one of the output circles—those on the right-hand side—then a "wire" will come out of it, which will attach to the nearest input port (you don't have to click on an input port precisely)—see Figure 52, where I've started dragging a wire out of the output port from *Parameter* towards the input port for *Variable*.

Figure 52: Wire being drawn out of output port



When you release the mouse button, the wire "snaps" to the nearest input port, which is that for *Variable*—see Figure 53. From now on, *Variable*'s value will be whatever *Parameter*'s value is.

Figure 53: Parameter output wired to Variable input



Of course, you'll want to use mathematical operators to create more complicated definitions, and in *Minsky* you can simply type simple mathematical operators—addition, multiplication, division and subtraction—directly onto the canvas: you don't have to use the drop-down menus on the icon bar.[7]

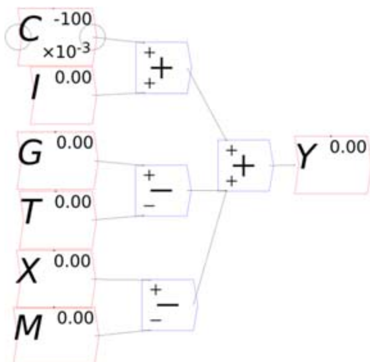
Let's see what the equation for GDP looks like in *Minsky*, using the standard symbols economists use:

$$Y = C + I + (G - T) + (X - M) \tag{1.1}$$

[7] The one complication here is that a minus sign (-) is firstly treated as a text entry, because we realise that sometimes modelers want to enter a negative constant: so if you want to enter a minus operator on the canvas, press "-" followed by pressing the Enter key or clicking on OK. To enter a negative constant, say -42, type -42 in the text entry box and then press the Enter key.

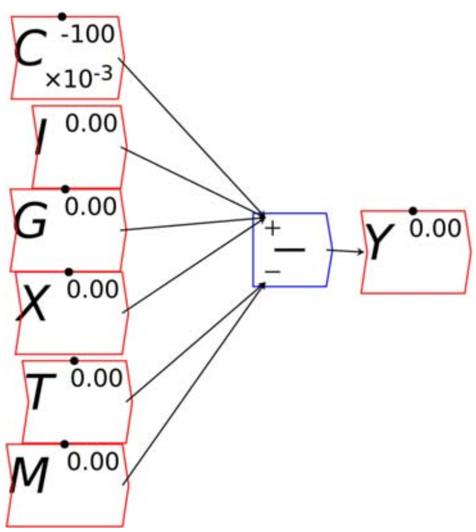
In Minsky, this looks like Figure 54:[8]

Figure 54



You will notice one unusual thing about Figure 54: there are *two* inputs to the bottom input port of the "+" key that defines *Y*. This is a common theme in *Minsky*, called "overloading": if an operator can sensibly accept more than one input, then it does. The reason we do this is that system dynamics diagrams—which are effectively flowcharts that map across to equations—can get very messy, with lots of wires which can ultimately produce a "spaghetti diagram" effect. We aim to minimize clutter on the canvas, so it's quite possible to replace the four addition operators in Figure 54 with just one, as shown in Figure 55.

Figure 55

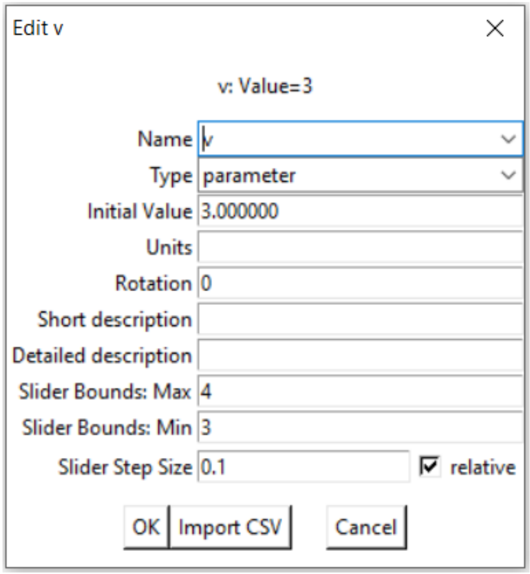


You may also have noticed the black dot on top of the *Variable* and *Parameter* blocks. This enables you to change the values of a parameter during a simulation. There are two ways to do this: by using the mouse to drag the dot to the left to reduce the value, and to the right to increase it; and by pressing the up key to increase the value, or the down key to reduce it, while the mouse cursor is hovering over

[88] This isn't to say that *Minsky*'s layout is better: I think it's actually harder to read than a standard equation in this example. However, it can be more intuitive to use a flowchart format when you're laying out causal relationships, as I do later. We also hope to enable both ways of displaying equations on the canvas in future versions of *Minsky*: both flowchart and standard mathematics. That, as always, is dependent on getting more funding to write the necessary code.

the parameter. The maximum, minimum and step size are all set on the Edit dialog box—see Figure 56.

Figure 56: The edit dialog box for v, showing the slider Max, Min and Step Size

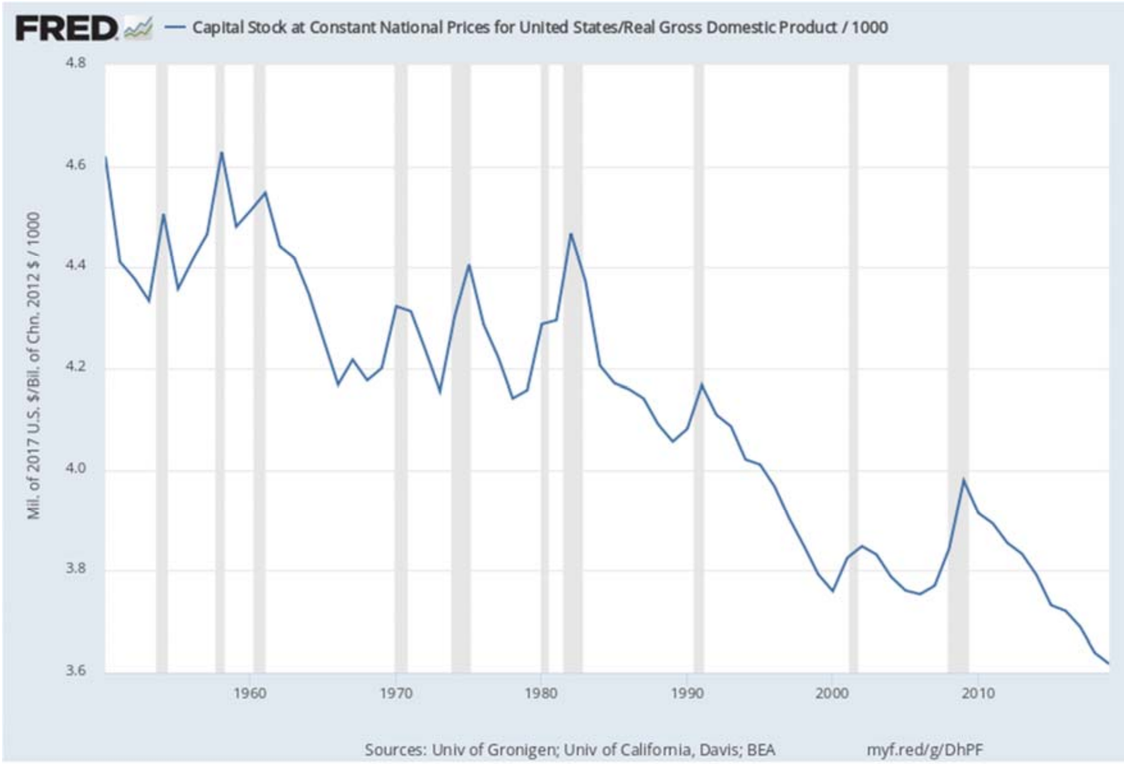


For example, you might use a "Leontief" production function, where output *Y* is defined as minimum of the capital stock *K* divided by a capital-output ratio v, and an output—to-labour ratio a times labor L:

$$Y = \min\left(\frac{K}{v}, a \cdot L\right) \tag{1.2}$$

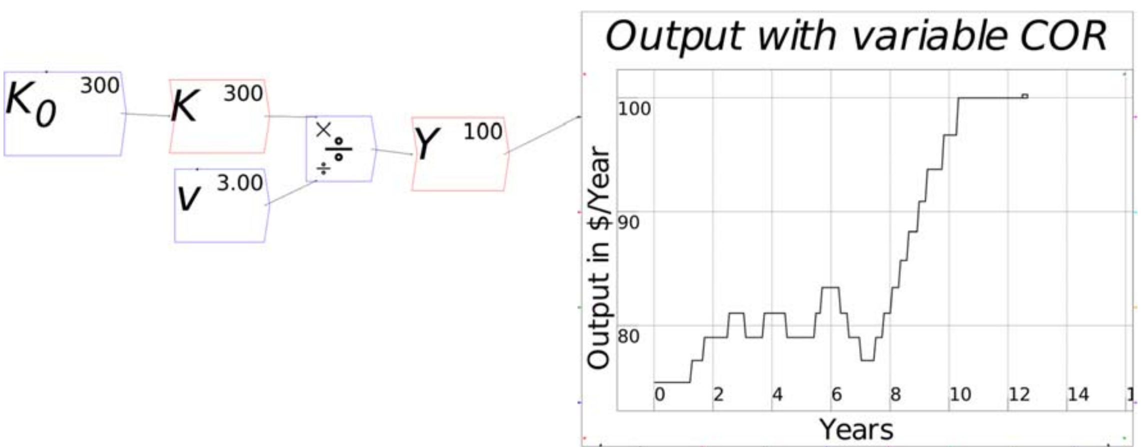
Post-Keynesian models generally treat the capital-output ratio as a constant with a value of between 2 and 4. However, economic data implies that this is a variable with a decreasing trend over time (within a very small range), and that it rises during recessions—see Figure 57.

Figure 57: Capital stock at 2017 prices divided by GDP at 2012 prices (www.myf.red/g/DhPF)



I'll explain what the capital-output ratio (*COR*) actually is, and give an explanation for this trend, in the Energy chapter. For now, this implies that the practice of treating the ratio as a constant is generally defensible—the range is small, and the measurement of capital stock is compromised anyway (Sraffa 1960; Pasinetti 1969; Harcourt 1972)—but it would be wise to be able to vary the parameter and see what happens. Figure 58 shows the effect of varying the value of *v* from 4 to 3 during a simulation.

Figure 58: Output with varying capital-output ratio



5.1 Text Formatting

Minsky supports text formatting, including $_{\text{Subscripts}}$, $^{\text{Superscripts}}$, and Greek letters α, β, etc., using the *LaTeX* mathematical formatting conventions. The basic formatting codes are:

- Underscore _, which subscripts the next character;

- Caret ^, which superscripts the next character;
- Brackets { }, which apply the underscore and caret to a string of characters; and
- Backslash \, which initiates a Greek character, using the English-language expression for the Greek letter. So typing \lambda into the text input dialog box and pressing Enter will generate the Greek letter λ.

For example, if you wish to distinguish Real GDP from Nominal GDP, you can create variables GDP_{Real} and $GDP_{Nominal}$ using these conventions. This improves the readability of the model, compared to standard text-only systems, which to my knowledge are all that are provided by the other system dynamics programs. Figure 59 shows some examples of *LaTeX* formatting in *Minsky*.

Figure 59: Some examples of LaTeX formatting in Minsky

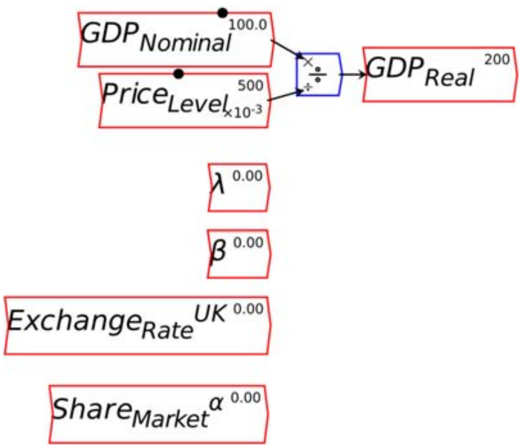


Figure 60 shows the most commonly used Greek characters supported by Minsky, and the English word that LaTeX displays as a Greek letter if you precede it by a backslash key (\).

Figure 60: A partial list of Greek characters supported by Minsky & the English word used for it[9]

α	alpha	μ	mu	Γ	Gamma
β	beta	ν	nu	Δ	Delta
γ	gamma	ξ	xi	Θ	Theta
δ	delta	π	pi	Λ	Lambda
∈	epsilon	ρ	rho	Ξ	Xi
ζ	zeta	σ	sigma	Π	Pi
η	eta	τ	tau	Σ	Sigma
θ	theta	υ	upsilon	Υ	Upsilon
ι	iota	φ	phi	Φ	Phi
κ	kappa	χ	chi		
λ	lambda				

5.2 Multiple copies of variables and parameters

Once you've defined a variable or parameter, you can copy it and use it anywhere else on a diagram. So, for example, if you use the Greek letter lambda (λ) to indicate the employment rate, then you can

[9] For the full list, see https://github.com/highperformancecoder/minsky/blob/master/engine/latexMarkup.cc.

make a copy of λ and use it elsewhere in your model as an input to a wage determination model—a so-called "Phillips Curve".

5.2.1 *A Keen Rant*: Rehabilitating Bill Phillips

Before I illustrate building a Phillips Curve in *Minsky*, it's important to rehabilitate the reputation of the man behind the name of the curve, the New Zealand engineer-turned-economist Bill Phillips.

Few people have been as badly misrepresented by Neoclassical economists as Bill Phillips: a courageous and innovative man has been reduced to a caricature of the empirical study he undertook over one weekend, to validate a hypothesis he made about a nonlinear relationship between the intensity of economic activity and the rate of change of input prices (Phillips 1958). Frankly, the Neoclassical caricature of Phillips is probably worse than their caricature of Keynes (Hicks 1937).

At least with Keynes, Neoclassicals couldn't completely ignore his outstanding contributions to the politics and economics of his time. As a leading civil servant, Keynes attended the *Treaty of Versailles*, witnessed its distortion by France into a means to destroy its long-standing enemy Germany, and raised the alarm that the Treaty's onerous terms would almost certainly lead to another war in *The Economic Consequences of the Peace* (Keynes 1920). He was a scion of English society, and while Hicks's IS-LM model eviscerated Keynes's *General Theory* (Keynes 1936), it didn't eviscerate the man himself.

Phillips, on the other hand, had a unremarkable birth as the son of a New Zealand farmer, trained as an engineer, and spent most of WWII in a Japanese prisoner-of-war camp. But in that camp, among many other outstanding deeds, he risked his life to fashion a radio out of parts he stole from the commandant's office, so that his fellow prisoners could hear British and American news reports on the progress of the War, rather than merely being force fed Japanese propaganda (Leeson 1994, pp. 606-608).

On his release, Phillips decided to use his engineering training to bring economics out of its Dark Ages of equilibrium thinking—using precisely the same modelling techniques that are now used in system dynamics programs like *Minsky*. The paper from which the model in Figure 61 is taken, "Stabilisation Policy in a Closed Economy" (Phillips 1954), pre-dates Forrester's initial proposal of system dynamics by 2 years (Forrester 2003 [1956]), and the practical development of system dynamics software by about six years. So Phillips was well ahead of his time. And, *of course*, his innovative work was ignored by mainstream economists.

Phillips's hypothesized relationship between the level of economic activity and the rate of change of money wages (not prices!) was supposed to fit into the dynamic model shown in Figure 61, where there would not be a simple "trade-off" between inflation and unemployment, as his statistical work was bowdlerized down to,[10] but a dynamic feedback process that would be difficult, though not necessarily impossible, to stabilize.

[10] I have to concede that Phillips did make one statement in his statistical paper that was easily interpreted as offering politicians a "menu" trading off unemployment against inflation: "Ignoring years in which import prices rise rapidly enough to initiate a wage-price spiral, which seem to occur very rarely except as a result of war, and assuming an increase in productivity of 2 per cent per year, it seems from the relation fitted to the data that if aggregate demand were kept at a value which would maintain a stable level of product prices the associated level of unemployment would be a little under 2 ½ per cent. If, as is sometimes recommended, demand were kept at a value which would maintain stable wage rates the associated level of unemployment would be about 5 ½ per cent" (Phillips 1958, p. 299). But the overall context of his paper, and of his macroeconomic modelling, was one of dynamic feedbacks, and the difficulty of stabilizing the economy.

Figure 61: Phillips's engineering diagram layout of an economic model with his hypothesized Phillips curve relationship inset (Phillips 1954, p. 309)

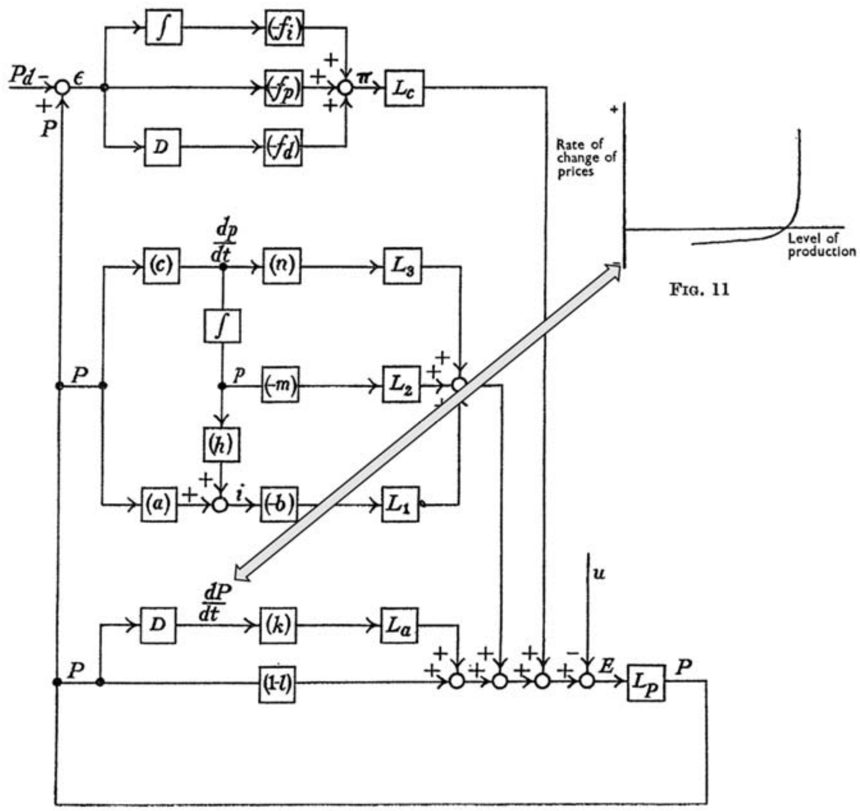


Fig. 12

5.3 Plots in *Minsky*

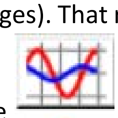
To illustrate the Phillips Curve in *Minsky*, we need to plot the input (for which I'll use the employment rate, rather than the unemployment rate) and the output (the rate of change of wages). That requires adding a plot widget to the canvas, and there are two ways to do this: click on the  icon on the toolbar; or press the @ key while on the canvas. We borrow a trick from Mathcad here: the @ symbol "looks like" a plot (use your imagination!; it's not as obvious as using * for "multiply", but it will do), so we use that as a keyboard shortcut.

Figure 62 shows the default shape of the plot widget after you've either clicked on the plot icon, or typed the @ key on the canvas. Also shown, in left to right order from the toolbox, are: the spreadsheet widget; the other toolbox icons that generate a single object (lock, note, and time at the left hand end of the toolbox; switch, Godley Table, integral and differential at the right hand end), plus all the drop-down menus shown as "tear-offs". Notice the dotted line at the top of the fundamental constants drop-down menu? There's one for each, you "tear off" the menu, so that it remains permanently available while you work on a model, and it can be located anywhere on your screen.

Figure 62: The "fundamental constants" menu on the toolbar, with the other menus as tear-offs

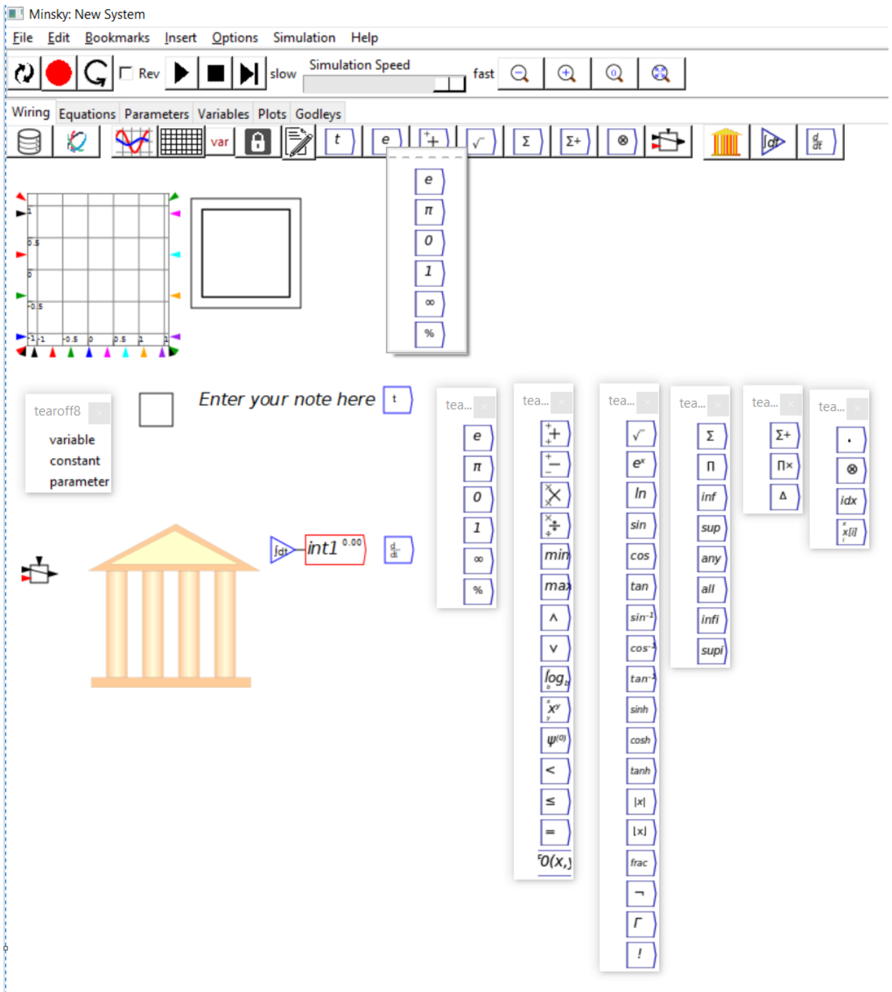
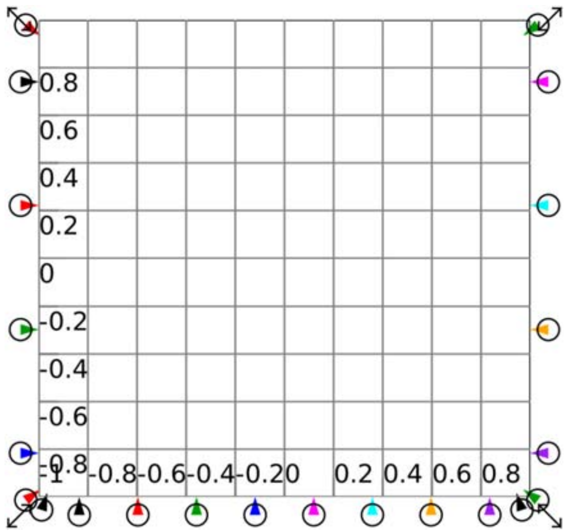


Figure 63 shows a plot with its resize arrows visible: these are four arrows, one on each corner. If you click and drag on one of them, you can resize the plot (a similar feature exists on all objects in a *Minsky* model: look for a mini-arrow when your mouse hovers over any element on the canvas).

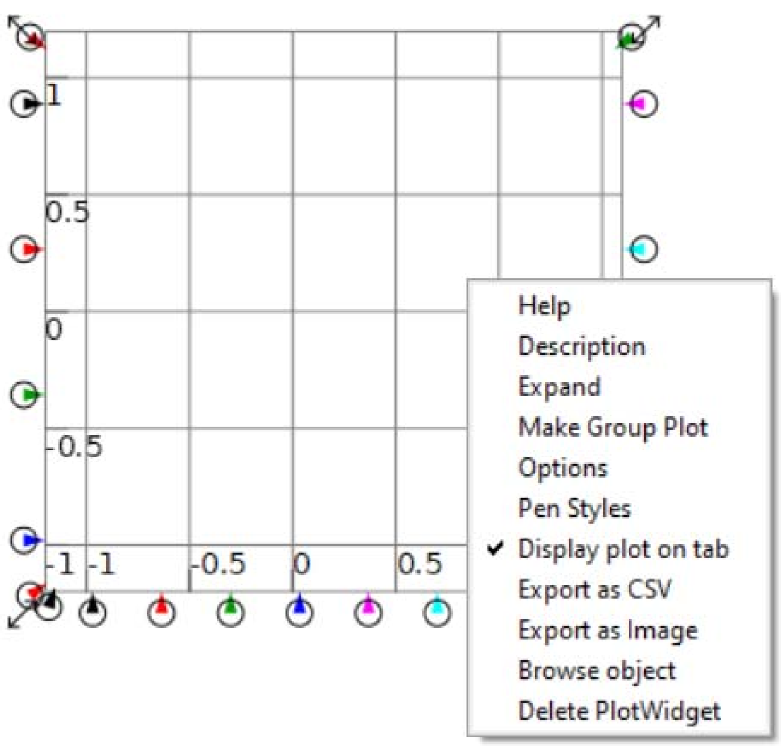
The coloured input ports are also highlighted (you can see this yourself by hovering your mouse over a plot). These are used to determine the upper and lower bounds for each axis (the angled inputs) and to attach variables for plotting (the horizontally aligned port on the two Y-axes, and the vertical one on the X-axis).

Figure 63: A plot with its resize handles visible



Plots are labelled using the "Options" element on the right-click mouse menu—see Figure 64. Minsky makes very heavy use of the right mouse button: right-click while hovering over a plot, and this menu will appear.

Figure 64: The right-click menu for Plots



"Options" and "Pen Styles" control the appearance of the Plot—see Figure 65.

Figure 65: Options and Pen style dialog boxes

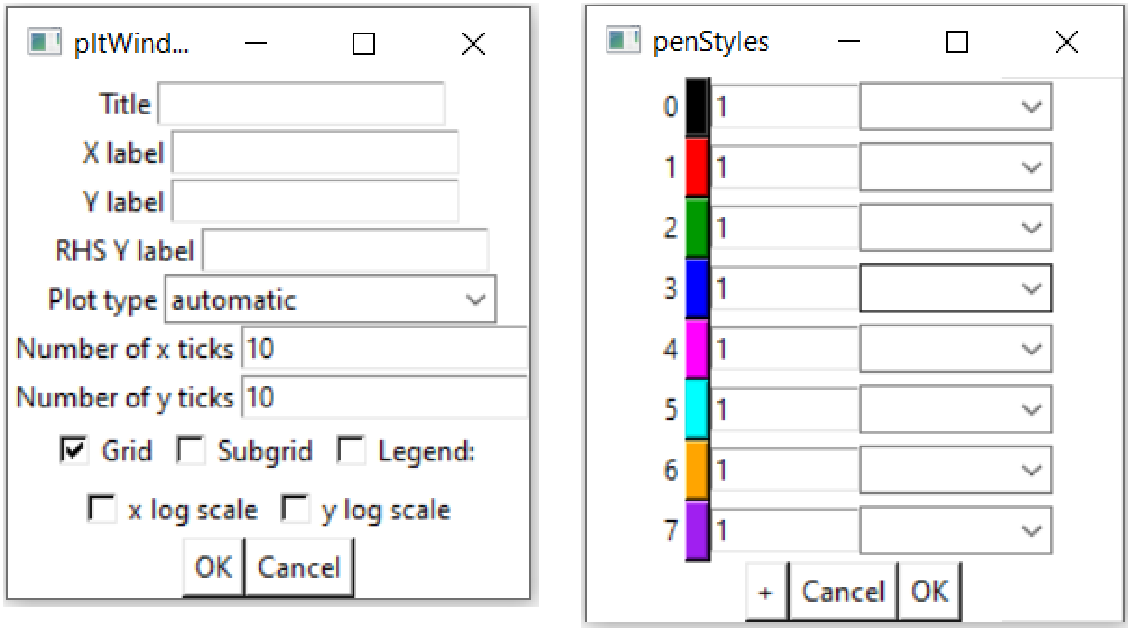
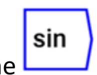
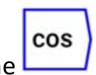


Figure 66 uses the inputs of time 

, and the two functions sine 

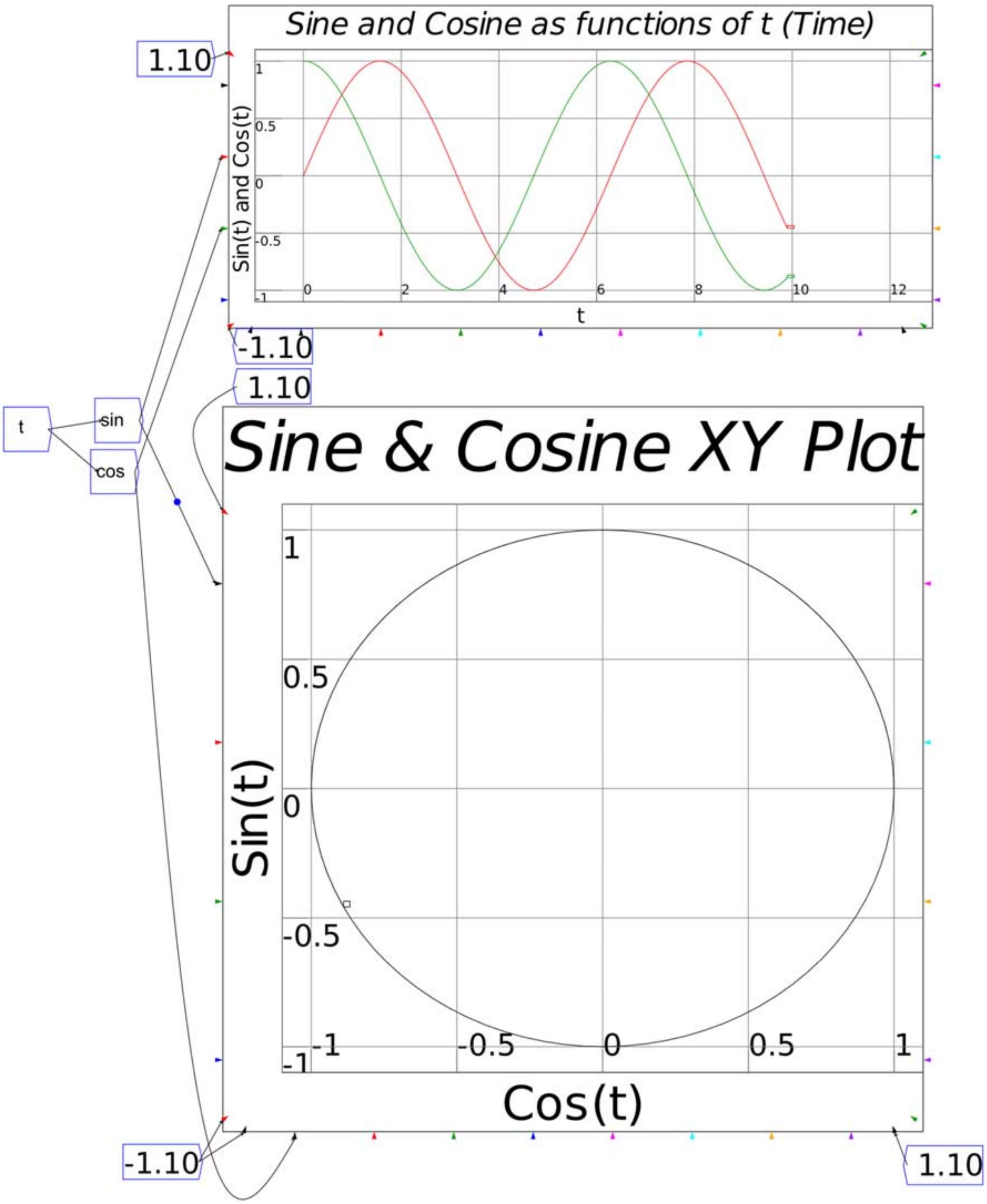
and cosine 

from the functions drop-down menu. Wire the output from t to the inputs to sine and cosine as shown, then attach them to the plots. As illustrated, more than one output wire can be dragged from an output port and attached to input ports elsewhere in a model.

The top plot in Figure 66 illustrates the default behaviour of a plot: if a variable is wired up to one of the four input ports on the left hand side of a plot, but nothing is wired to one of the eight input ports on the bottom, then “time” is treated as the input on the x-axis and the behaviour of the variable over time is plotted. The bottom plot shows that if you attach an input to the black input on the x-axis, and another to the black input on the y-axis, Minsky plots x against y, as shown in Figure 68. You can create xy plots of different colours by using matching colour inputs on the horizontal and vertical axes.

Notice also that several of the connecting lines in Figure 66 are curved. Lines can be turned into curves by clicking and dragging the blue dot that will appear when your mouse hovers over a line. Multiple points of curvature can be added to create any curve shape, by clicking and dragging somewhere on the line away from the existing blue dot(s).

Figure 66: Using Minsky's Plot widget



5.4 Building a "Phillips Curve" in Minsky

Now back to the "Phillips Curve". Phillips insisted that the function relating the rate of change of money wages to the level of unemployment would be nonlinear, and that it would have three causal factors—the level of unemployment, the rate of change of unemployment, and "the rate of change of retail prices, operating through cost of living adjustments in wage rates" (Phillips 1958, p. 283). The model in Figure 68 is a linear model with only one input, but these limitations are easily overcome later. For now, I'm just using a linear model here to keep it simple early on. You should build this model yourself in *Minsky* before continuing.

The model introduces one more feature of Minsky, the percent operator: this takes an input and multiplies it by 100. It's the last entry on the "fundamental constants" toolbar dropdown, which is headed by the operator *e* for the value of the transcendental number *e*. Click on *e* and the drop-down menu shown in Figure 62 will appear; click on 

and that will be attached to the mouse pointer; move to where you want to place it on the canvas and click the mouse, and it will be inserted there.

Then wire the model up as shown in Figure 68, using the parameter values shown in Figure 67.

Figure 67: Parameter values in the model in Figure 68 (this Figure was generated by choosing Export Canvas while on the Parameters Tab)

Name	Definition	Initial Value	Short Description	Long Description	Slider Step	Slider Min	Slider Max	Value
λ_{test}		0.630000			0.01	0.5	0.8	0.63
S_{λ}		10	Slope of Phillips Curve		1	0	20	10
Z_{λ}		0.6	Employment rate where wage change is zero		0.01	0	0.7	0.6
Z_{λ}		0.6			0.01	0	0.7	0.6
S_{λ}		10			1	0	20	10

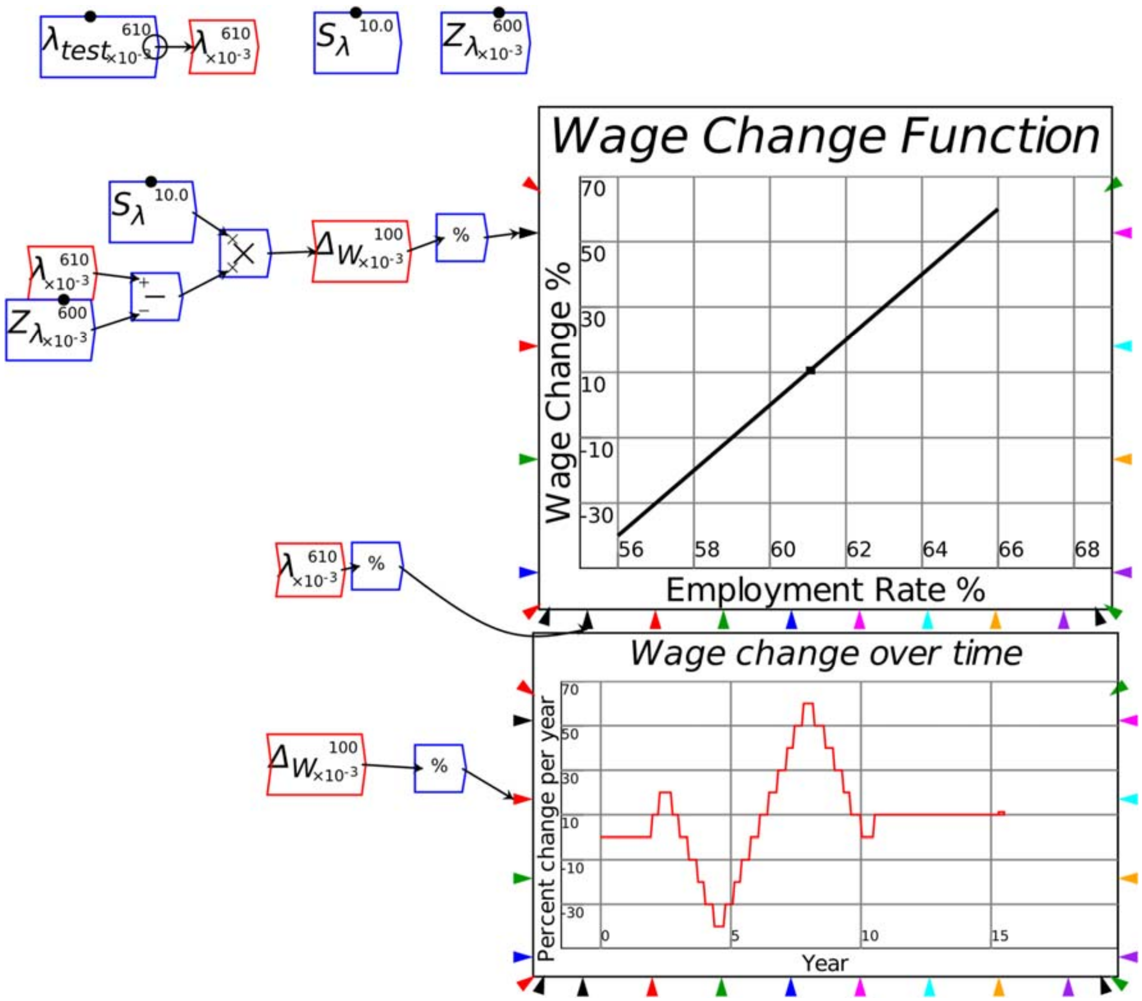
Table 1 shows what you have to type to get the elements shown on the canvas in Figure 68

Table 1: Variable and parameter names and how to type them

What is displayed on the canvas	What you type to get it
λ	\lambda
λ_{test}	\lambda_{test}
S_{λ}	S_λ
Z_{λ}	Z_λ
Δ_w	\Delta_w

To simulate this equation, vary the value of the parameter λ_{test} using the arrow keys or the mouse. As you do, the line shown in the plot in Figure 68 will be drawn.

Figure 68: A linear "Phillips Curve" in Minsky



Minsky generates the equations of its models in *LaTeX*. You can export these from the program via the File menu option “Export Canvas”, which has six options: SVG (a generic vector graphics format that I’m using to produce the Figures in this book); *PDF*; *EPS* (Postscript); *EMH* (Enhanced Metafile, a Windows vector graphics format); *LaTeX*; and *Matlab*. Choose *LaTeX* and you’ll save a file with a *.tex suffix, which you can load into a *LaTeX*-aware mathematics formatting application (which includes Word itself as of 2017). The equations behind Figure 68 are shown in Equation (1.3):

$$\begin{aligned}\lambda_S &= 10\\ \lambda_Z &= 0.6\\ \lambda_{\text{test}} &= 0.63\\ \Delta_W &= \lambda_S \times (\lambda - \lambda_Z)\end{aligned} \tag{1.3}$$

This takes us about as far as we can go without discussing how to handle time in dynamic modeling.

6 System Dynamics Basics

6.1 A *Keen Rant*: How *not* to handle time

The vast majority of economic models, whether Neoclassical (Sargent and Stachurski 2020b, a) or Post Keynesian (Godley and Lavoie 2007b), use what economists term “periods”. A recent example is the debate in Post Keynesian macroeconomics between Claudio Sardoni (Sardoni 2019) and Marc Lavoie and Gennaro Zezza (Lavoie and Zezza 2020), where both sides advocate what they term a “sequential” approach, in preference to “equilibrium” analysis. Sardoni states, quite correctly, that

> sequential analysis represents a clearer conceptual framework to cope with processes that occur in time. The analysis of the multiplier effects of investment is one of the cases in which the occurring of events in time should not be ignored. (Sardoni 2019, p. 243)

But he also comments, immediately before this, *and also quite correctly*, that:

> Keynes may have been right to underline the difficulties of sequential analysis and, in particular, the difficulty to provide a precise definition of the length of periods. (Sardoni 2019, p. 243)

The resolution to this paradox is itself paradoxical: ***there is no “period***”. There are instead, economic processes which, at a “micro” level, are discrete acts—each individual act of consumption, investment, borrowing, etc. Each of them takes a different amount of time to complete, and each recurs at a different frequency: no one individual act of consumption is timed precisely with others, nor each act of investment: they are *asynchronous*. All these individual “periods”, when viewed from the perspective of an aggregate economic system, overlap, and a macro-level period cannot be defined.

While it would be feasible to model these as discrete processes in a multi-agent model,[11] at the level of aggregate macroeconomic modelling, asynchronous microeconomic processes are best treated as happening in what mathematicians call “continuous” time, as opposed to “discrete” time. This in turn means that the proper mathematical technology for dynamic economic modelling is not the “difference equation”, but the “differential equation”.

Therefore, equations like the “discrete-time” Equation (1.4), from the seminal Godley and Lavoie paper “Fiscal policy in a stock-flow consistent (SFC) model” (Godley and Lavoie 2007a, p. 84, Equation 19), which defines government debt as a difference equation:

$$GD \equiv GD_{-1} + DEFICIT \tag{1.4}$$

should instead be written as a differential equation in “continuous time”:

$$\frac{d}{dt}GD \equiv DEFICIT \tag{1.5}$$

While “sequential analysis” is indeed preferable to equilibrium analysis, continuous time modelling is preferable to both. There are, of course, rejoinders to this, which I have heard many, many times from my Post Keynesian colleagues (especially from Marc Lavoie: we are good friends, and, when our schedules and geography permit, tennis rivals/buddies, as well as intellectual collaborators).

[11] I briefly discuss multi-agent modelling in (Keen 2021, p. 149).

The commonest defense of "sequential analysis" is that economic data is periodic, and therefore economic models should use equivalent periods. This is a fallacy, as stated bluntly by the father of System Dynamics, the engineer Jay Forrester, when he first reported on his study of economic models to his Faculty Seminar at MIT in 1956:

> The incremental time intervals for which the variables of a model are solved step-by-step in time must be much shorter than often supposed… This solution interval is unrelated to the interval at which national statistics and economic indicators are measured… (Forrester 2003, pp. 337-345)

Forrester backgrounded this didactic statement in his textbook *Industrial Dynamics*:

> A discontinuous model, which is evaluated at infrequent intervals, such as an economic model solved for a new set of values annually, should never be justified by the fact that data in the real system have been collected at such infrequent intervals. The model should represent the continuously interacting forces in the system being studied. The frequency with which measurements on the real system may happen to have been taken is not relevant to the frequency with which internal dynamic performance must be calculated. (Forrester 2013, p. 65)

Another frequently made rejoinder is that economic decisions, such as investment, are based on lagged data, rather than current data, and therefore period analysis is needed to capture these lags. For example, Godley & Lavoie 2007 assume:

> that governments react to lagged inflation rates, rather than to actual or expected inflation rates, on the realistic grounds that fiscal policy may have a reaction time somewhat longer than monetary policy. (Godley and Lavoie 2007a, p. 92)

Therefore, they use the two equations shown in Equation (1.6) to represent "real pure government expenditures" g, and the "growth rate of real pure government expenditures", gr_G, where the rate of growth of government expenditure is a function of "the growth rate of potential output" gr, the change in the lagged inflation rate $\Delta\pi_{-1}$, and the deviation of the lagged inflation rate from the target inflation rate π^T:

$$\begin{aligned} g &= g_{-1} \cdot (1 + gr_G) \\ gr_G &= gr - \beta_1 \cdot \Delta\pi_{-1} - \beta_2 \cdot (\pi_{-1} - \pi^T) \end{aligned} \tag{1.6}$$

In fact, lags are easily represented in differential equations, using what is known as a "first-order time lag", to relate the delayed perception of the rate of inflation to the actual, instantaneous rate of inflation π. I'll use π_L rather than π_{-1} for the time-lagged inflation rate, since a time lag can be any length, not merely "one period". The time-lagged inflation rate is defined by its rate of convergence to the actual inflation rate, which is given by the "time constant" τ_π (which, in an elaborate model, can be a variable if desired) which measures the length of time, in years, that it takes for the percieved rate of inflation π_L to converge to the actual rate of inflation π. If $\tau_\pi = 0.5$, this is a 6-month lag; if $\tau_\pi = 1$, a year, and so on. This rate of convergence is given by the differential equation shown in Equation (1.7):

$$\frac{d}{dt}\pi_L = -\frac{1}{\tau_\pi} \cdot (\pi - \pi_L) \tag{1.7}$$

Similarly, the growth rate of government expenditure is expressed as a differential equation:

$$\frac{d}{dt} g = gr_G \times g \tag{1.8}$$

The variable growth rate gr_G can now defined as something like Equation (1.9), or it could be replaced with its own differential equation.

$$gr_G = gr - \beta_1 \cdot \frac{d}{dt} \pi_L - \beta_2 \cdot \left(\pi_L - \pi^T \right) \tag{1.9}$$

This approach is *vastly superior* to the discrete approach to time lags (which is more correctly called a *time-delay*, rather than a time-lag), for many reasons.

Time-lags are flexible. Your lag can be a fraction of a year, or multiple years, or even an irrational number if you wish: it doesn't have to be 1,2, 3 "time periods", as in conventional economic modeling. And of course, I'm being generous in saying that! Economic models use a time delay of "1 period" for almost everything. In Lavoie and Godley 2007, interest payments have a lag of -1 (equation 1); spending is negatively related to the interest rate with a lag of -1 (equation 2); taxes on wealth are lagged -1 (equation 7). This is typical. Factors which in the real world occur at vastly different frequencies—consumption, for example, has a much higher frequency than investment—are all corralled into the same arbitrary frequency.

Therefore, the *time-delays* (not time-lags) in discrete time economic models—which is to say, the majority of economic models—are spurious. They have nothing to do with the actual characteristics of time-dependent actions in the real economy. Time lags, on the other hand, can be derived from empirical data. They are also easy to edit: a time lag is a simple scalar, and if you find that you're using the wrong value—say, data shows that the time lag in investment is actually 1.5 years when your model uses 3 years—then all you have to alter is that number. On the other hand, if discrete-time economic models did time delays properly, they would have different delays for consumption (short) versus investment (long). This simply isn't done. If it were, and then empirical data indicated that the delay was different to what the model used, a wholesale re-writing of the model is necessary.

The final objection made to using continuous time is, how then do we derive the values for parameters in such models, and test them, when the economic data we have is in discrete time format (quarterly or yearly)? This is in fact a valid issue, since it does take care to do this properly. Once we've been through the basics of system dynamics and the models in *Manifesto*, I'll cover methods to derive parameters for continuous time models from discrete time data.

The bottom line is, ***stop using difference equations for economic models***! They are simply the wrong technology for the macro modelling of asynchronous micro processes in general, let alone economics in particular. Difference equations are really only appropriate for macro-level modelling when the micro-level processes that generate it are synchronized. This is the case for, for example, the birth dynamics of Christmas Island Red Crabs, which give birth on the same day, so that the sheer number born on that day overwhelms predators, and enables the survival of the species (Adamczewska and Morris 2001). So, if you're modelling the life cycle of Christmas Island Red Crabs, go right ahead and use difference equations. But if you're modelling anything else…, then ***don't use them***.

Given how inappropriate difference-equation models are for modelling the economy, and yet how much they are used by economists, *Minsky* ***deliberately*** does not support time-delays:

"***friends don't let friends use periods***". We may need to introduce time-delays at some point, to enable the importing of models from other system dynamics programs, but if so, they will exist solely for that purpose.

6.2 Mathematics and *Minsky*

One of the reasons that economists have used difference equations is because they're easy to write down: anyone can define a simple equation in terms of time differences, and it can easily be modelled with conventional software like a spreadsheet. You need specialist software (including mathematics programs like *R* and *Matlab*, in addition to system dynamics programs like *Minsky*) to simulate differential equations, and it is also much harder to think in terms of differential equations initially. For this reason, I recommend undertaking some study of differential equations, even though you can use *Minsky* without that training.

If you do study them, do a course given by mathematicians rather than economists, and make sure the tuition extends to third order nonlinear differential equations (or at least their qualitative features compared to 2^{nd} order equations), since, as I explain in *Manifesto,* 3-dimensional models are the foundation of complex systems modelling: as Li and Yorke put it, "Period Three Implies Chaos" (Li and Yorke 1975). Alternately, get a good textbook: my favourite, because it is so well written, and because it covers stability analysis, qualitative analysis, and the basics of the linear algebra needed for differential equations as well, is Braun's *Ordinary Differential Equations and their Applications* (Braun 1993).

6.3 Integrals versus differentials

Since system dynamics programs simulate systems that change over time, differential equations are fundamental to them. However, differentiation (working out the slope of a curve) is a much more volatile operation than integration (working out the area under a curve): the slope of a curve can vary dramatically over a short interval, but the area beneath it will change less dramatically. Approximating the slope of a curve numerically can result in large errors, so for this reason (and a few others), system dynamics programs work with integration rather than differentiation.

Therefore, if you start with a differential equation for population growth, like Equation (1.10):

$$\frac{d}{dt} Population = Births - Deaths \tag{1.10}$$

Where births and deaths are proportional to the existing population:

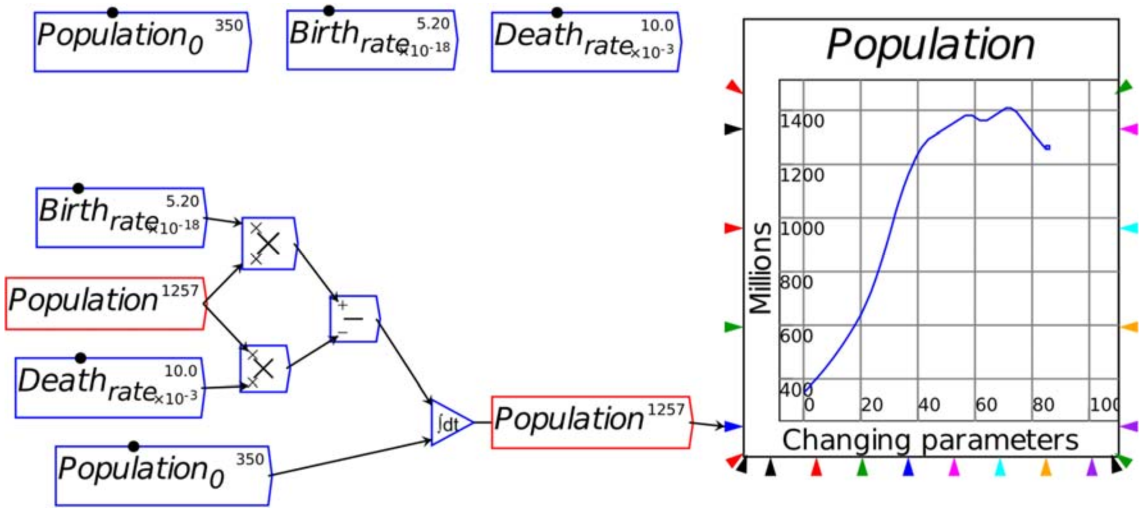
$$\begin{aligned} Births(t) &= Birth_{Rate} \cdot Population(t) \\ Deaths(t) &= Death_{Rate} \cdot Population(t) \end{aligned} \tag{1.11}$$

Then, in a system dynamics program, you would express Equation (1.10) in integral form by integrating both sides:

$$Population(t) = \int_0^t \left(Birth_{Rate} \cdot Population(s) - Death_{Rate} \cdot Population(s) \right) \cdot ds \tag{1.12}$$

In *Minsky*, this looks like Figure 69:

Figure 69: A simple equation for population, with the parameters being varied during the simulation



6.4 A first model, done two ways

Now let's build a first serious model using *Minsky*: Goodwin's growth cycle model (Goodwin 1967). Normally, a system dynamics model is designed by considering causal relationships between elements of a model, and then connecting them all into a causal loop. We'll do that in a moment, and also follow up with a second method, of deriving the model directly from macroeconomic definitions. This is to emphasize the point I made in *Manifesto* that Goodwin's model—and my extension of it to model Minsky's Financial Instability Hypothesis (Keen 1995, 2020)—are foundational models for a modern, complex systems approach to macroeconomics.

Figure 70 shows the opening paragraphs of Goodwin's paper, where he sets out the assumptions underlying his model (Goodwin 1967, p. 54). I'll follow this structure in deriving the model in a system dynamics way, though Goodwin's own derivation was closer to the second approach we'll use later. I should note that I found Goodwin's explanation of his model interesting but inscrutable when I first read it, and only properly understood the model—and its potential—when I read Blatt's masterful exposition of it in *Dynamic economic systems: a post-Keynesian approach* (Blatt 1983). If you're reading this book with serious intent—in that you plan to become proficient at system dynamics modelling in economics—then I strongly suggest that you buy a copy of Blatt's recently republished masterpiece.

Figure 70: Goodwin's statement of the assumptions from which his model is derived

A GROWTH CYCLE†

R. M. GOODWIN

Presented here is a starkly schematized and hence quite unrealistic model of cycles in growth rates. This type of formulation now seems to me to have better prospects than the more usual treatment of growth theory or of cycle theory, separately or in combination. Many of the bits of reasoning are common to both, but in the present paper they are put together in a different way.

The following assumptions are made for convenience:

(1) steady technical progress (disembodied);
(2) steady growth in the labour force;
(3) only two factors of production, labour and 'capital' (plant and equipment), both homogeneous and non-specific;
(4) all quantities real and net;
(5) all wages consumed, all profits saved and invested.

These assumptions are of a more empirical, and disputable, sort:

(6) a constant capital-output ratio;
(7) a real wage rate which rises in the neighbourhood of full employment.

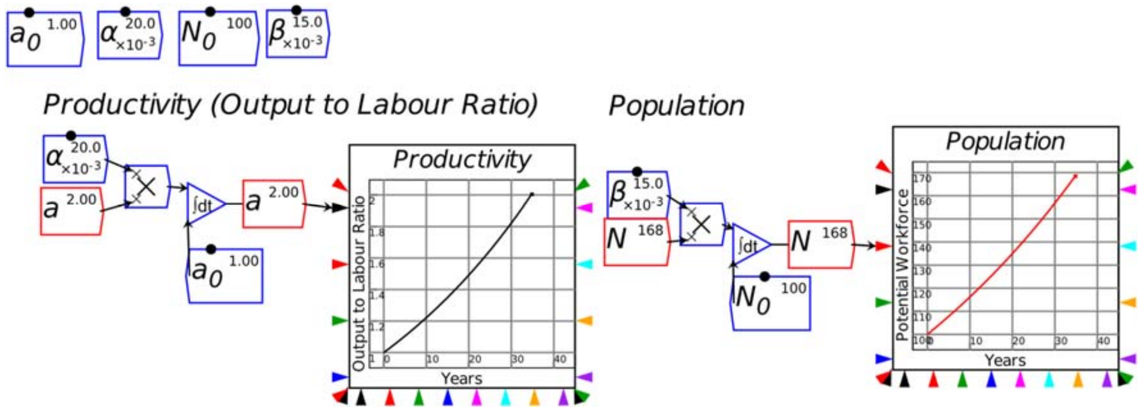
No. (5) could be altered to constant proportional savings, thus changing the numbers but not the logic of the system. No. (6) could be softened but it would mean a serious complicating of the structure of the model.

Working from Goodwin's exposition here—and using slightly different notation—his first two assumptions are constant exogenous growth of the output to labour ratio a and of population N. Using α for the rate of growth of the output to labour ratio and β for the rate of growth of population, that gives us these two equations:

$$\begin{aligned} \frac{1}{a}\frac{d}{dt}a &= \alpha \\ \frac{1}{N}\frac{d}{dt}N &= \beta \end{aligned} \tag{1.13}$$

In Minsky, these equations are entered as shown in Figure 71:

Figure 71: Exogenous growth rates of technology and population in Minsky

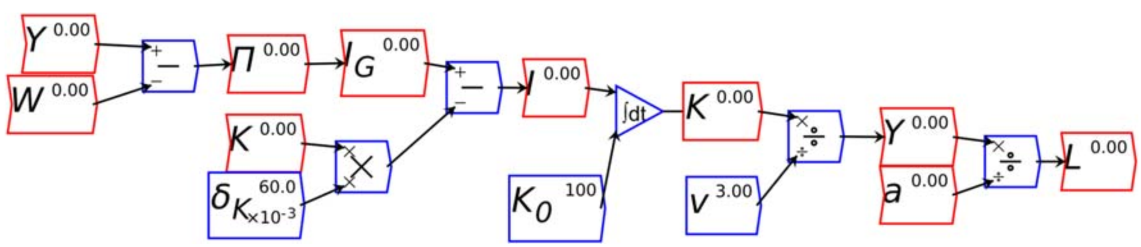


Goodwin's assumption 6 gives us a constant ratio v between capital K and output Y, while assumption 5 means that the level of gross investment I_G equals profits Π, which in this simple two-class (workers and capitalists) model equals output Y minus wages W. Goodwin neglected to include depreciation of capital, so I include that as well, defining net investment I to be equal to gross investment I_G minus depreciation, which is a constant δ_K times K:

$$\begin{aligned} v &= \frac{K}{Y} \\ \Pi &= Y - W \\ I_G &= \Pi \\ I &= I_G - \delta_K \cdot K \\ \frac{d}{dt}K &= I \end{aligned} \tag{1.14}$$

These equations can be used to commence building the model, as shown in Figure 72.

Figure 72: Partial Goodwin model, from the definition of profit to the determination of employment



Reading from left to right in Figure 72:

- Output Y minus Wages W determines profit: Π $(Y - W \to \Pi)$;
- all profit is invested: $(\Pi \to I_G)$;
- net investment I is gross investment I_G minus depreciation $\delta_K \cdot K$: $(I_G - \delta_K \cdot K \to I)$;
- net investment, integrated and added to the initial level of capital stock K_0, is the current capital stock: $\left(\int I \to K + K_0\right)$;
- Capital stock divided by the capital output ratio v is output: $\left(\frac{K}{v} \to Y\right)$; and
- Output divided by the output to labour ratio a is Labour: $\left(\frac{Y}{a} \to L\right)$.

This leaves just his assumption 7: "a real wage that rises somewhere in the neighbourhood of full employment" (Goodwin 1967, p. 54). I'll use $\lambda = \frac{L}{N}$ for the employment rate, but I'll relate this to the total population N, and not just the proportion of the total population that is employable, which is what Goodwin and Blatt used.[12] In generic mathematical notation, the Phillips Curve relationship is as stated two equivalent ways in Equation (1.15):

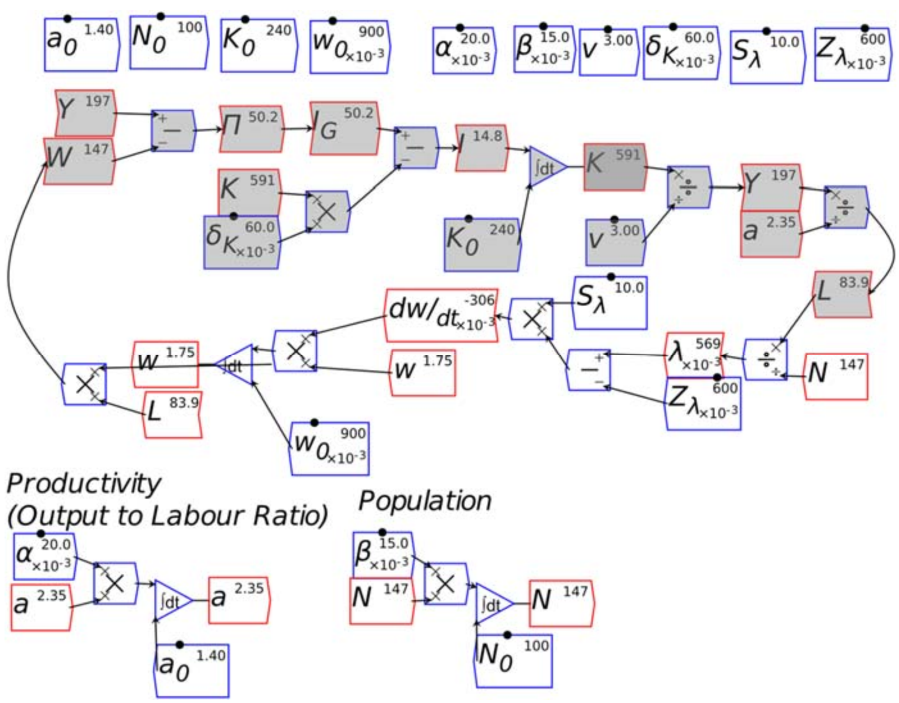
$$\begin{aligned} \frac{1}{w}\frac{d}{dt}w &= f(\lambda) \\ \frac{d}{dt}w &= w\cdot f(\lambda) \end{aligned} \tag{1.15}$$

We've already built a linear version of this, in Figure 68 and Equation (1.3). So all we need to do is add the equation defining λ as L/N, and then to define W as $w \cdot L$:

$$\begin{aligned} \lambda &= \frac{L}{N} \\ W &= w\cdot L \end{aligned} \tag{1.16}$$

This adds the terms in white in the causal diagram part of Figure 73.

Figure 73: The completed model, with the original terms in grey and the new ones in white[13]



[12] This approach yields a value of λ for stable wages of roughly 0.60, versus the value of about 0.96 that Blatt used, which arose from treating λ as one minus the unemployment rate $(1-u)$. Blatt's approach results in the "no-wage-change" value for λ being 0.96. If you use a linear function as an approximation of the Phillips Curve—which is what Goodwin did—then the model generated dynamics that give returns silly results like the employment rate exceeding 100%. This doesn't happen as easily if the stable wages value of λ is 0.60.

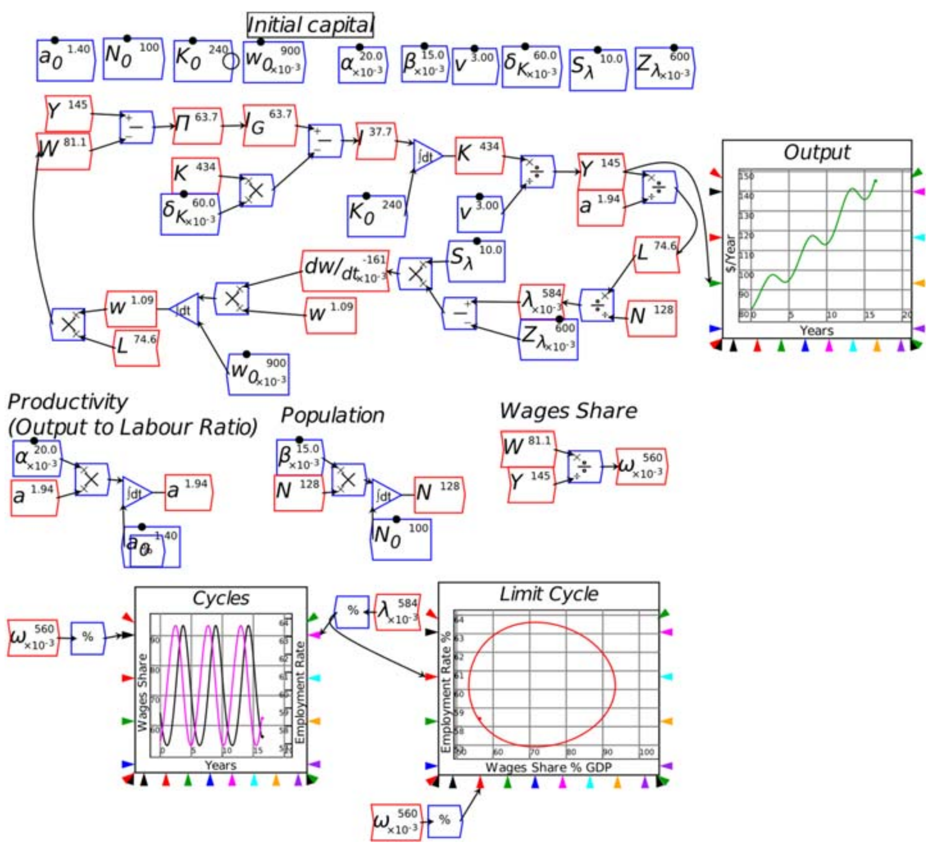
[13] Notice the strange wiring at the bottom left, where the wire crosses over the *w*? That's a bug: the rotation of the integral block didn't update where the output wire emanated from. It's a good example of the sort of debugging that is needed to make a computer program work well. We've since fixed it (see Figure 74). Repairing

Reading right to left in this section—since I have "flipped" the model components to close the causal loop:[14]

- Labour L divided by population N determines the employment rate λ;
- The employment rate fed into the "Phillips Curve" function determines the rate of change of wages $\frac{d}{dt}w$;
- Multiplied by the current wage, integrated and added to the initial wage w_0, this determines the wage rate w;
- From this point on, it's what one of my undergraduate maths lecturers described as "money for old rope":
    - The wage rate times L determines W;
    - Subtract W from Y, and the causal loop is closed.

With this completed, we can now see the dynamics of the Goodwin model. A few plots inserted and wired up to Y, ω and λ generate Figure 74.

Figure 74: Goodwin model with plots



There are many interesting features to this model that we'll explore later, but I want to address a criticism that I've frequently heard of this model, that it is in some way contrived or "ad hoc". In fact, as I noted in *Manifesto*, this model—and my "Minskian" extension of it to include private debt—can

errors like this, as well as adding new features, is a major reason why continued funding is needed to develop *Minsky*.

[14] This wasn't necessary—I could have designed the whole model left to right, and terminated it with another instance of W—but that resulted in a model whose elements were too small to read on an A4 wide page.

be derived directly from incontrovertible macroeconomic definitions. For this reason, I regard these two models as not "ad hoc", but as foundational models for a complex systems approach to macroeconomics. I'll explain why here as I redo the derivation of the model directly from macroeconomic definitions.

In English, the definitions behind the Goodwin model are:

$$\begin{aligned} Employment_{Rate} &\equiv \frac{Labour}{Population} \\ Wage_{Share} &\equiv \frac{Wages}{GDP} \end{aligned} \tag{1.17}$$

Using the symbols we've already employed in building the flowchart version of Goodwin's model, these are:

$$\begin{aligned} \lambda &\equiv \frac{L}{N} \\ \omega &\equiv \frac{W}{Y} \end{aligned} \tag{1.18}$$

I'll derive the dynamic model using the differentiation shortcuts that I noted in *Manifesto*:

- The percentage rate of change of a variable, say x, (expressed as a ratio rather than percentage) is $\frac{1}{x}\frac{dx}{dt}$;
- The notation mathematicians use for this expression is $\hat{x} \equiv \frac{1}{x}\frac{dx}{dt}$
- The percentage rate of change of a ratio is equal to the percentage rate of change of the numerator, minus the percentage rate of change of the denominator, so that $\widehat{\left(\frac{X}{Y}\right)} = \hat{X} - \hat{Y}$; and
- The percentage rate of change of a product is the sum of the percentage rates of change of the two parts of the product so that $\widehat{X \cdot Y} = \hat{X} + \hat{Y}$.

Putting the definitions in (1.18) into percentage rate of change format, and using the differentiation shortcuts, yields:

$$\begin{aligned} \hat{\lambda} &= \widehat{\left(\frac{L}{N}\right)} = \hat{L} - \hat{N} \\ \hat{\omega} &= \widehat{\left(\frac{W}{Y}\right)} = \widehat{W} - \hat{Y} \end{aligned} \tag{1.19}$$

In words, Equation (1.19) is saying that:

- The employment rate will rise if the workforce grows faster than population; and
- The wages share of GDP will rise if total wages rise faster than GDP.

At the moment, these are still true-by-definition statements. To get from here to a model, we need to introduce one more definition—the output to labour ratio $a \equiv Y/L$—and the assumption of a uniform wage rate w. This lets us make the following substitutions:

$$\begin{aligned} L &\equiv \frac{Y}{a} \\ W &= w \cdot L \end{aligned} \tag{1.20}$$

Substituting these definitions into the expression for λ in (1.19) yields the following for λ:

$$\begin{aligned} \hat{\lambda} &= \hat{L} - \hat{N} \\ &= \widehat{\frac{Y}{a}} - \hat{N} \\ &= \hat{Y} - \hat{a} - \hat{N} \end{aligned} \tag{1.21}$$

And for ω:

$$\begin{aligned} \hat{\omega} &= \hat{W} - \hat{Y} \\ &= \widehat{w \cdot L} - \hat{Y} \\ &= \hat{w} + \hat{L} - \hat{Y} \\ &= \hat{w} + \widehat{\frac{Y}{a}} - \hat{Y} \\ &= \hat{w} + \hat{Y} - \hat{a} - \hat{Y} \\ &= \hat{w} - \hat{a} \end{aligned} \tag{1.22}$$

As did Goodwin, we'll assume a constant rate of technological growth and a constant rate of population growth. This lets us make the substitutions:

$$\begin{aligned} \hat{a} &= \alpha \\ \hat{N} &= \beta \end{aligned} \tag{1.23}$$

Our almost completed model is now:

$$\begin{aligned} \hat{\lambda} &= \hat{Y} - \alpha - \beta \\ \hat{\omega} &= \hat{w} - \alpha \end{aligned} \tag{1.24}$$

Now we need to expand $\hat{Y}$ and $\hat{w}$. To do this, we need two more of Goodwin's simplifying assumptions:

- A constant capital to output ratio $v = \frac{K}{Y}$ (which I discuss further in Chapter 10 on page 159 *et seq.*); and
- An investment function (with depreciation, which Goodwin omitted). Goodwin assumed that all profits were invested.

Because v is assumed to be a constant, the percentage rate of change of Y is identical to the percentage rate of change of K:

$$\begin{aligned}\hat{Y} &= \widehat{\left(\frac{K}{v}\right)} \\ &= \widehat{K} - \hat{v} \\ &= \widehat{K} - 0 \\ &= \widehat{K}\end{aligned} \tag{1.25}$$

Therefore once we work out $\widehat{K}$, we can substitute this for $\widehat{Y}$, otherwise known as the rate of economic growth. We also insert Goodwin's assumption that all profits are invested, so that $I_G = \Pi$.

$$\begin{aligned}\frac{d}{dt}K &\equiv I_G - \delta_K \cdot K = \Pi - \delta_K \cdot K \\ \widehat{K} &\equiv \frac{1}{K}\frac{d}{dt}K = \frac{\Pi}{K} - \delta_K \\ \widehat{K} &= \left(\frac{Y-W}{v \cdot Y} - \delta_K\right) \\ \widehat{K} &= \frac{1-\omega}{v} - \delta_K \\ \widehat{Y} &= \frac{1-\omega}{v} - \delta_K\end{aligned} \tag{1.26}$$

That leaves just the rate of change of wages $\widehat{w}$, which is the "Phillips Curve". Using the same linear function as in Figure 68 give us:

$$\widehat{w} = S_\lambda \cdot (\lambda - Z_\lambda) \tag{1.27}$$

Substituting (1.26) and (1.27) into (1.24) yields the reduced-form version of Goodwin's model:

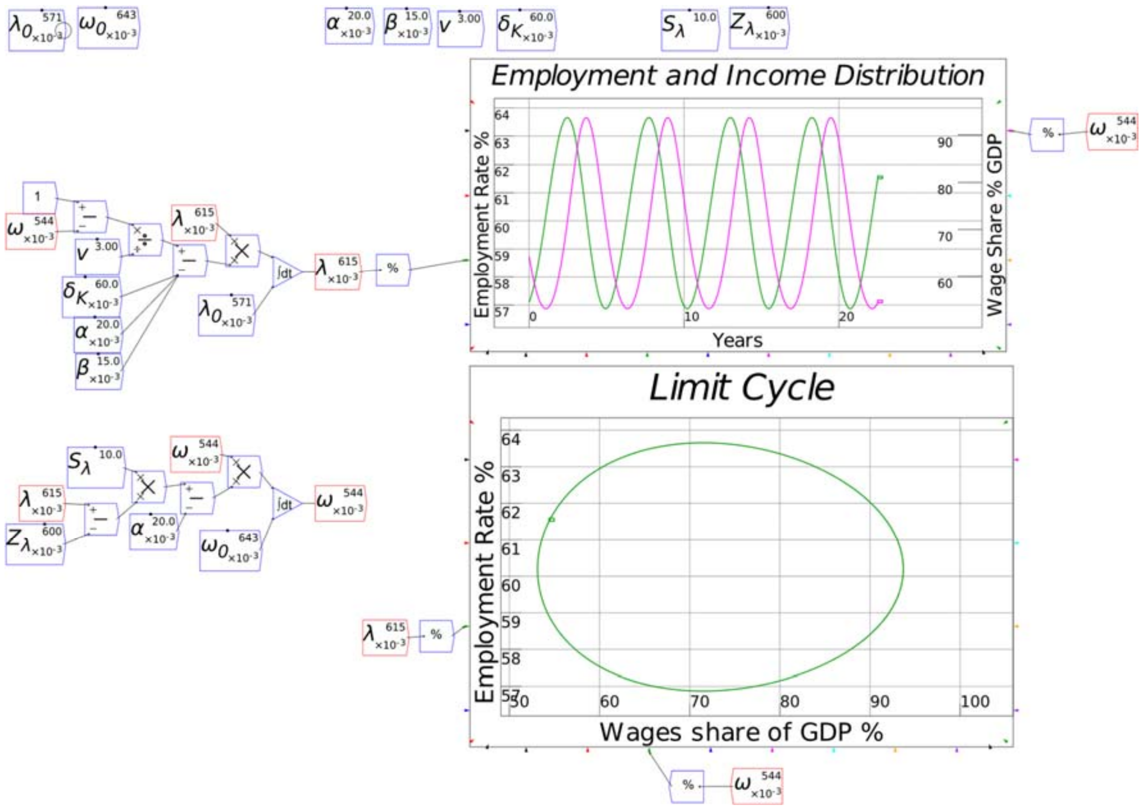
$$\begin{aligned}\hat{\lambda} &= \left(\frac{1-\omega}{v} - \delta_K\right) - \alpha - \beta \\ \widehat{\omega} &= S_\lambda \cdot (\lambda - Z_\lambda) - \alpha\end{aligned} \tag{1.28}$$

Expressed in differential equation form, this is:

$$\begin{aligned}\frac{d}{dt}\lambda &= \lambda \cdot \left(\left(\frac{1-\omega}{v} - \delta_K\right) - \alpha - \beta\right) \\ \frac{d}{dt}\omega &= \omega \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) - \alpha\right)\end{aligned} \tag{1.29}$$

This model, using the same parameter values as the previous model (plus initial conditions similar to the initial values for λ and ω) yields the same dynamics as Figure 74:

Figure 75: The Goodwin model in reduced form



This is a foundational model for macroeconomics, firstly, because it can be derived directly from incontestable macroeconomic definitions and a set of reasonable simplifying assumptions, and secondly, because the simplifying assumptions themselves suggest ways in which the model can be generalized and extended.

The assumption that all profits are invested, for example, is defensible as a first-order approximation (in the Taylor series sense) in that investment is ultimately a function of profit; but the obvious extension is that capitalists invest more than profits during a boom, and less than profits during a slump.[15] This generates a “finance … demand for money” (Keynes 1937, p. 247), which in an “endogenous money” world in which bank loans create money, adds to aggregate demand and income when the change in debt is positive, and subtracts from it when it is negative (Keen 2020). As I explain in Chapter 9.3 on page 149, replacing “capitalist invest all their profits” with this more realistic assumption is how I started the development of my model of Minsky’s Financial Instability Hypothesis (Keen 1995).

There are therefore at least two methods to go about designing dynamic, complex-systems models of the economy: the conventional flowchart method, and deriving a model from definitions using calculus. Both approaches have their strengths: the flowchart method is easier, while the definitional approach gives you some insights into how a model might be extended—by, for another example,

[15] Other factors, such as a desired level of capacity utilization, can be added. Matheus Grasselli and colleagues are working on fitting my Minsky extension of Goodwin to US data, and preliminary results indicate that the rate of investment should be modelled as depending upon the wages share of GDP (which includes the profit share and hence the profit rate as an argument), the employment rate (which can be shown to be a proxy for capacity utilization) and loans and deposit ratios of corporations.

replacing the single-commodity definitions of K and Y with multiple commodities and input-output dynamics. The closed form solution is also more appropriate for stability analysis. Personally, I find myself using the two approaches symbiotically as I build models.

But there's one thing I could never model properly with flowcharts: the dynamics of the monetary system. My first successful attempts to model monetary dynamics used systems of equations in the mathematics program *Mathcad*, with a matrix keeping track of relationships between accounts (Chapman and Keen 2006). But this only generated "static" plots of the models, when I wanted to also show the system changing through time, as I could do with the system dynamics program *Vissim*. However, every time I tried to put one of my models into *Vissim*, I'd make a mistake—by changing one equation (say for debt) but not a related one (for money), or putting the wrong sign in one equation, and so on. In 2008, I realized that I could generate the equations directly from the matrix (which I originally did in the program Mathcad). This became the inspiration for creating Godley Tables, and funding from INET in 2012 finally turned that into reality.

I've learnt a lot about money from building *Minsky*, and extending the capabilities of its Godley Tables—so much so that I now know that some of what I wrote about money in *Debunking Economics* (Keen 2011) was wrong. I will start work on a 3rd (and final!) edition after completing this book. And now to Minsky's *raison d'etre*, Godley Tables.

7 Godley Tables

Minsky's double-entry bookkeeping tables are named after Wynne Godley, for three reasons:

- Godley, in collaboration with Francis Cripps, was the originator of the concept of using double-entry bookkeeping tables to ensure stock-flow consistent modelling;
- I spent six very pleasant months at the Levy Institute in 2000, writing the first edition of *Debunking Economics* while on sabbatical leave, and I learnt a great deal from interacting with Wynne at that time, including the crucial role of double-entry bookkeeping in ensuring stock-flow consistency; and
- Because non-Neoclassical economics needs to preserve the names of its heroes. If we leave history to the victors—which, in the sad case of economics, means leaving it to Neoclassical economists—then their names will be forgotten. Hence the name of *Minsky* itself, *Godley Tables* for our double-entry bookkeeping tool, and—if I can raise further development funding after our £200,000 grant from *Friends Provident Foundation* runs out—*Moore Tables* to show the macroeconomics of inter-sectoral flows, to honour Basil Moore (Moore 1979, 1988b).

Godley Tables in *Minsky* differ from the flow matrix tables in Godley's own work. Whereas both the rows and columns in his tables summed to zero "on the principle that every flow comes from somewhere and goes somewhere" (Godley 1999, p. 394), the rows in a Godley Table sum to zero, but the sum of the columns adds up all the flows into and out of a given account, and therefore tells you the *rate of change* of the account the column represents.

Therefore, when you fill out the rows in a Godley Table, you are actually building a set of differential equations with which to model an integrated financial system. The rule enforced by the Godley Table, that each row must sum to zero, ensure that these differential equations are stock-flow consistent.

7.1 Creating a Godley Table

There are 2 ways to insert a Godley Table onto the canvas: click on the Godley Table icon on the toolbar, and then click on the canvas where you wish to place it; or choose "Godley Table" from the main menu item "Insert".

When you first create a Godley Table, you get a bank icon on the canvas—see Figure 76.

Figure 76: A blank Godley Table icon

Double-click on the icon, or click the right-mouse button and choose "Open Godley Table", and Figure 77 will appear, inside a new Window.

Figure 77: A blank Godley Table opened in an editing window

The top row labels each account as either an Asset, a Liability, or Equity—the difference between Assets and Liabilities. The final column, labelled $A - L - E$, applies the "golden rule of accounting", that Assets minus Liabilities minus Equity equals zero, to each row in the table.

Immediately below this line has buttons to add or delete columns. There is one set of buttons for each of Asset, Liability and also the Equity columns, if you enable multiple equity columns from the Options menu (if you don't, there are no buttons for the Equity column). The $+$ key adds a new column to the right, the – key deletes the current column, and the arrow keys move the selected column one position to the left ← or right →.

The third line starts with the top left cell in the table, which notes that the columns are "Stock variables", while the rows are flows between these stock variables. The columns to the right are where you type the names of the accounts (the down-triangle icon is discussed  later).

The row below this shows the initial conditions for the accounts—the amount of money in each account when a simulation commences—which must also follow the $A - L - E$ rule that the numerical sum of these conditions must be zero. At the left-hand end of this line is a plus key, which creates the first row. Once you have done this, plus, minus, up and down symbols appear to allow you to add and delete rows, and move them up and down.

While there can be numerous entries in a row, the norm is two, which must sum to zero according to the rule $Assets - Liabilities - Equity = 0$, which is checked by the $A - L - E$ column. The entries are symbolic: words, not numbers. These words can include the formatting tricks discussed in the first chapter—subscripts, superscripts, grouped text and Greek letters—and if $Spend$ you can have $0.7 \times Spend$ in one column and $0.3 \times Spend$ in another.

Now let's use *Minsky* to build the simplest possible models of a monetary economy, starting with a model of a pure credit economy in which all money is created by bank loans.

7.2 The simplest possible monetary model of a pure credit economy

Figure 78 shows a simple model with credit (bank-created) money only, with six flows: lending to firms, interest payments, debt repayment, wages, workers' consumption, and bank purchases from firms.

Figure 78: A simple Godley Table

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	*Loans*	*Firms*	*Workers*	*Banks*	*0*
Initial Conditions	110	90	10	10	*0*
Firm borrowing	$Lend_F$	$Lend_F$			*0*
Interest payments		$-Interest_F$		$Interest_F$	*0*
Debt repayment	$-Repay_F$	$-Repay_F$			*0*
Hire workers		*-Wages*	*Wages*		*0*
Workers consume		$Consume_W$	$-Consume_W$		*0*
Bank purchases		Buy_B		$-Buy_B$	*0*

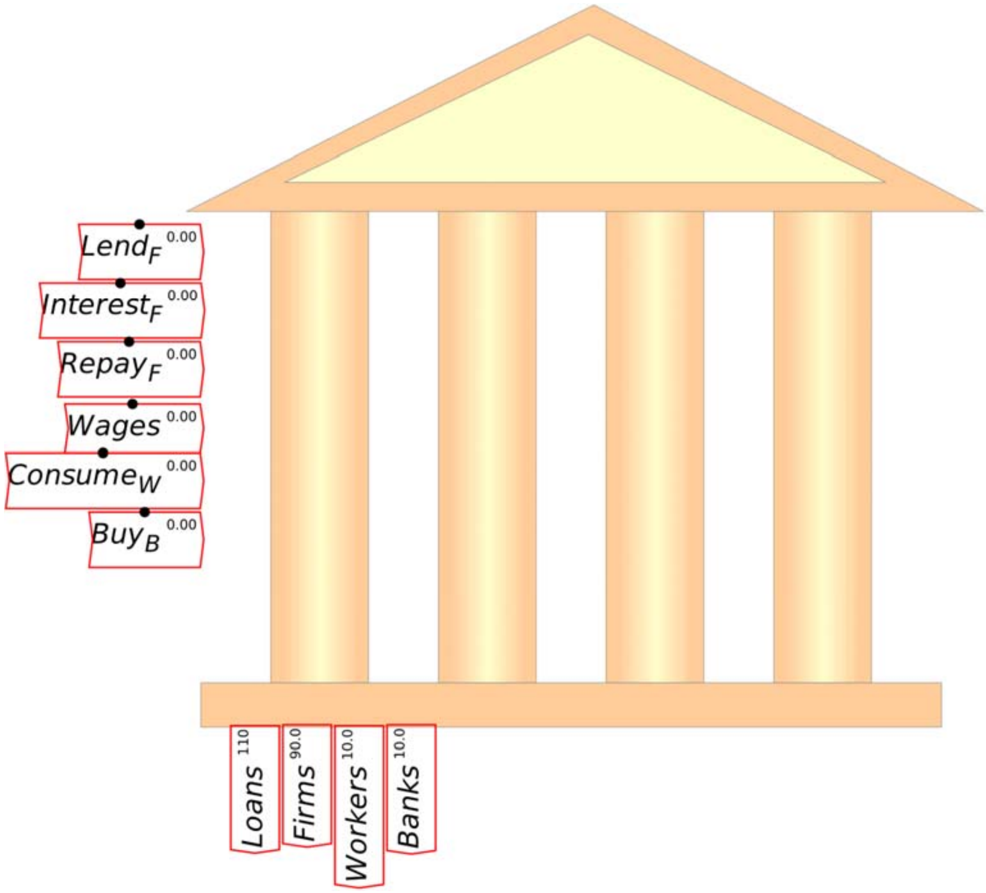
Minsky takes these entries and creates a set of ordinary differential equations, which you can see either by clicking on the Equations Tab, or by choosing “Export Canvas” from the File menu, and then choosing the file type to be LaTeX (*.tex). Equation (1.30) shows the differential equations for Figure 78.

$$
\begin{aligned}
\frac{d\mathrm{Banks}}{dt} &= \mathrm{Interest}_F - \mathrm{Buy}_B \\
\frac{d\mathrm{Firms}}{dt} &= \mathrm{Lend}_F + \mathrm{Consume}_W + \mathrm{Buy}_B - \left(\mathrm{Interest}_F + \mathrm{Repay}_F + \mathrm{Wages}\right) \\
\frac{d\mathrm{Loans}}{dt} &= \mathrm{Lend}_F - \mathrm{Repay}_F \\
\frac{d\mathrm{Workers}}{dt} &= \mathrm{Wages} - \mathrm{Consume}_W
\end{aligned}
\tag{1.30}
$$

Though this is quite a simple “toy” model, the same process enables huge, detailed models of the financial system to be built, with complete confidence that these equations are stock-flow consistent.

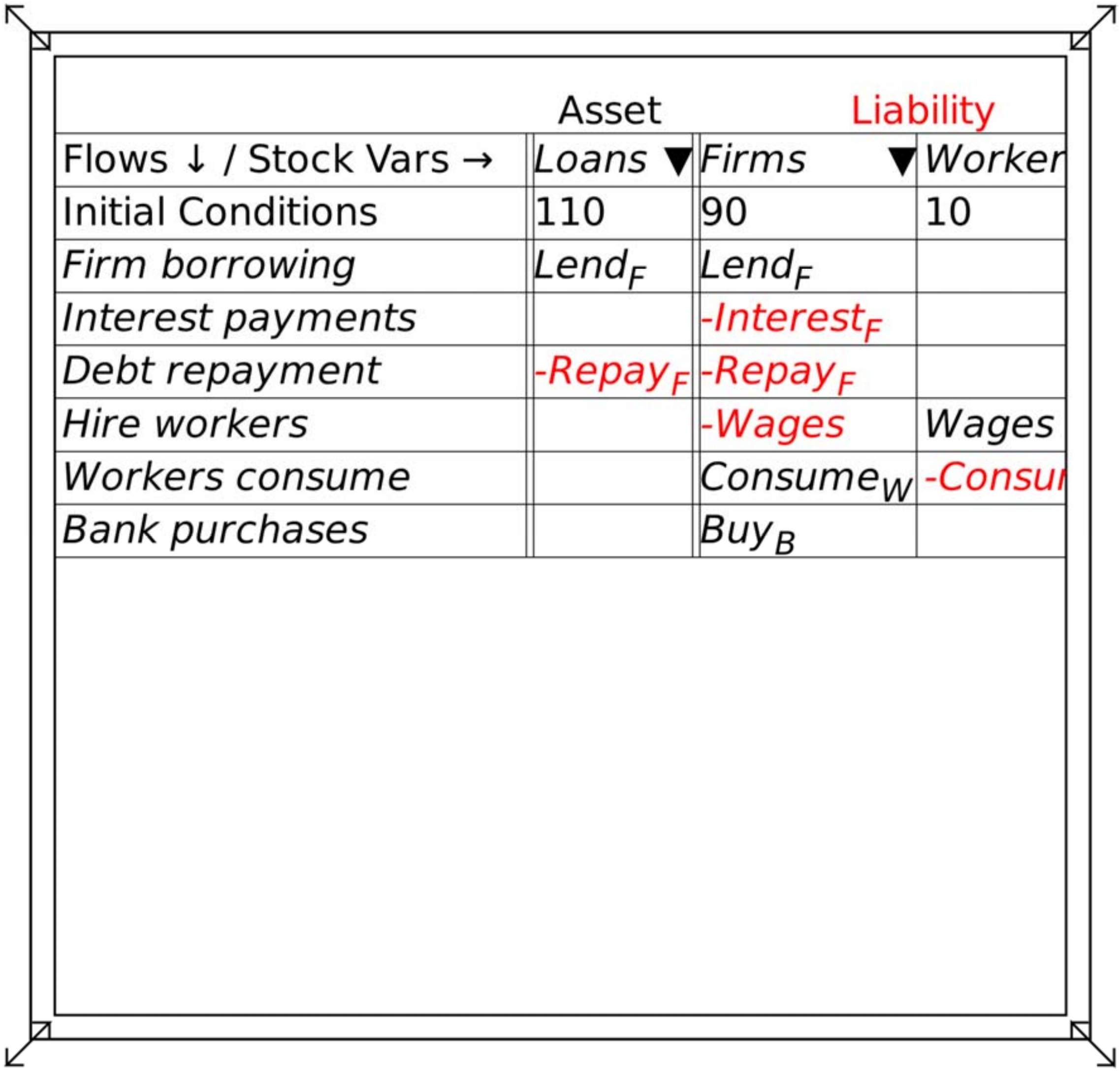
Once you have made entries in a Godley Table, the Godley icon on the canvas changes to show the flows as inputs on the left-hand side, and the stocks as outputs on the bottom:

Figure 79: A Godley Table after stocks and flows have been defined



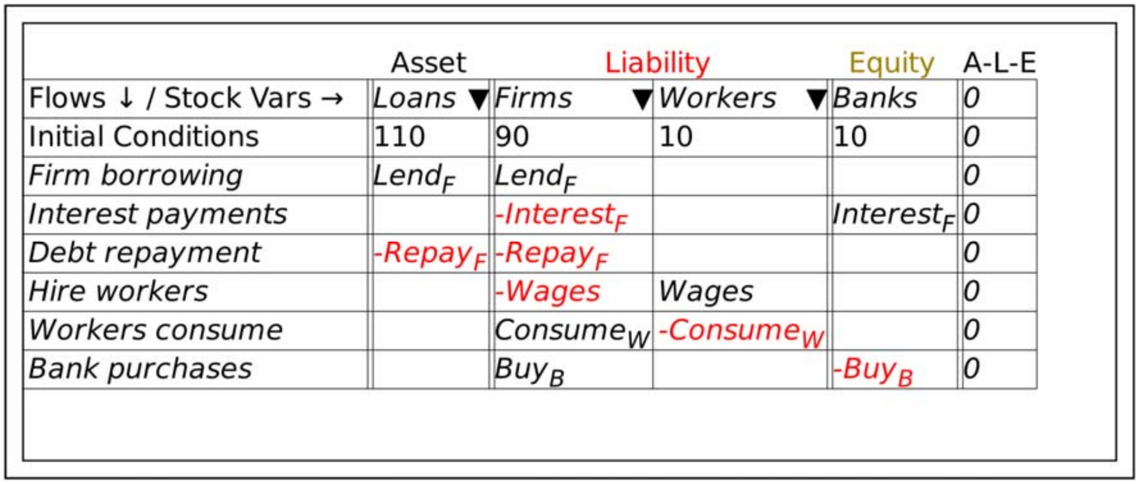
You can also alter the view so that you see the actual Godley Table on the canvas. Choose "Editor Mode" from the right-click menu, and the table will display as shown in Figure 80.

Figure 80: The Godley Table in Editor Mode, before resizing



The “bounding box” around the Table initially starts at the same size as the Godley Icon, so you need to drag one of the four corner handles to stretch the window to see the whole table (we are still working on this new *Editor Mode* feature, and at present the arrows are a bit hard to select: try clicking very close to the box end, rather than the arrow end). This may seem inconvenient—why not resize it automatically?—but if that were done by default, the table could well obscure parts of an existing model. So we left the resizing exercise to the user (we may scale the table to fit the bounding box size in a future release of *Minsky*). Figure 81 shows the Table after resizing.

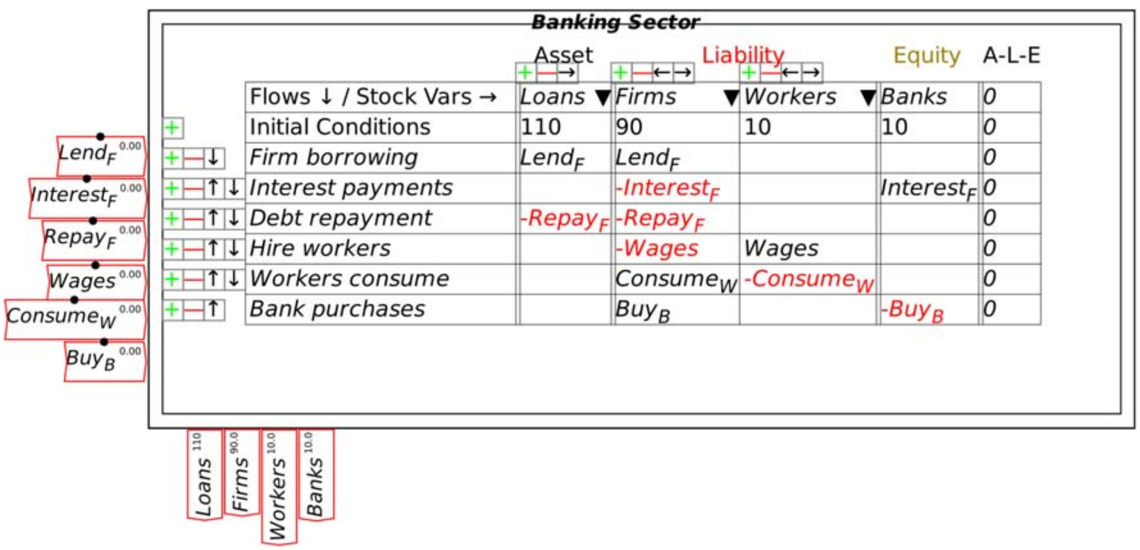
Figure 81: The Editor Mode view resized to fit the whole table



	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	*Loans*	*Firms*	*Workers*	*Banks*	*0*
Initial Conditions	110	90	10	10	*0*
Firm borrowing	$Lend_F$	$Lend_F$			*0*
Interest payments		$-Interest_F$		$Interest_F$	*0*
Debt repayment	$-Repay_F$	$-Repay_F$			*0*
Hire workers		*-Wages*	*Wages*		*0*
Workers consume		$Consume_W$	$-Consume_W$		*0*
Bank purchases		Buy_B		$-Buy_B$	*0*

As the name of this display mode implies, you can edit the table here rather than in a separate window, but you have to activate the row and column buttons that are shown automatically in the separate window. You can also turn on showing the stocks and flows attached to the table via the “Display Variables” option on the right-click menu—see Figure 82 (I have also added a title to the Table, using the right-click menu option “Title”).

Figure 82: The Table showing editing buttons and the Stock and Flow widgets



Banking Sector

	Asset	Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	*Loans*	*Firms*	*Workers*	*Banks*	*0*
Initial Conditions	110	90	10	10	*0*
Firm borrowing	$Lend_F$	$Lend_F$			*0*
Interest payments		$-Interest_F$		$Interest_F$	*0*
Debt repayment	$-Repay_F$	$-Repay_F$			*0*
Hire workers		*-Wages*	*Wages*		*0*
Workers consume		$Consume_W$	$-Consume_W$		*0*
Bank purchases		Buy_B		$-Buy_B$	*0*

One thing Russell Standish and I have focused upon in designing *Minsky* is enabling quality documentation of a model by *Minsky* itself. This includes the capacity to export a Table in either CSV or LaTeX format, via the “Export to File” option on the right-click menu (and also on the file menu from within a Godley Table window). Figure 83 shows the LaTeX output for the Table in Figure 82.

Figure 83: A screenshot of the LaTeX rendition of a Godley Table

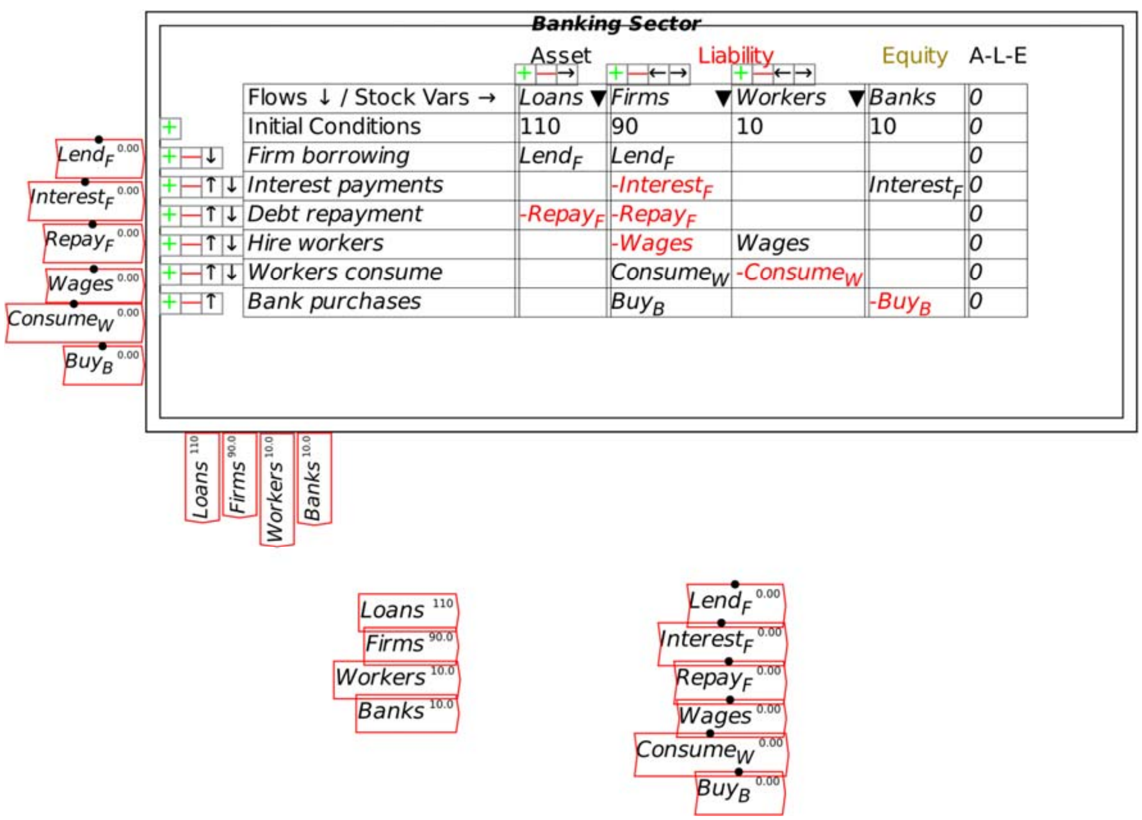
Flows ↓ / Stock Variables →	$Loans$	$Firms$	$Workers$	$Banks$
Asset Class	asset	liability		equity
Initial Conditions	110	90	10	10
Firm borrowing	$Lend_F$	$Lend_F$		
Interest payments		$-Interest_F$		$Interest_F$
Debt repayment	$-Repay_F$	$-Repay_F$		
Hire workers		$-Wages$	$Wages$	
Workers consume		$Consume_W$	$-Consume_W$	
Bank purchases		Buy_B		$-Buy_B$

7.3 Defining the flow elements of a Godley Table

The Godley Table itself clarifies issues in monetary economics, even without a simulation—especially when it is linked to other Godley Tables in an integrated model. But it also makes the modelling of monetary dynamics so much easier than with flowchart programs, as noted above.

To turn a Godley Table into a model, the flows in it have to be defined on the canvas itself, using the same flowchart logic as in the previous section. There are two ways to get the stock and flow variables in a Godley Table onto the canvas: individually, by right-clicking on the flow or stock variables attached to either the icon (Figure 79) or the table (Figure 82); or by right-clicking on the table and choosing “Copy Flow Variables” and “Copy Stock Variables” commands which copies all the relevant variables at once. Figure 84 shows the canvas after they have all been copied.

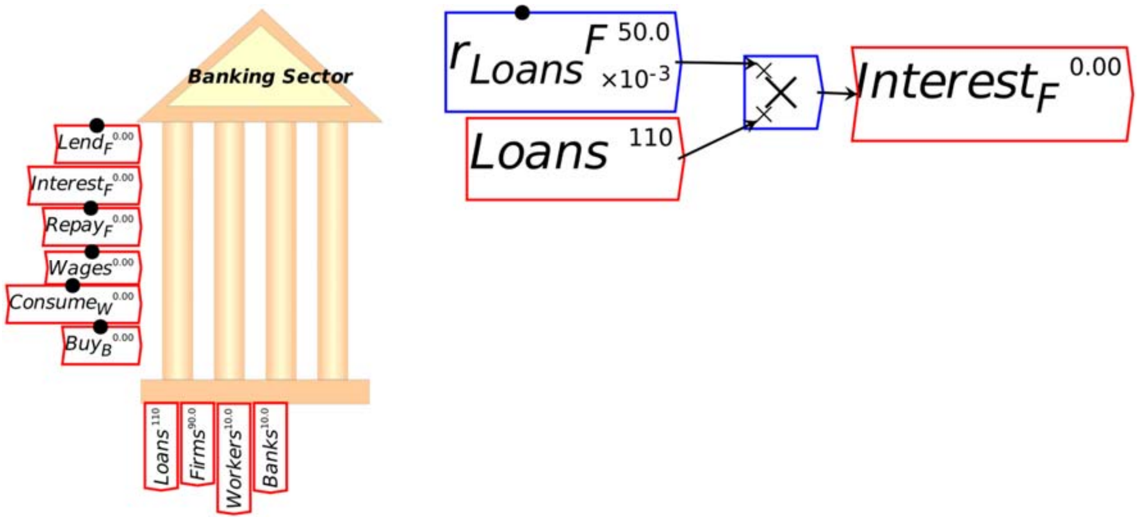
Figure 84: All the stock and flow variables from a Godley Table copied to the canvas



How you define a model is up to you, but you can only define it using the variables and stocks you currently have, or transformations of them—so if you want to define investment flows in a model as a function of capacity utilization, for example, or the wage level as a function of the level of employment, then you need to add those variables to your system. Here I'll just demonstrate defining a model from the elements of the Godley Table itself.

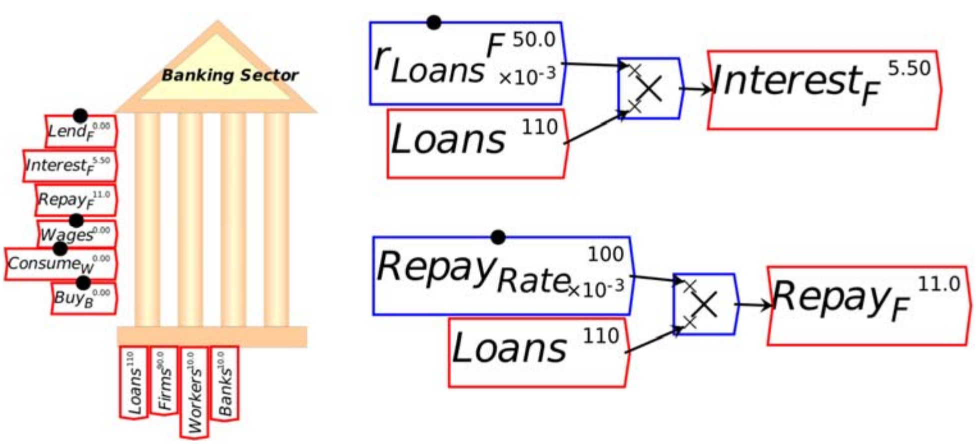
The simplest flow to define is $Interest_F$, which is the rate of interest on loans multiplied by the current level of Loans. In Figure 85 I add a new parameter, $r_{Loans}{}^F$, for the rate of interest on loans to firms, multiply that by the stock variable Loans, and this defines $Interest_F$ (I have also shrunk the Godley Table using its bounding box arrows).

Figure 85: Defining interest payments (without hitting the "Recalc" button before exporting the figure)



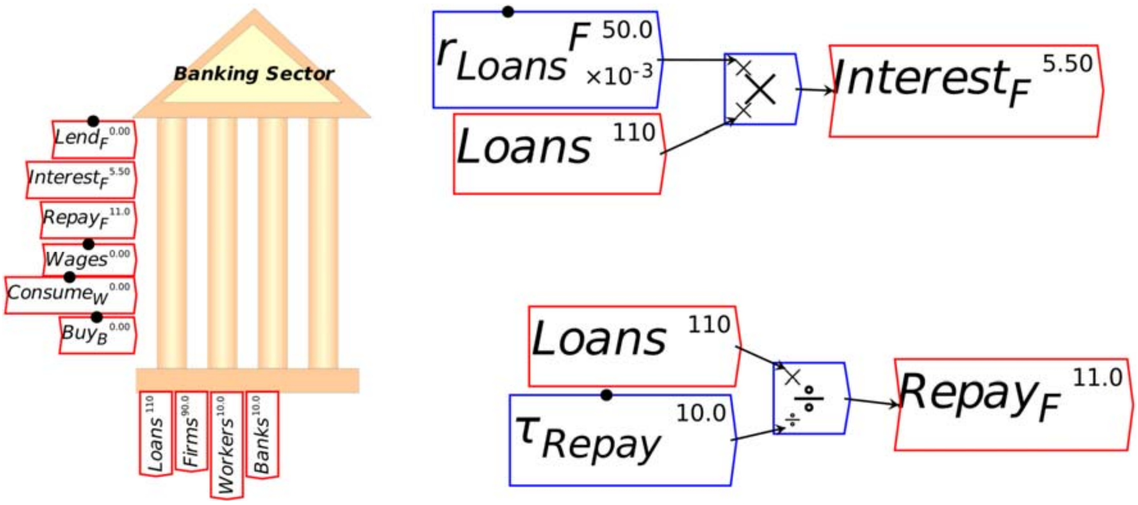
To define the other flows, I use the mechanism I explained earlier in these models—the first-order time lag. As well as being useful to define a lagged variable, such as lagged inflation as a function of the actual rate of inflation, it can be used to explain a flow as a time-based response to a stock. For example, the level of repayment of existing debt will be roughly proportional to the current level of loans—it can be a large proportion or a small one, but there will be some proportionality. This could be done using a simple scalar—say, for example 10%, so that 10% of loans are repaid every year, as in Figure 86.

Figure 86: Repayment modelled using a repayment rate (after hitting the "Recalc" button)



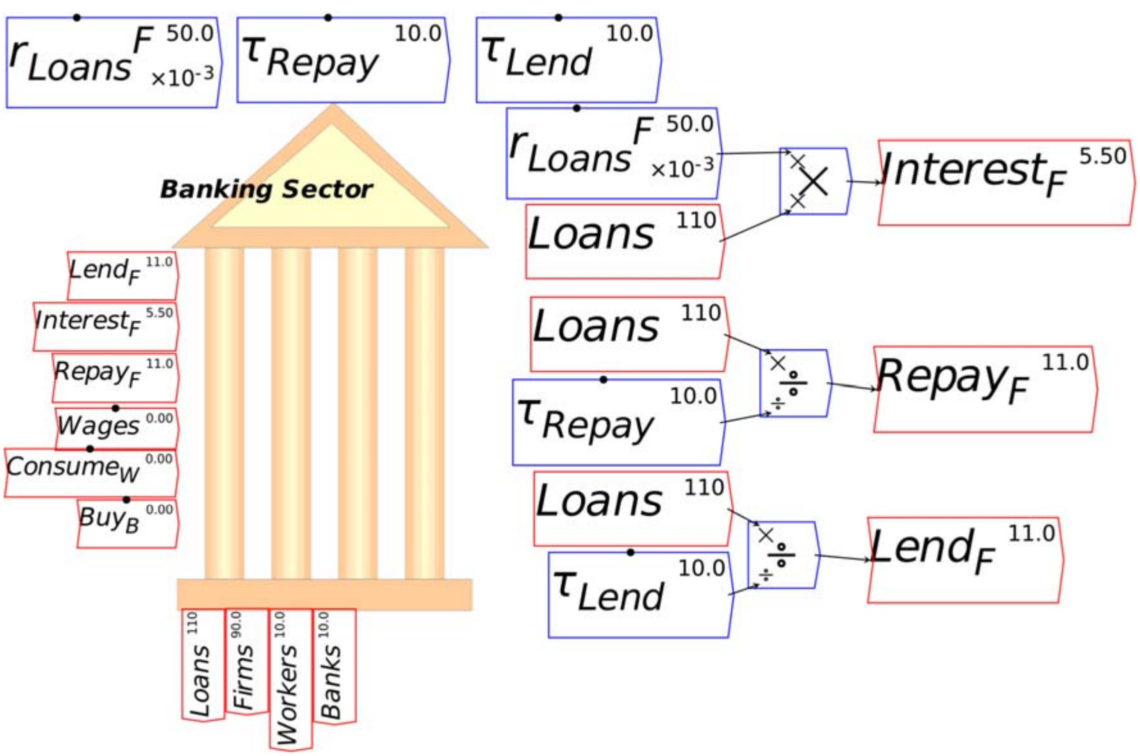
I prefer to use a time constant instead, because then the value of the time constant is easily understood in terms of the time dimension of the model. If I give that time constant a value of 10, as in Figure 87, then I get the same numerical result as in Figure 86, but the number 10 stands for the number of years it would take to reduce the debt to zero if this rate of repayment were sustained.

Figure 87: Using a time constant instead for the rate of debt repayment



A similar definition for the rate of new lending tells you how long this rate of lending would take to double the debt—see Figure 88, where I've also copied the model's parameters to the top of the canvas, where they can be varied easily during a simulation.

Figure 88: Lending also with a time constant, plus copying parameters into a "control panel"



This simple model, without any physical output or price component, needs a definition of GDP as well in terms of financial flows only. We can't just add up consumption and investment using this model, because the only non-financial flows in it are *Wages* as the income of workers, and consumption by workers ($Consume_W$) and bankers (Buy_B): there's no definition of profit as the income of capitalists, nor is there any definition of investment—which involves capitalists buying from other capitalists, and is therefore subsumed within the single column for the Firm sector. So, given how this simple model is constructed, investment—as well as profit, and consumption by capitalists—doesn't appear at all, and it therefore has to be inferred as a residual.

My approach in these simple models (without an integrated model of the physical economy as well) has been to take a leaf out of *Das Kapital*—specifically, Volume II, Chapter 7, "The Turnover Time and the Number of Turnovers":

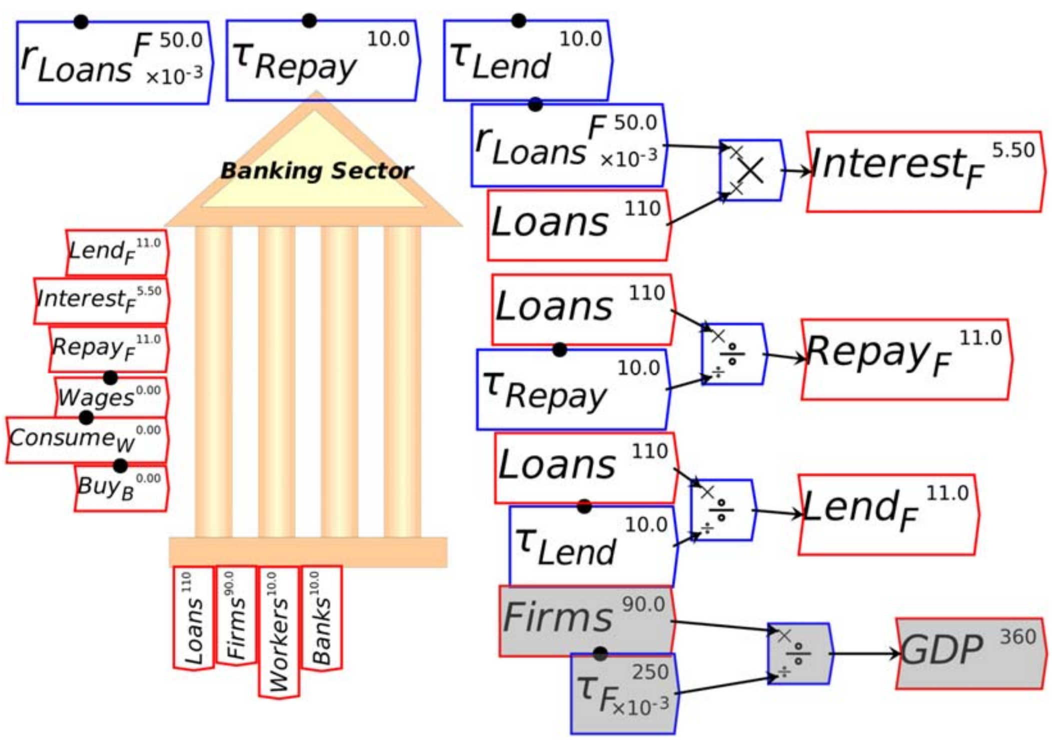
> We have seen that the entire time of turnover of a given capital is equal to the sum of its time of circulation and its time of production. It is the period of time from the moment of the advance of capital-value in a definite form to the return of the functioning capital-value in the same form. (Marx and Engels 1885)

In this model, GDP is derived from the amount of money in the Firm sector, and its turnover rate:

$$GDP = \frac{Firms}{\tau_F} \tag{1.31}$$

This equation, in flowchart form, is highlighted in grey in Figure 89.

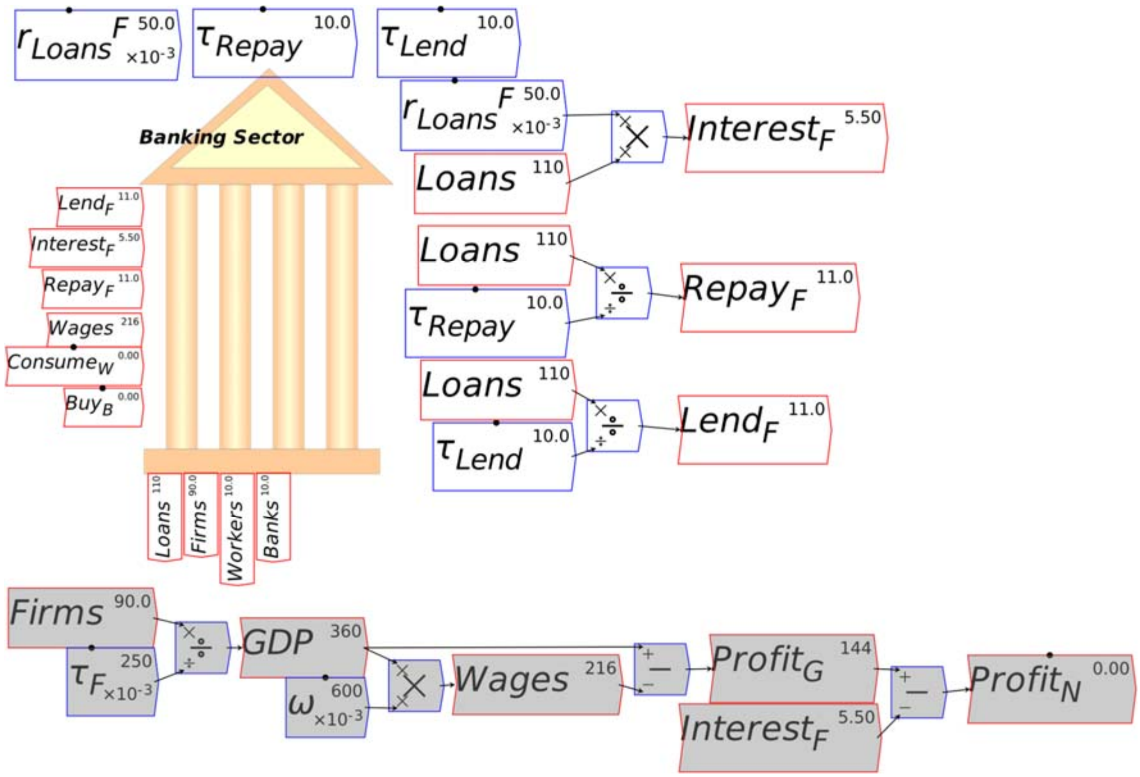
Figure 89: GDP as the turnover of money per year in the firm sector



With GDP defined this way, inter-firm spending (which in this simple model, includes investment and consumption by capitalists, since I haven't separated out capitalists as a different financial entity to

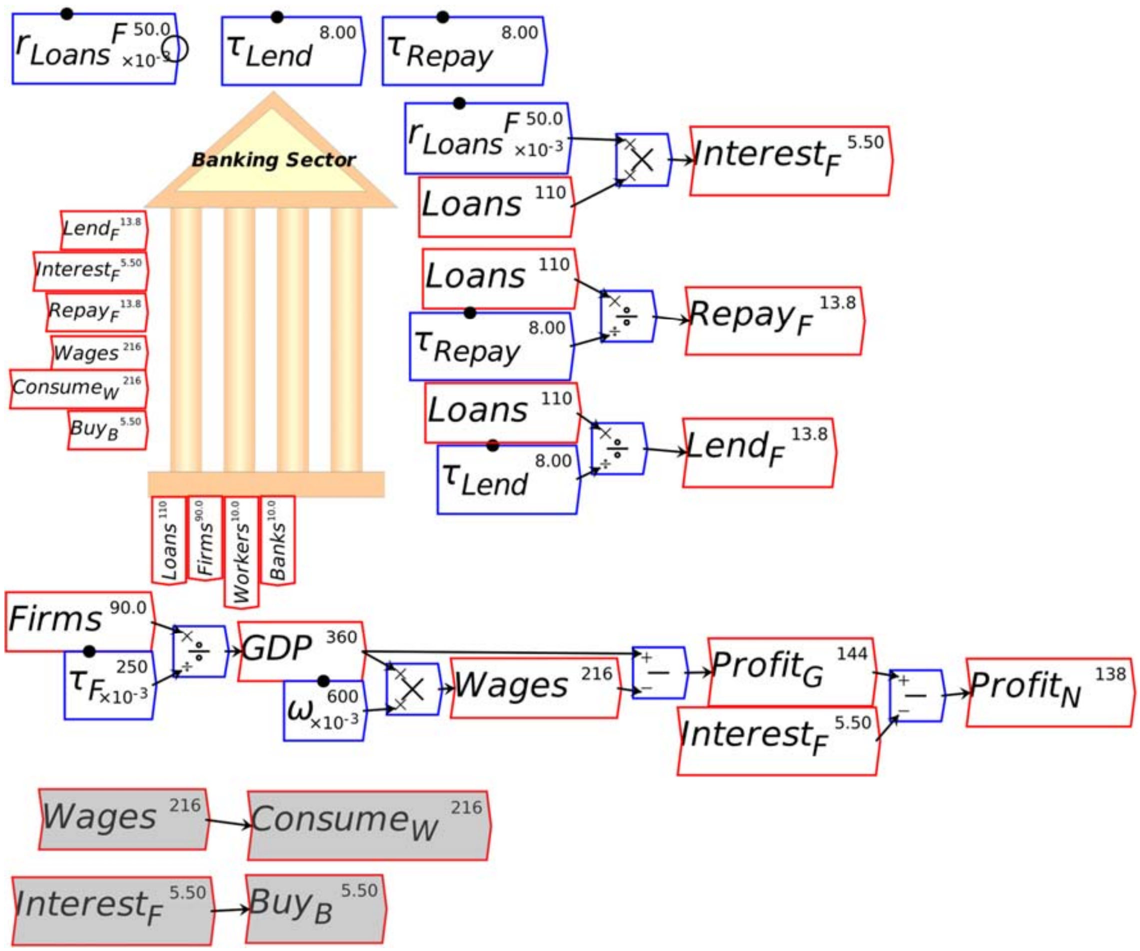
Firms in this model) is the residual between GDP and wages. This residual includes profits, dividends, etc.—again, aspects of a capitalist system that you could explicitly model in a more elaborate model.

I also use a very simple assumption to determine wages: I make the distribution of income a parameter, so that $\omega\%$ of GDP goes to workers and $(1-\omega)\%$ goes to capitalists as gross profit, with this minus interest payments being net profit (in an integrated physical-monetary economy model, wage determination would be driven by bargaining power, as in the Goodwin model).



This leaves consumption by workers and bankers to be defined. You could, of course, make a "Kaleckian" assumption that workers simply consume their wages, and equivalently, that bankers spend their interest income. For the sake of illustration, I'll do that first (in Figure 90), and compare the results to a model with time constants, based on the amount of money in the workers' and bankers' accounts.

Figure 90: "Kaleckian" assumptions on consumption by workers and bankers



This Kaleckian assumption effectively making workers and bankers passive parts of the system, rather than active parts: whatever they receive as Wages ($216/Year) or Interest ($5.50/Year) goes out as consumption. On the other hand, if you base spending upon the amounts in their bank accounts, divided by time constants that reflect that workers are living close to “hand to mouth” whereas bankers have large buffers compared to their spending, then you have a small time constant for Workers and a large time constant for Bankers. Changing the distribution of income between workers and bankers will therefore change the amount turning up in the Firms account, thus changing GDP.

Figure 91: Consumption based on time constants

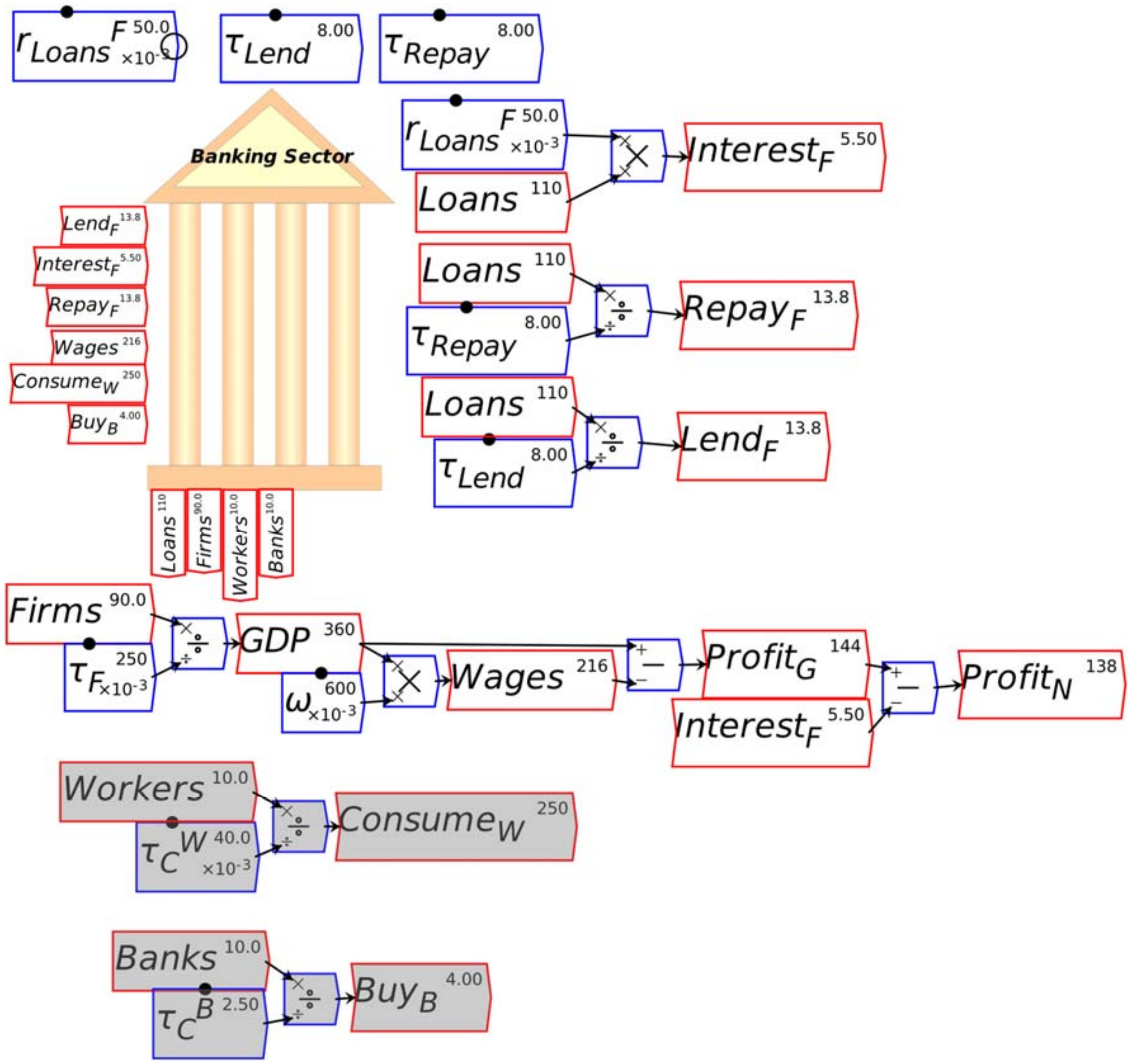
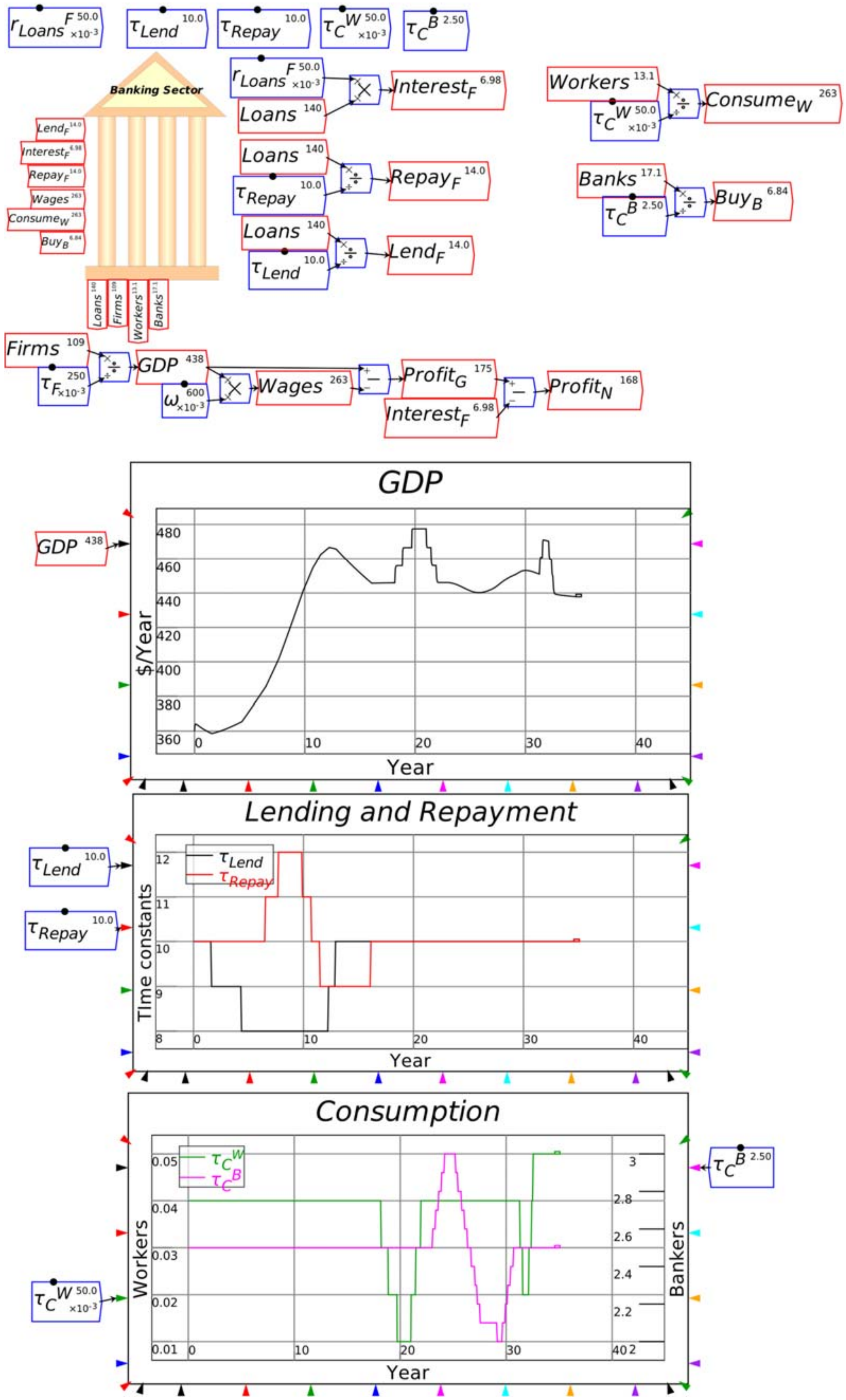


Figure 91 also shows the advantages of dividing by a time constant to define a flow, rather than multiplying by an equivalent constant. The value for the time constant tells you how long, in fractions of a year, that the social class could consume before running out of money: 1/25th of a year for Workers (otherwise known as a fortnight), 2.5 years for Bankers. The size of the time constant is readily interpreted as an indicator of the relative income and wealth of the two social classes.

Figure 92 illustrates the impact of varying the time constants in the model. If the time constant for lending is smaller than that for repayment, then there is net debt and money creation by the banking sector, and GDP rises. If repayment is faster than new lending, then there is net money destruction and GDP falls. Changes in the workers' time constant have more impact than changing that for bankers—workers spend their accounts much more quickly than bankers, because they have to. So though their bank accounts are the same size in Figure 92, workers generate far more spending than do bankers ($250/Year versus $4/Year).

Figure 92: Varying time constants for lending, repayment, worker & banker consumption



By using the "Editor Mode" display of the Godley Table, and choosing "Godley Table Show Values" from the Preferences form of the Options main menu (see Figure 93), you can see the amounts passing between the accounts in this model in the Godley Table itself—see Figure 94.

Figure 93: The Preferences form from the Options menu

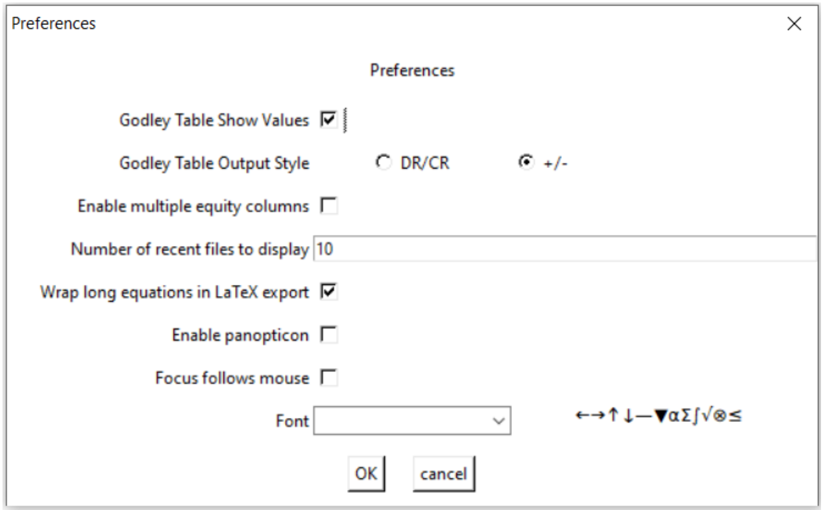
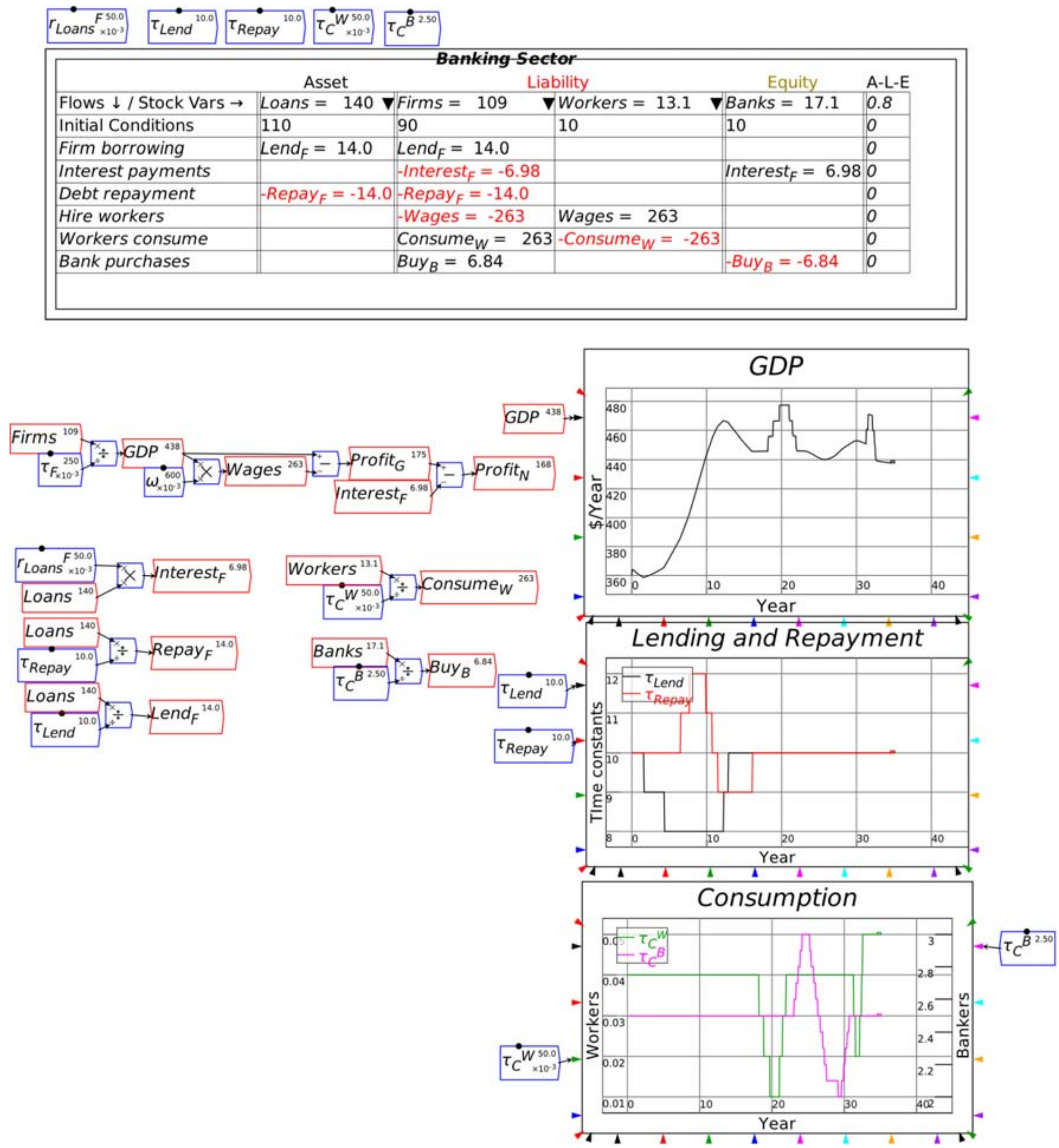


Figure 94: Editor Mode display with numerical values shown in the Godley Table



Flows ↓ / Stock Vars →	Loans = 140	Firms = 109	Workers = 13.1	Banks = 17.1	A-L-E 0.8
	Asset	Liability		Equity	
Initial Conditions	110	90	10	10	0
Firm borrowing	$Lend_F$ = 14.0	$Lend_F$ = 14.0			0
Interest payments		-$Interest_F$ = -6.98		$Interest_F$ = 6.98	0
Debt repayment	-$Repay_F$ = -14.0	-$Repay_F$ = -14.0			0
Hire workers		-Wages = -263	Wages = 263		0
Workers consume		$Consume_W$ = 263	-$Consume_W$ = -263		0
Bank purchases		Buy_B = 6.84		-Buy_B = -6.84	0

The last step in putting together a comprehensive model is to show the financial system from the point of view of Firms and Workers, as well as from the Banking sector. To do this, insert two more Godley Tables on the canvas, label one *Firms* and the other *Workers*, and then use the down-arrow on the columns (▼) to search for Liabilities that haven't yet been defined as Assets, and vice versa. The Firm sector has one Asset—its deposit account *Firms*—and one Liability—*Loans*. When these are added to its Godley Table, *Minsky* automatically fills in the rows where there are already operations on both accounts (*Lend*$_F$ and *Repay*$_F$), while leaving those where there is an operation on one but not the other unbalanced: the sum of $A-L-E$ shows the flow that hasn't yet been allocated to an account—see Figure 95.

Figure 95: The Firm sector's Godley Table with Assets & Liabilities added

		Asset	Liability	Equity	A-L-E
	Flows ↓ / Stock Vars →	*Firms*	*Loans*		-20
+	Initial Conditions	90	110		-20
+ − ↓	Bank purchases	Buy_B			Buy_B
+ − ↑ ↓	Workers consume	$Consume_W$			$Consume_W$
+ − ↑ ↓	Interest payments	$-Interest_F$			$-Interest_F$
+ − ↑ ↓	Firm borrowing	$Lend_F$	$Lend_F$		0
+ − ↑ ↓	Debt repayment	$-Repay_F$	$-Repay_F$		0
+ − ↑	Hire workers	$-Wages$			$-Wages$

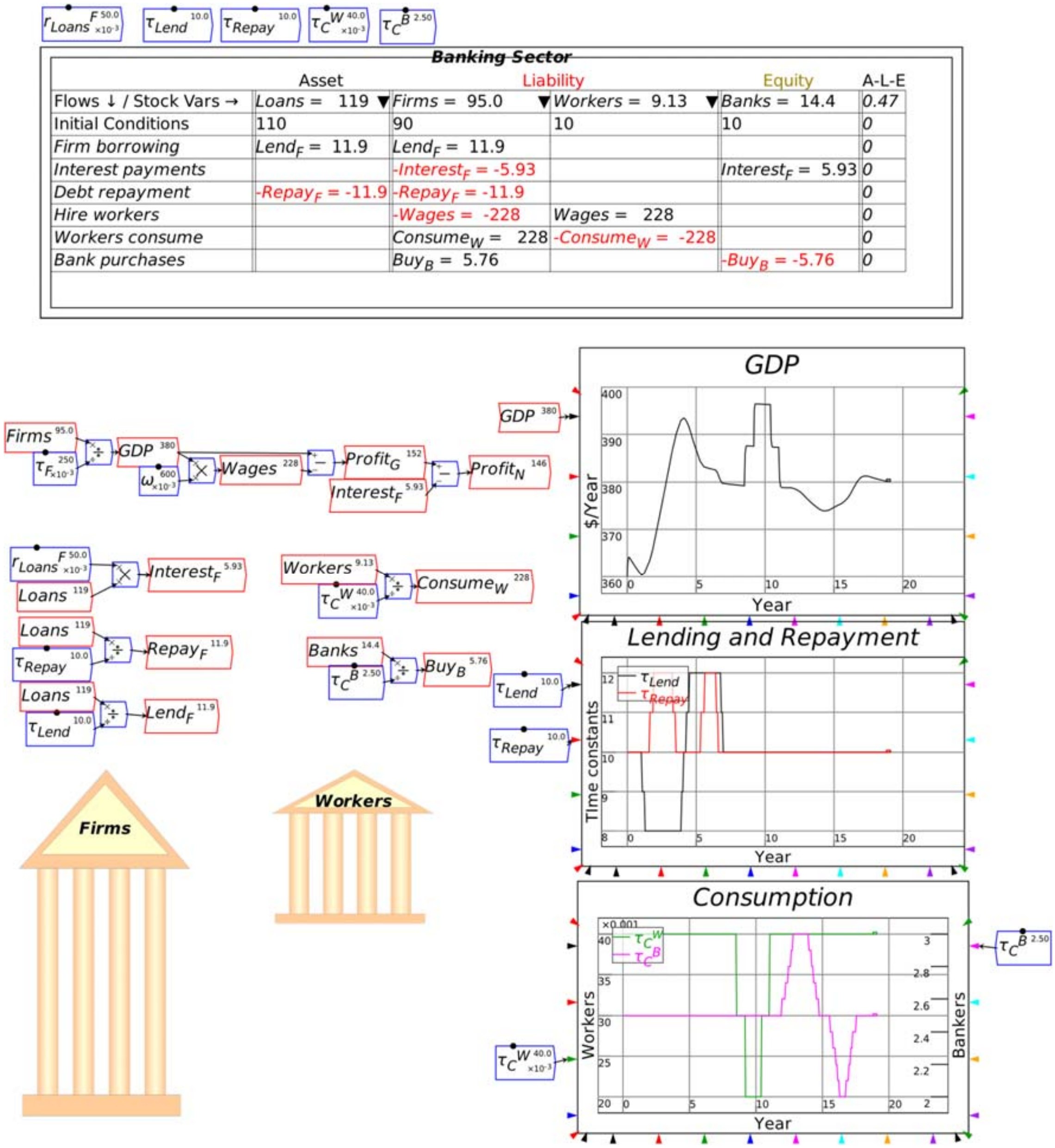
To fully specify the model, you need to define an Equity column for the Table. I used $Firm_E$ as the name for "Firm Equity" and made the matching entry needed so that $A - L - E = 0$ on every row—see Figure 96. In this simple model, all those entries go in the Equity column, but that isn't necessarily the case in a more complex model. You might, for example, have $Cash_W$ as an asset of the Workers, so that withdrawing money from the Banking sector reduced the Workers' deposit account (*Workers*) and increased their cash account $Cash_W$, without altering their Equity.

Figure 96: The model from the Firm sector's point of view

		Asset	Liability	Equity	A-L-E
	Flows ↓ / Stock Vars →	*Firms* = 109	*Loans* = 140	$Firms_E$	-31
+	Initial Conditions	90	110	-20	0
+ − ↓	Bank purchases	Buy_B = 6.84		Buy_B = 6.84	0
+ − ↑ ↓	Workers consume	$Consume_W$ = 263		$Consume_W$ = 263	0
+ − ↑ ↓	Interest payments	$-Interest_F$ = -6.98		$-Interest_F$ = -6.98	0
+ − ↑ ↓	Firm borrowing	$Lend_F$ = 14.0	$Lend_F$ = 14.0		0
+ − ↑ ↓	Debt repayment	$-Repay_F$ = -14.0	$-Repay_F$ = -14.0		0
+ − ↑	Hire workers	$-Wages$ = -263		$-Wages$ = -263	0

Figure 97 shows the complete model, with all accounts recorded in the respective Godley Tables.

Figure 97: The complete model, which is still very simple, with financial dynamics only



I hope that's enough background to enable you to use Godley Tables in your own modelling. Now let's use Minsky to show why, when it comes to money, Paul Krugman doesn't know what he's talking about.

7.4 A *Keen Rant*: Revisiting the Keen-Krugman Debate using Minsky

Almost a decade ago now, Paul Krugman gave me a birthday present by citing me in his New York Times column on March 27th, 2012.[16]

[16] My birthday is March 28th, and since I lived in Sydney then, I first saw his column when I was alerted to it on my birthday by a Facebook message—see **Error! Reference source not found.**.

Entitled "Minsky and Methodology (Wonkish)", the post began as a present should—it was very nicely wrapped:

Figure 98: The opening to Krugman's first post


The Opinion Pages

The Conscience of a Liberal

PAUL KRUGMAN

Minsky and Methodology (Wonkish)

MARCH 27, 2012 12:09 PM 134

Steve Keen has a new post up (with a link to a new paper) about Minksyan (Minskyite?) economics, and how people like me get it wrong. Good for him; debates like this are always productive, and I wish domestic responsibilities weren't keeping me from going to the Berlin conference.

Unfortunately, once I opened the present, it was all downhill. He wrote a series of seven posts,[17] ending with "Oh My, Steve Keen Edition", whose final line was "Nick [Rowe] uses a four-letter word to describe this; I can't, because this is the Times."

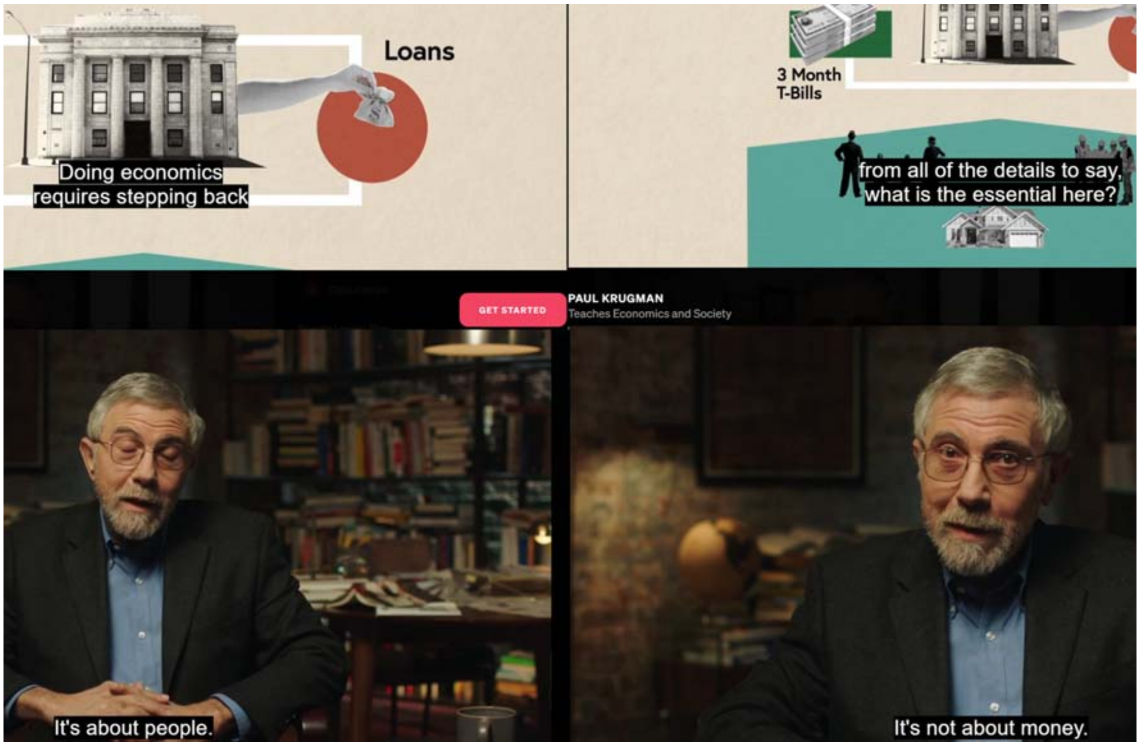
In between the nice introduction and the derogatory denouement, there was something that is far too rare in economics today, a "debate" between opposing schools of thought in economics over a fundamental issue. I put "debate" in inverted commas because we never spoke, and while I read his posts, he didn't read mine.[18] But the juxtaposition of opposing views was something that rarely happens in economics, so in that sense, it qualifies as a debate.

The topic of the debate was "Do banks, debt and money matter in macroeconomics?" Krugman's position was "No" back then, and it's still "No" today: in the 2021 promotional video for his Masterclass on economics, he says "It's about people. It's not about money".

[17] Krugman's posts on me https://krugman.blogs.nytimes.com/2012/03/27/minksy-and-methodology-wonkish/; https://krugman.blogs.nytimes.com/2012/03/27/banking-mysticism/; https://krugman.blogs.nytimes.com/2012/03/30/banking-mysticism-continued/; https://krugman.blogs.nytimes.com/2012/04/01/tobin-brainard-1963/; https://krugman.blogs.nytimes.com/2012/04/02/things-i-should-not-be-wasting-time-on/; https://krugman.blogs.nytimes.com/2012/04/02/a-teachable-money-moment/; https://krugman.blogs.nytimes.com/2012/04/02/oh-my-steve-keen-edition/.

[18] My posts on Krugman http://www.debtdeflation.com/blogs/2012/03/29/krugman-on-or-maybe-off-keen/; http://www.debtdeflation.com/blogs/2012/04/02/blog-observations-on-krugman/; http://www.debtdeflation.com/blogs/2012/04/03/oh-my-paul-krugman/; http://www.debtdeflation.com/blogs/2012/04/04/krugman-apologises/; http://www.debtdeflation.com/blogs/2012/04/09/capital-account-interview-on-the-keen-krugman-brawl/.

Figure 99: Screenshots from Krugman's promotional video



Yes it is about money, as I'll now explain using Minsky.[19] Firstly, here are the substantive parts of Krugman's first post in 2012, where he set out very well the basic assumptions of the "Loanable Funds" model of banking. I've highlighted the key passages in italics:

> I always try to find the simplest representation I can of whatever story I'm trying to tell about the economy. The goal, in particular, is to identify which assumptions are really crucial — and in so doing to catch yourself when you're making implicit assumptions that can't stand clear scrutiny.
>
> Keen doesn't seem to be doing that. His paper contains a number of assertions about what is crucial, without much explanation of why these things are crucial. And I guess I just don't see it.
>
> *In particular, he asserts that putting banks in the story is essential. Now, I'm all for including the banking sector in stories where it's relevant; but why is it so crucial to a story about debt and leverage?*
>
> Keen says that it's because once you include banks, lending increases the money supply. OK, but why does that matter? He seems to assume that aggregate demand can't increase unless the money supply rises, but that's only true if the velocity of money is fixed; so have we suddenly become strict monetarists while I

[19] I received the INET grant that enabled me to create Minsky in September 2011, so Minsky was in its infancy back then, and in particular, we hadn't yet implemented Godley Tables: all Minsky could do back then was model simple ordinary differential equations using the flowchart paradigm of conventional system dynamics programs—See https://www.ineteconomics.org/research/grants/extending-macroeconomics-and-developing-a-dynamic-monetary-simulation-tool. So I couldn't use Minsky to illustrate my argument back then (the original version is still accessible from https://sourceforge.net/projects/minsky/files/Windows%20Binaries/).

> wasn't looking? In the kind of model Gauti and I use, lending very much can and does increase aggregate demand, so what is the problem?
>
> Keen then goes on to assert that lending is, by definition (at least as I understand it), an addition to aggregate demand. I guess I don't get that at all. *If I decide to cut back on my spending and stash the funds in a bank, which lends them out to someone else, this doesn't have to represent a net increase in demand.* Yes, in some (many) cases lending is associated with higher demand, because resources are being transferred to people with a higher propensity to spend; but Keen seems to be saying something else, and I'm not sure what. *I think it has something to do with the notion that creating money = creating demand, but again that isn't right in any model I understand*. (Krugman 2012b. Emphasis added)

The key technical issue here is *what do banks do*? According to Krugman, banks take in deposits from some customers, and lend them out to others:

> If I decide to cut back on my spending and stash the funds in a bank, which lends them out to someone else, this doesn't have to represent a net increase in demand…

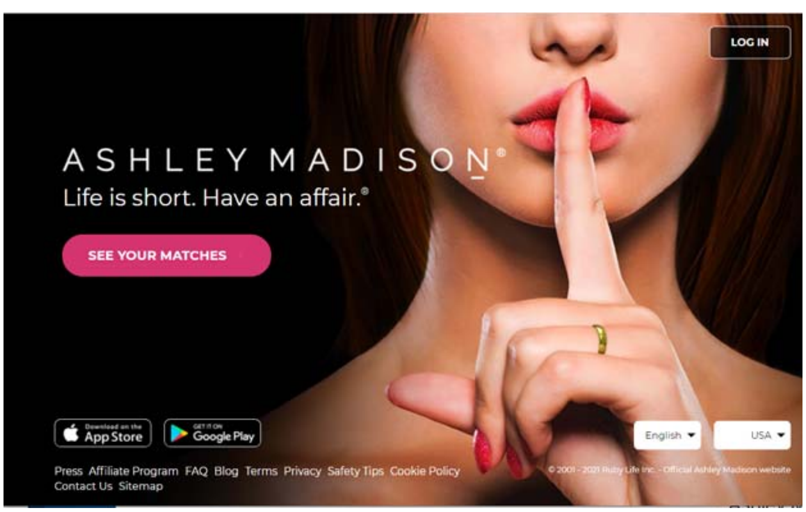
This is not, of course, what banks actually do, as we now can state with the authority of the Bank of England:

> This article explains how the majority of money in the modern economy is created by commercial banks making loans. Money creation in practice differs from some popular misconceptions — *banks do not act simply as intermediaries, lending out deposits that savers place with them*, and nor do they 'multiply up' central bank money to create new loans and deposits. (McLeay et al. 2014. Emphasis added)

But it's worth putting Krugman's misconception into Minsky to show that, if Neoclassicals were right about what banks do, then they would also be right to ignore banks in their macroeconomic models.

Fundamentally, as the Bank of England notes, Neoclassicals believe that banks act "simply as intermediaries". I call it the "Ashley Madison theory of banking"—see Figure 100 if you haven't heard of Ashley Madison before.

Figure 100: Ashley-Madison as an intermediary



Ashley Madison doesn't actually provide sex: instead, it lets men who want sex find women who want sex, and charges a fee for the introduction service. Similarly, in the Neoclassical mind, banks don't actually provide money: instead, they let people with more money than they need at the moment (savers) meet people with less money than they need (borrowers). The savers lend money to the borrowers, and the bank charges a fee for the introduction. No money is created because of the new debt—just ask Paul Krugman:

> Think of it this way: when debt is rising, *it's not the economy as a whole borrowing more money*. It is, rather, a case of less patient people—people who for whatever reason want to spend sooner rather than later—borrowing from more patient people. (Krugman 2012a, pp. 146-147. Emphasis added)[20]

The easiest way to model what Krugman—and all Neoclassicals, with almost the sole exception of the Bank of England economist Michael Kumhof—think banks do is to model the *literal* case of savers lending money directly to borrowers, through the deposit facilities provided by banks. Figure 101 shows the banking sector's view of that person-to-person case, where the "less patient people" are factories, and the "more patient people" are rentiers who both invest in and lend to factories. The bank has only one asset—Reserves, which match the sum of the deposits of Impatient people, Patient people and Workers, plus the Banking sector's Equity.[21] The first four rows show financial operations—lending, paying interest, repaying debt, and paying the bank's "intermediation" fee. Then we have paying wages to workers, which enables production; dividend payments to shareholders (those "patient people" again), and finally consumption of the output of the factories managed by the "impatient people" by "patient people", workers and bankers.

Figure 101: Patient lends to impatient via the banking system

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Impatient*	*Patient*	*Workers*	*Bank$_E$*	*0*
Initial Conditions	200	40	140	2	18	*0*
Lend Money		*Lend*	*-Lend*			*0*
Pay Interest		*-Interest*	*Interest*			*0*
Repay Loans		*-Repay*	*Repay*			*0*
Pay Bank Fee		*-Fee*			*Fee*	*0*
Pay Wages		*-Wages*		*Wages*		*0*
Pay Dividends		*-Dividends*	*Dividends*			*0*
Capitalists consume		*Consume$_C$*	*-Consume$_C$*			*0*
Workers consume		*Consume$_W$*		*-Consume$_W$*		*0*
Bankers consume		*Consume$_B$*			*-Consume$_B$*	*0*

Notice that while lending shows up in the banking sector's Godley Table, the actual debt owed doesn't, because in this model, the debt is not an asset of the banking sector: instead, it's an asset for the "Patient" people and a liability for the "Impatient" people. So to see the debt itself (which I labelled as "Loans" in this model), you have to create additional tables for Patient, Impatient and Workers. Figure 102 shows all the Godley Tables in this model—as noted in the chapter on Godley Tables, all

[20] I will never cease to be amused by Neoclassical protestations that their approach is "value-free", while at the same time they use pejorative terms all the time: "perfect" competition, Pareto "optimal", etc. And here, Krugman gives us "patient" versus "impatient" people…

[21] In most models, for the same of simplicity, I treat Bank Equity as short-term "at call" funds, ignoring that banks have long-term equity (which includes long-term debt). But Minsky supports multiple Equity columns, so if you wish you can model Bank Equity as including both at-call and long-term components: just choose "Enable multiple Equity columns" from the Preferences form on the Options menu.

you have to do is create a new table and then use the down-arrow on the columns (▼) to search for Liabilities that haven't yet been defined as Assets, and vice versa.

Figure 102: All the Godley Tables for "Patient to Impatient" lending

Banking Sector

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves* ▼	*Impatient* ▼	*Patient* ▼	*Workers* ▼	*Bank$_E$*	*0*
Initial Conditions	200	40	140	2	18	*0*
Lend Money		*Lend*	*-Lend*			*0*
Pay Interest		*-Interest*	*Interest*			*0*
Repay Loans		*-Repay*	*Repay*			*0*
Pay Bank Fee		*-Fee*			*Fee*	*0*
Pay Wages		*-Wages*		*Wages*		*0*
Pay Dividends		*-Dividends*	*Dividends*			*0*
Capitalists consume		*Consume$_C$*	*-Consume$_C$*			*0*
Workers consume		*Consume$_W$*		*-Consume$_W$*		*0*
Bankers consume		*Consume$_B$*			*-Consume$_B$*	*0*

Patient

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Patient* ▼	*Loans* ▼	▼	*Patient$_E$*	*0*
Initial Conditions	140	0		140	*0*
Pay Interest	*Interest*			*Interest*	*0*
Lend Money	*-Lend*	*Lend*			*0*
Repay Loans	*Repay*	*-Repay*			*0*
Pay Dividends	*Dividends*			*Dividends*	*0*
Capitalists consume	*-Consume$_C$*			*-Consume$_C$*	*0*

Impatient

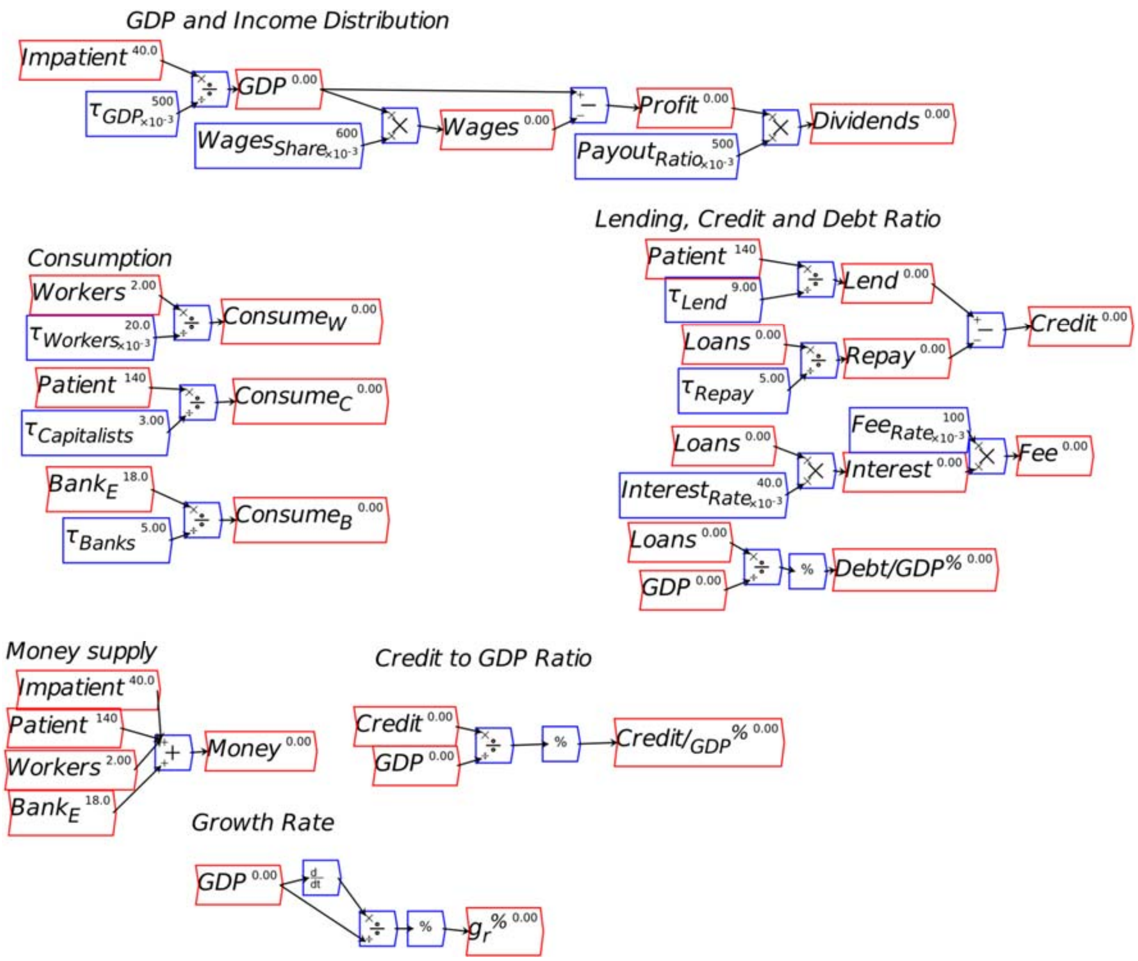
	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Impatient* ▼	*Loans* ▼	*Impatient$_E$*	*0*
Initial Conditions	40	0	40	*0*
Lend Money	*Lend*	*Lend*		*0*
Pay Interest	*-Interest*		*-Interest*	*0*
Repay Loans	*-Repay*	*-Repay*		*0*
Pay Dividends	*-Dividends*		*-Dividends*	*0*
Capitalists consume	*Consume$_C$*		*Consume$_C$*	*0*
Pay Bank Fee	*-Fee*		*-Fee*	*0*
Bankers consume	*Consume$_B$*		*Consume$_B$*	*0*
Pay Wages	*-Wages*		*-Wages*	*0*
Workers consume	*Consume$_W$*		*Consume$_W$*	*0*

Workers

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	*Workers* ▼	▼	*Workers$_E$*	*0*
Initial Conditions	2		2	*0*
Pay Wages	*Wages*		*Wages*	*0*
Workers consume	*-Consume$_W$*		*-Consume$_W$*	*0*

To complete the model, I made very similar definitions to the model developed in Chapter 7—see Figure 103. The main differences are that Krugman's silly "Impatient" term takes the place of the Firm sector there.

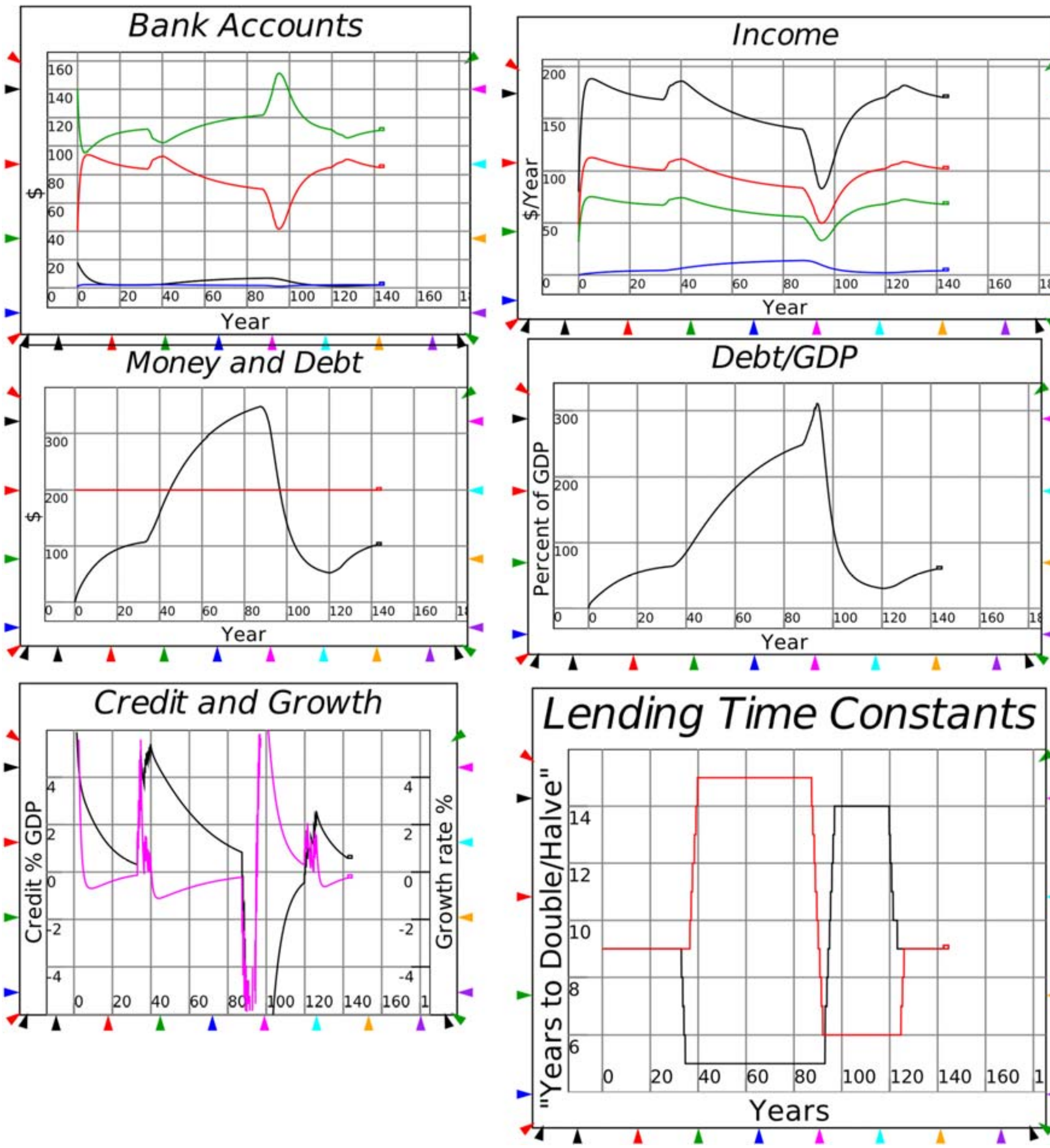
Figure 103: The definitions of flows in the Loanable Funds model



With those definitions made, the model can be run, and the parameters that control lending and repayment varied while the model runs, to see the impact of higher and lower levels of credit and debt on this toy economy. While there clearly is some impact, some things don't change: as Krugman put it himself, "when debt is rising, it's not the economy as a whole borrowing more money": changes in debt have no impact on the money supply. There are variations in GDP and incomes as a result of the variations in the time constants for lending and repayment, but they are both minor and transient. *If this model described the real world accurately*, then it would make sense to leave banks, debt and money out of macroeconomics: to cite Krugman once more, "I'm all for including the banking sector in stories where it's relevant; but why is it so crucial to a story about debt and leverage?"

To answer his question, we have to take account of the real-world situation that banks lend to non-banks, so that Loans are an asset of the Banking Sector, and not of "Patient People". Before I show how to do this, note one aspect of Figure 104: given the parameters in the model, a higher debt to GDP ratio is associated with a lower GDP—see the Income and Debt/GDP plots on the right hand side of Figure 104.

Figure 104: Dramatic changes in Debt/GDP, minor transient changes in GDP



To change this model so that Banks, rather than "Patient People", lend to "Impatient People", you have to:

- Shift the Loans column from Patient's Godley Table to the Bank's;
- Delete the financial operations on the Patient Godley Table: the first three rows for Interest payments, lending and repayment all go, leaving just two rows—receiving Dividends and consuming;
- Make room for a new Asset on the Banking Sector Godley Table by clicking on the green plus icon below the Asset label;
- Click on the  , which will show Loans as a Liability that hasn't yet been classified as an Asset (when you delete the Loans column from Patient's Godley Table, Loans remains in the model as a Liability of Impatient), and select Loans;

- Minsky then brings across the two operations that affect Loans, Lend and Repay;
- The Banking sector Godley Table will still show Interest as a transfer out of Impatient's account, but it doesn't go anywhere; Type "Interest" into the $Bank_E$ column, to show that interest payments increase the (at-call) equity of the banking sector.

That's it: strictly speaking we should also change how lending is determined, since the Loanable Funds model shows lending as being based on amount of money in Patient's deposit account, but this is enough to see if this simply structural change to the model—as opposed to a behavioural change—has any impact on the dynamics.

Figure 105: Altering the Godley Tables of Loanable Funds to fit the real world

Banking Sector

	Asset			Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves	Loans	Impatient	Patient	Workers	$Bank_E$	0
Initial Conditions	200	0	40	140	2	18	0
Lend Money		Lend	Lend				0
Pay Interest			-Interest			Interest	0
Repay Loans		-Repay	-Repay				0
Pay Bank Fee			-Fee			Fee	0
Pay Wages			-Wages		Wages		0
Pay Dividends			-Dividends	Dividends			0
Capitalists consume			$Consume_C$	$-Consume_C$			0
Workers consume			$Consume_W$		$-Consume_W$		0
Bankers consume			$Consume_B$			$-Consume_B$	0

Patient

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Patient		$Patient_E$	0
Initial Conditions	140		140	0
Pay Dividends	Dividends		Dividends	0
Capitalists consume	$-Consume_C$		$-Consume_C$	0

Impatient

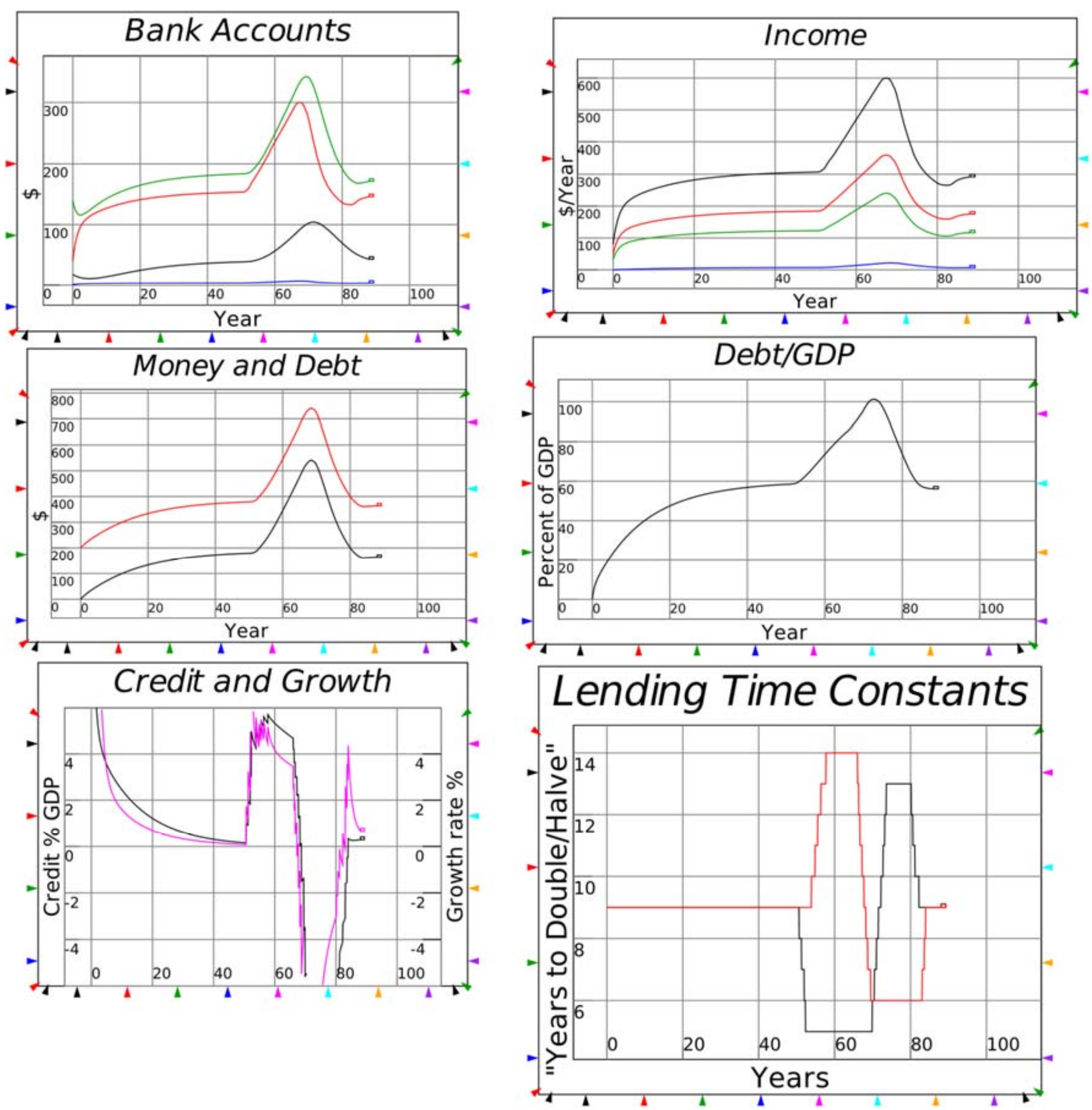
	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Impatient	Loans	$Impatient_E$	0
Initial Conditions	40	0	40	0
Lend Money	Lend	Lend		0
Pay Interest	-Interest		-Interest	0
Repay Loans	-Repay	-Repay		0
Pay Dividends	-Dividends		-Dividends	0
Capitalists consume	$Consume_C$		$Consume_C$	0
Pay Bank Fee	-Fee		-Fee	0
Bankers consume	$Consume_B$		$Consume_B$	0
Pay Wages	-Wages		-Wages	0
Workers consume	$Consume_W$		$Consume_W$	0

Workers

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Workers		$Workers_E$	0
Initial Conditions	2		2	0
Pay Wages	Wages		Wages	0
Workers consume	$-Consume_W$		$-Consume_W$	0

You bet it does: see Figure 106. Debt creates money, so the money supply rises when debt rises, and falls when it falls; a rising debt to GDP ratio is associated with a rising GDP (and the obverse for falling debt); credit growth, which was out of synch with GDP growth in the Loanable Funds model, now parallels—and in fact causes—the growth of GDP. We are in a completely different world to the Neoclassical model of loanable funds—and it happens to be the real world we actually inhabit. Loanable Funds is a misleading fantasy.

Figure 106: Dramatic changes in Debt/GDP, dramatic changes in GDP



So too are all the models that go with it—especially the model of "Fractional Reserve Banking". Here Mankiw's Macroeconomics textbook gives a good outline of the fantasy. It starts with banks as just warehouses for deposits

> We begin by imagining a world without banks. In such a world, all money takes the form of currency, and the quantity of money is simply the amount of currency that the public holds. For this discussion, suppose that there is $1,000 of currency in the economy.

> Now introduce banks. At first, suppose that banks accept deposits but do not make loans. The only purpose of the banks is to provide a safe place for depositors to keep their money.
>
> *The deposits that banks have received but have not lent out are called reserves.* Some reserves are held in the vaults of local banks throughout the country, but most are held at a central bank, such as the Federal Reserve. In our hypothetical economy, all deposits are held as reserves: banks simply accept deposits, place the money in reserve, and leave the money there until the depositor makes a withdrawal or writes a check against the balance. This system is called 100–percent–reserve banking. (Mankiw 2016, p. 89. Emphasis added)

Mankiw displays this using a T-Account:

Figure 107: Mankiw's model of Full Reserve Banking

Firstbank's Balance Sheet			
Assets		Liabilities	
Reserves	$1,000	Deposits	$1,000

When Mankiw introduces lending, it is *lending from reserves*:

> Now imagine that banks start to use some of their deposits to make loans… The banks must keep some reserves on hand so that reserves are available whenever depositors want to make withdrawals. But as long as the amount of new deposits approximately equals the amount of withdrawals, a bank need not keep all its deposits in reserve. Thus, bankers have an incentive to make loans. When they do so, we have fractional–reserve banking, a system under which banks keep only a fraction of their deposits in reserve. (Mankiw 2016, pp. 89-90)

Mankiw then shows the bank as lending out 80% of its reserves:

Figure 108: Mankiw's model of Fractional Reserve Lending

Firstbank's Balance Sheet			
Assets		Liabilities	
Reserves	$200	Deposits	$1,000
Loans	$800		

Let's compare this model of banks to what banks actually do, in Minsky—see Figure 109.

Figure 109: Comparing actual banking with the Fractional Reserve Banking Model

First Bank

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Reserves_{FB}$ ▼	$Loans_{FB}$ ▼	$Deposits_{FB}$ ▼		*0*
Initial Conditions	1000	0	1000		*0*
How banks actually lend		*Lend*	*Lend*		*0*
Mankiw's Lend from Reserves	$-Lend_{Reserves}$	$Lend_{Reserves}$			*0*

The first line shows what banks actually do to make a loan: they add an amount to the borrower's deposit account, and simultaneously record a debt by the borrower to the bank for precisely the same amount. Lending creates deposits directly, which is creating money directly—there's no need for the iterative process alleged in the Fractional Reserve Banking model.

The second line shows the first stage of Fractional Reserve Banking model, but it is clearly incomplete: it shows a transfer of Assets from Reserves to Loans, but where is the money for the borrower?

The only way to show the loan actually giving money to the borrower is if the loan is in cash: the borrower walks out of the bank with a debt to the bank as shown in Figure 109, and an equivalent amount of cash, as shown in Figure 110.

Figure 110: Completing the first round of Fractional Reserve Banking

First Bank

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Reserves_{FB}$	$Loans_{FB}$	$Deposits_{FB}$		0
Initial Conditions	1000	0	1000		0
Mankiw's Lend from Reserves	$-Lend_{Reserves}$	$Lend_{Reserves}$			0

Borrower 1

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Cash_1$	$Loans_{FB}$		0
Initial Conditions	0	0		0
Mankiw's Lend from Reserves	$Lend_{Reserves}$	$Lend_{Reserves}$		0

This alone is enough reason to reject the model: it's very easy to say "use some of their deposits to make loans" as Mankiw does, but when one models what that means in strict double-entry format, lending from reserves only works if *all* loans are in cash.[22] In the real world, almost all loans are made by crediting a deposit account.[23]

So why do Neoclassicals stick with an unrealistic and complicated model, in place of a realistic and simpler one? Because with the more complicated model, they can ignore banks and money and debt in their macroeconomics, and claim that the money supply is controlled by government policy—government reserve creation times the "money multiplier"—rather than determined by bank lending. In part, this is ideology disguised as science, but it's also the standard reaction of a discipline to a discovery that contradicts a core belief. As the physicist who discovered quantum mechanics put it:

> "a new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." (Planck 1949, pp. 33-34)

Krugman's reaction to the Bank of England report that rejected these textbook models is par for the course here. The Bank of England paper said nothing that hadn't been said by many non-mainstream economists in the previous five decades (Moore 1979, 1988a; Graziani 1989; Holmes 1969), but it said it with the authority of a body that Neoclassical economists could not ignore:

> *The reality of how money is created today differs from the description found in some economics textbooks*:
>
> - Rather than banks receiving deposits when households save and then lending them out, *bank lending creates deposits*.

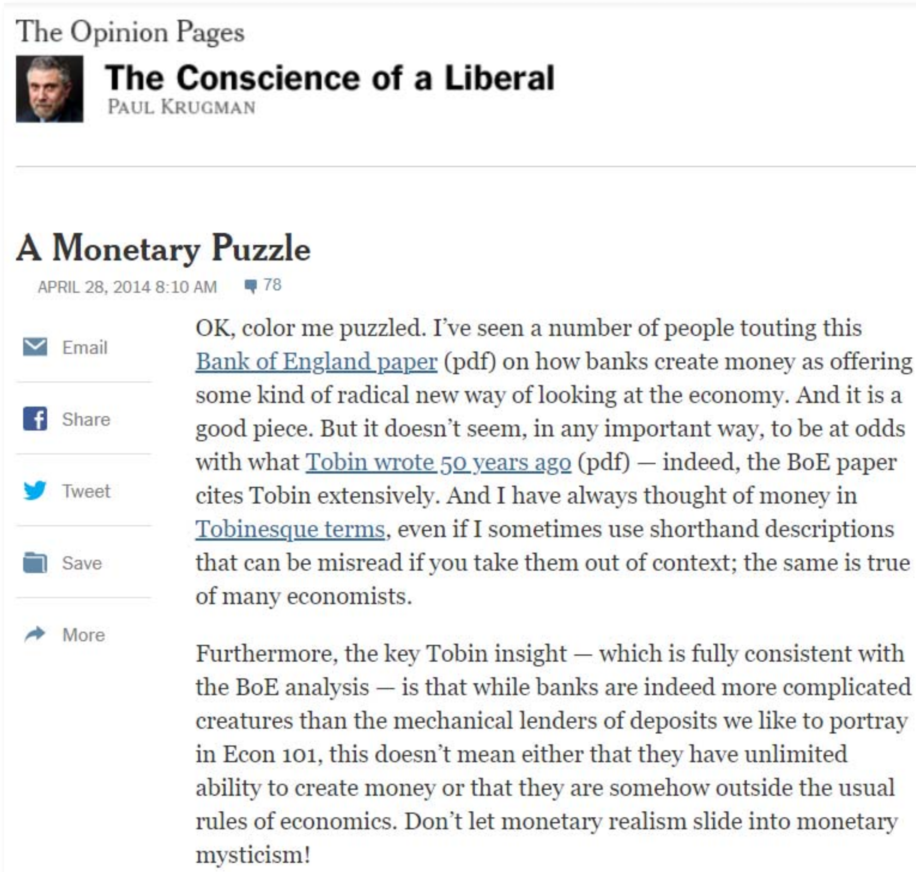
[22] Or some other negotiable instrument like a bank cheque.

[23] It doesn't have to credit the depositor's account *per se*: if you use your credit card to shop, the deposit account that will be credited will be the merchant's, while the increased debt will be recorded against your credit card account. But the essence of the entire process is contained in that one line in a Godley Table.

- In normal times, the central bank does not fix the amount of money in circulation, nor is central bank money 'multiplied up' into more loans and deposits. (McLeay et al. 2014a, p. 1. Emphasis added)

Pretty definitive, right? So how did Krugman react to it? See Figure 111

Figure 111: Krugman's reaction to the Bank of England paper



The Opinion Pages

The Conscience of a Liberal

PAUL KRUGMAN

A Monetary Puzzle

APRIL 28, 2014 8:10 AM 78

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OK, color me puzzled. I've seen a number of people touting this Bank of England paper (pdf) on how banks create money as offering some kind of radical new way of looking at the economy. And it is a good piece. But it doesn't seem, in any important way, to be at odds with what Tobin wrote 50 years ago (pdf) — indeed, the BoE paper cites Tobin extensively. And I have always thought of money in Tobinesque terms, even if I sometimes use shorthand descriptions that can be misread if you take them out of context; the same is true of many economists.

Furthermore, the key Tobin insight — which is fully consistent with the BoE analysis — is that while banks are indeed more complicated creatures than the mechanical lenders of deposits we like to portray in Econ 101, this doesn't mean either that they have unlimited ability to create money or that they are somehow outside the usual rules of economics. Don't let monetary realism slide into monetary mysticism!

That reaction can be summarized as "I've read it. So what?". He did not even consider that he should model what this meant for money creation, let alone macroeconomics as a whole. The same applies to Mankiw, whose textbook post-dates the Bank of England paper, and yet repeats the myths that the Bank of England debunked.

Let's leave these barter mystics behind, and consider the actual macroeconomics of money.

8 Money

My main motivation for inventing and designing Minsky was to enable proper modelling of money. Since then, *Modern Monetary Theory* (*MMT*) has risen to prominence, and Minsky is ideally suited to analyzing the claims and counter-claims made about *MMT*. The core claim is that government spending precedes taxation: that rather than having to tax to be able to spend, governments create money first by spending, and tax it back later. One of the best ways to illustrate this is to take a situation where there was no monetary system, and have money introduced. As Graeber (Graeber 2011) emphasized, this was not what normally happened in history—the assumption first enunciated by Smith that barter was the rule before money was introduced is a myth. But there are instances where one political system has collapsed, and the monetary system with it, followed later by the development of a new monetary system in the context of forging a new political system. The legal scholar Christine Desan identified instances of this in England after the collapse of the Roman Empire:

> The new narrative explains how each of the capacities associated with money—its function as a unit of account, mode of payment, and medium of exchange—is, at base, a mode of governing. The unit of account, first, arises when a stakeholder takes something that is not fungible—the in-kind service owed by individuals or families—and marks it with a token. Accounts that rely on the "convergence story" of money's creation often simply assume the existence of a unit of account because it is so difficult to understand how people who are engaged only in bipolar exchanges can create a term for value that is shared among them all. But establishing a unit of account is a critical accomplishment that demands an explanation. The capacity of an object to furnish homogeneous comparative terms—a unit of account—to evaluate other objects supplies the terms for "counting" value, i.e., price. That unit is used both as the basis of accounting systems and as the metric into which circulating coin or currency can be converted. Once we admit the agency of a stakeholder common to those engaged in bipolar exchanges, the accomplishment becomes intelligible.
>
> In early medieval England, rulers chose to make the basic unit of account—the penny—out of silver. That choice gave silver a price. For example, a weighed pound of silver of specified fineness might be exchanged for 230 pennies at the mint—the "mint price" received when an individual took that amount of bullion in to be coined. The mint made perhaps 242 pennies out of the bullion, kept 12 for the moneyer and the king, and returned the remainder. The "price" of silver was tied, by definition, to the value of the tribute or tax obligation: pennies made by the mint were the tokens used by the king to pay for resources advanced to him. At the time the tax was due, each penny carried value towards extinguishing the tax obligation.115 Note that without violence to that reality, observers could assume that coin expressed the value of the silver it contained: at tax time, the arrangement itself identified the value that a penny held for extinguishing the fiscal obligation with the value of silver. In fact, we might say that the silver coin had become a material proxy for the tax obligation. (Money therefore also furnished a "store of value," another function often attributed to money.) It was not, however, the content of coin that gave it a priced value, but the system that made coin into money. (Desan 2015, p. 58)

Desan singled out the example of the 8th century King of Mercia, Offa, whose coinage was particularly well designed—see Figure 112.

Figure 112: A silver penny from Offa's reign

The main aspect to the design wasn't the art on the coin itself, but the role of the coin in defining the Kingdom itself: what once were payments-in-kind to the King became payments in coin, while the coin came to be used in person-to-person trade in the Kingdom as well:

> in-kind payment of rents began to be converted in part into cash payments during the 8th century, a trend that would continue in later centuries, and the late 8th century Mercian king Offa extended tribute obligations to virtually "everyone" in his territory… tokens clearly acted as a "mode of payment" to the government when they were returned in lieu of tribute or other obligation. As we saw above, the tokens invited use as a mode of payment in private deals as well. (Desan 2015, pp. 57-59)

The first fully-fledged Minsky model in *Manifesto* simulates this "ab initio" creation of a monetary system.

8.1 Modelling the Origins of Fiat Money in *Minsky*: pp. 33-39 of *Manifesto*

File: http://www.profstevekeen.com/wp-content/uploads/2021/05/Figure03DesanOffaCoins-1.mky

To create a monetary system based on coins, firstly you have to create the coins. Assuming that the King starts with enough silver to make the initial coins, the most sensible entity to start with is the authority he directs to make the coins, The Mint.[24]

The steps in this process are:

- The Mint creates the coins;
- The Mint gives the coins to the Treasury;
- The Treasury spends the coins to procure goods from lords and peasants, where the purchase replaced the pre-monetary practice of compulsory acquisition at the point of a sword (the in-kind payment of rents noted by Desan above);
- The coins are then used by lords to pay peasants to produce output, which is sold to both lords and peasants, and the government (this is a bit of a fiction: back in the 700s, peasants were indentured to their feudal lords, but it's for the purposes of illustration); and
- Finally, coins are taxed from the lords and peasants by the Treasury.

Figure 113 shows these steps in The Mint's Godley Table.

Figure 113: The Mint's view of King Offa's establishment of a monetary economy

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Mint}$	$Coins_{Treasury}$	$Coins_{Lords}$	$Coins_{Peasants}$	$Mint_{Equity}$	0
Initial Conditions	0	0	0	0	0	0
Create coins	$Mint_{Coins}$				$Mint_{Coins}$	0
Give Coins to Treasury		$Issue_{Coins}$			$-Issue_{Coins}$	0
Spend on Peasants		$-Spend_{Peasants}$		$Spend_{Peasants}$		0
Spend on Lords		$-Spend_{Lords}$	$Spend_{Lords}$			0
Hire Peasants			$-Wages$	$Wages$		0
Peasants Consume			$Consume_{Peasants}$	$-Consume_{Peasants}$		0
Tax Peasants		$Tax_{Peasants}$		$-Tax_{Peasants}$		0
Tax Lords		Tax_{Lords}	$-Tax_{Lords}$			0

As I note in *Manifesto*, Godley Tables don't show actual coins, but are an accountant's record of where the coins are at some point in time (the stocks called $Coins_{Mint}$, $Coins_{Treasury}$, etc. in Figure 113), and the rate at which they're moving from one account to another per year (the flows called $Mint_{Coins}$, $Spend_{Peasants}$, etc., in Figure 113):

> Think of the entries as records in a spreadsheet file, rather than the things themselves, whether these be grams of gold in a vault, penny coins in your pocket, or electronic dollars stored in a bank database. (Keen 2021, p. 27)

This "spreadsheet" shows the distribution of coins from the Mint's point of view. Once you have defined it, it also tells you how many more tables you need to complete the model: three, one each for the Liabilities of the Mint. Figure 114 shows the model after those three tables have been created,

[24] You could start your model earlier if you wish—Desan explains how rulers acquired silver once they started making coins—just start from the assumption that The Mint has all the silver it needs, and then later add buying silver from the public in the manner that Desan explains. "In early medieval England, rulers chose to make the basic unit of account—the penny—out of silver. That choice gave silver a price. For example, a weighed pound of silver of specified fineness might be exchanged for 230 pennies at the mint—the "mint price" received when an individual took that amount of bullion in to be coined. The mint made perhaps 242 pennies out of the bullion, kept 12 for the moneyer and the king, and returned the remainder. The "price" of silver was tied, by definition, to the value of the tribute or tax obligation: pennies made by the mint were the tokens used by the king to pay for resources advanced to him. At the time the tax was due, each penny carried value towards extinguishing the tax obligation" (Desan 2015, p. 58)

but before they are populated with stocks and flows by using the ▼ tool in each table to identify Liabilities that haven't yet been defined as Assets.

Figure 114: Introducing Godley Tables for the other 3 entities in the model

Mint

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{MInt}$ ▼	$Coins_{Treasury}$ ▼	$Coins_{Lords}$ ▼	$Coins_{Peasants}$ ▼	$Mint_{Equity}$	0
Initial Conditions	0	0	0	0	0	0
Create coins	$Mint_{Coins}$				$Mint_{Coins}$	0
Give Coins to Treasury		$Issue_{Coins}$			$-Issue_{Coins}$	0
Spend on Peasants		$-Spend_{Peasants}$		$Spend_{Peasants}$		0
Spend on Lords		$-Spend_{Lords}$	$Spend_{Lords}$			0
Hire Peasants			-Wages	Wages		0
Peasants Consume			$Consume_{Peasants}$	$-Consume_{Peasants}$		0
Tax Peasants		$Tax_{Peasants}$		$-Tax_{Peasants}$		0
Tax Lords		Tax_{Lords}	$-Tax_{Lords}$			0

The Treasury has both the Asset of $Coins_{Treasury}$, and the Liability of $Coins_{Mint}$ (the Mint's Asset has to be a Liability for another entity in the model). If you open the Treasury Godley Table, and use the ▼ tool on both the Asset and Liability side of its ledger, you will generate the table you see in Figure 115:

Figure 115: The Treasury's Godley Table after allocating the Mint's assets and liabilities

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Treasury}$ ▼	$Coins_{MInt}$ ▼		0
Initial Conditions	0	0		0
Give Coins to Treasury	$Issue_{Coins}$			$Issue_{Coins}$
Spend on Lords	$-Spend_{Lords}$			$-Spend_{Lords}$
Spend on Peasants	$-Spend_{Peasants}$			$-Spend_{Peasants}$
Tax Lords	Tax_{Lords}			Tax_{Lords}
Tax Peasants	$Tax_{Peasants}$			$Tax_{Peasants}$
Create coins		$Mint_{Coins}$		$-Mint_{Coins}$

The next step is matching the flows with changes to the Equity of the Treasury—which I define as $Treasury_E$—see Figure 116

Figure 116: Treasury Godley Table finalised. Peasants and Lords to go

Mint

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{MInt}$	$Coins_{Treasury}$	$Coins_{Lords}$	$Coins_{Peasants}$	$Mint_{Equity}$	0
Initial Conditions	0	0	0	0	0	0
Create coins	$Mint_{Coins}$				$Mint_{Coins}$	0
Give Coins to Treasury		$Issue_{Coins}$			$-Issue_{Coins}$	0
Spend on Peasants		$-Spend_{Peasants}$		$Spend_{Peasants}$		0
Spend on Lords		$-Spend_{Lords}$	$Spend_{Lords}$			0
Hire Peasants			$-Wages$	$Wages$		0
Peasants Consume			$Consume_{Peasants}$	$-Consume_{Peasants}$		0
Tax Peasants		$Tax_{Peasants}$		$-Tax_{Peasants}$		0
Tax Lords		Tax_{Lords}	$-Tax_{Lords}$			0

Treasury

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Treasury}$	$Coins_{MInt}$	$Treasury_E$	0
Initial Conditions	0	0	0	0
Give Coins to Treasury	$Issue_{Coins}$		$Issue_{Coins}$	0
Spend on Lords	$-Spend_{Lords}$		$-Spend_{Lords}$	0
Spend on Peasants	$-Spend_{Peasants}$		$-Spend_{Peasants}$	0
Tax Lords	Tax_{Lords}		Tax_{Lords}	0
Tax Peasants	$Tax_{Peasants}$		$Tax_{Peasants}$	0
Create coins		$Mint_{Coins}$	$-Mint_{Coins}$	0

Peasants

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →				0
Initial Conditions				0

Lords

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →				0
Initial Conditions				0

All that is left to complete the structure of coin accounts in this model is to repeat this process for the Peasants and Lords—see Figure 117.

Figure 117: The complete set of Godley Tables for the King Offa model

Mint

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{MInt}$	$Coins_{Treasury}$	$Coins_{Lords}$	$Coins_{Peasants}$	$Mint_{Equity}$	0
Initial Conditions	0	0	0	0	0	0
Create coins	$Mint_{Coins}$				$Mint_{Coins}$	0
Give Coins to Treasury		$Issue_{Coins}$			$-Issue_{Coins}$	0
Spend on Peasants		$-Spend_{Peasants}$		$Spend_{Peasants}$		0
Spend on Lords		$-Spend_{Lords}$	$Spend_{Lords}$			0
Hire Peasants			$-Wages$	$Wages$		0
Peasants Consume			$Consume_{Peasants}$	$-Consume_{Peasants}$		0
Tax Peasants		$Tax_{Peasants}$		$-Tax_{Peasants}$		0
Tax Lords		Tax_{Lords}	$-Tax_{Lords}$			0

Treasury

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Treasury}$	$Coins_{MInt}$	$Treasury_E$	0
Initial Conditions	0	0	0	0
Give Coins to Treasury	$Issue_{Coins}$		$Issue_{Coins}$	0
Spend on Lords	$-Spend_{Lords}$		$-Spend_{Lords}$	0
Spend on Peasants	$-Spend_{Peasants}$		$-Spend_{Peasants}$	0
Tax Lords	Tax_{Lords}		Tax_{Lords}	0
Tax Peasants	$Tax_{Peasants}$		$Tax_{Peasants}$	0
Create coins		$Mint_{Coins}$	$-Mint_{Coins}$	0

Lords

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Lords}$		$Lords_E$	0
Initial Conditions	0		0	0
Peasants Consume	$Consume_{Peasants}$		$Consume_{Peasants}$	0
Spend on Lords	$Spend_{Lords}$		$Spend_{Lords}$	0
Tax Lords	$-Tax_{Lords}$		$-Tax_{Lords}$	0
Hire Peasants	$-Wages$		$-Wages$	0

Peasants

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Peasants}$		$Peasants_E$	0
Initial Conditions	0		0	0
Peasants Consume	$-Consume_{Peasants}$		$-Consume_{Peasants}$	0
Spend on Peasants	$Spend_{Peasants}$		$Spend_{Peasants}$	0
Tax Peasants	$-Tax_{Peasants}$		$-Tax_{Peasants}$	0
Hire Peasants	$Wages$		$Wages$	0

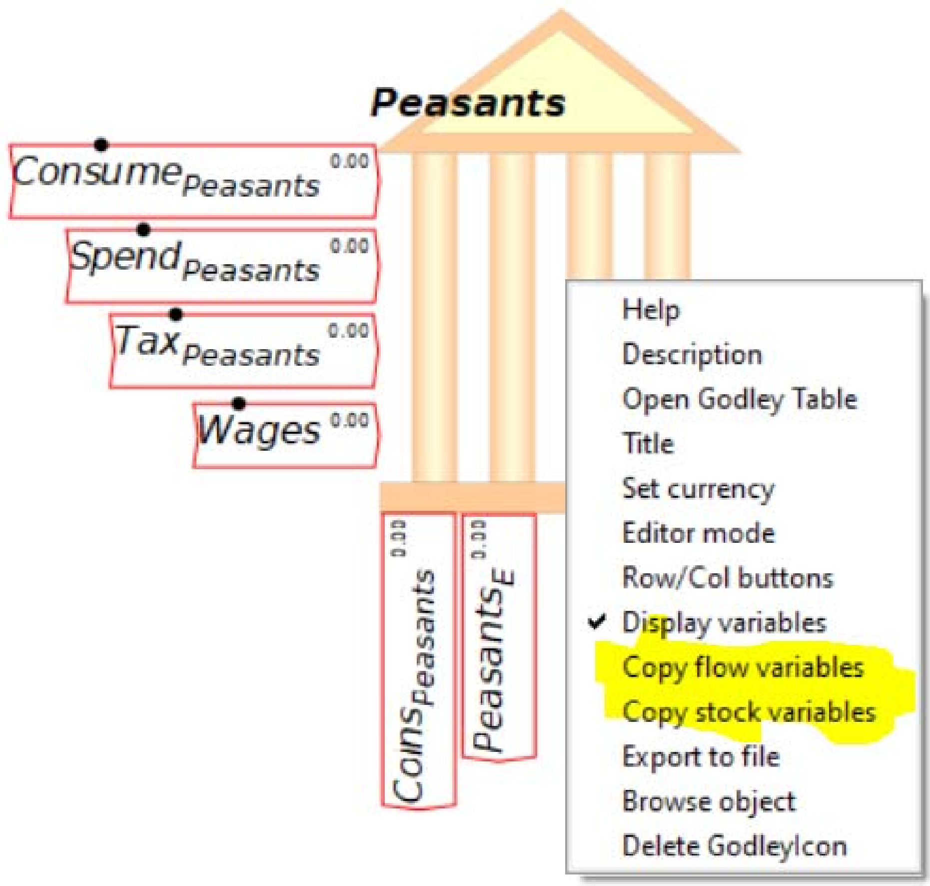
I find this structure alone, in any model, to be very informative. We'll see a better example with the next model, of modern-day fiat money. But for now, it is obvious that act of minting the coins sends the Treasury into negative equity, while the issuance of those coins to it by the Mint puts it back into zero equity—if the coins just remained in the Treasury. But of course, the Mint and the Treasury are two wings of the government, so the sum of their two operations is zero. In effect, the fact that the government can do this—create liabilities and assets within itself, and then have those liabilities accepted by other entities in the society ("Would you prefer a coin in your hand, or a sword at your throat, in exchange for those chickens?")—is the essence of what gives the government's balance sheet a unique status in a monetary economy.

To complete the model, we need to define the flows, and the initial flow here is the minting of coins. I'm using 1000 coins to match Milton Friedman's mythical "Optimum Quantity of Money" model, in which

> (12) All money consists of strict fiat money, i.e., pieces of paper, each labelled "This is one dollar." (Friedman 1969, p. 3)

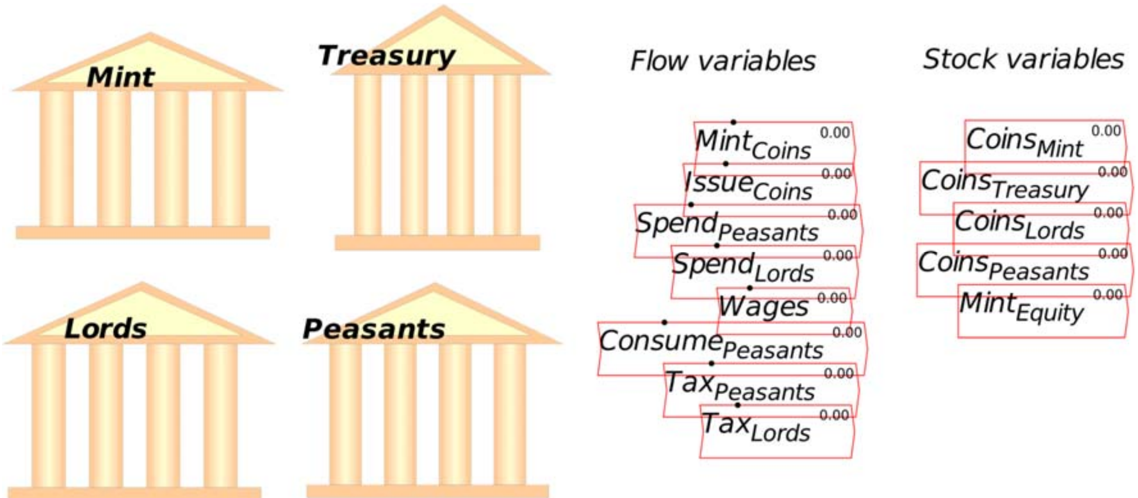
The first stage of modelling the flows is to take the stocks and flows from the Godley Tables themselves and place them on the canvas. This is done using the right-mouse menu—see Figure 118.

Figure 118: Copy stock and flow variables from Godley Tables to the canvas



In Figure 119, I've copied the stocks and flows from the Mint's Godley Table, placed them on the canvas, put the Mint's table back in icon rather than Edit view, and turned off display of variables on each Godley Table to save space. This is feasible now that Minsky has a "Godleys" tab that lets you see all the Godley Tables at once. The stocks and flows overlap onscreen somewhat because when I resized the Mint's icon, it rescaled the layout of the variables as well. This is technically a bug—it would be better if the symbols didn't overlap. But it's not fatal, so we'll leave this bug in place until we're rolling in development funding.

Figure 119: Stocks and flows extracted from the Mint's Godley Table



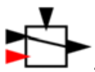
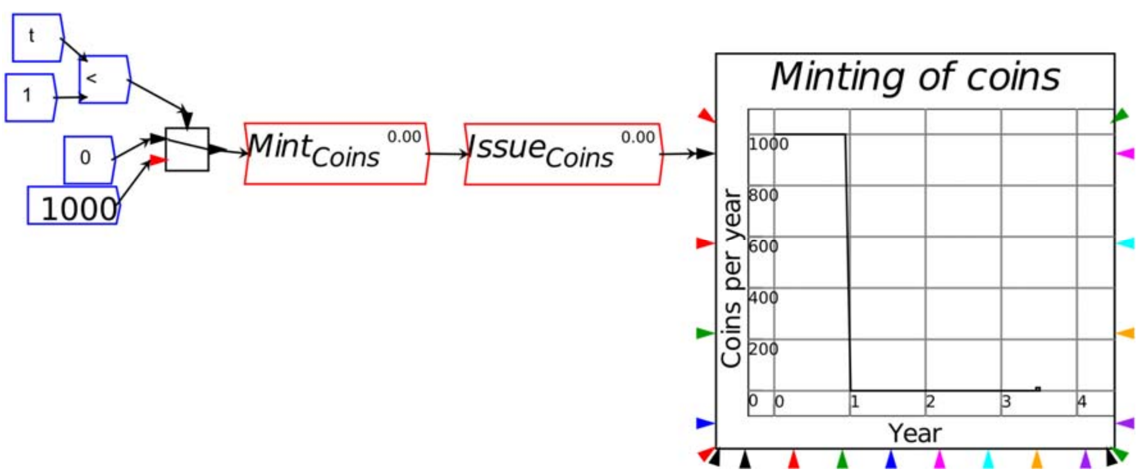
The first activity to define is the minting of coins, and here I use a simple but very useful feature of Minsky, the switch . This takes a logical operator in at the top, and has two options on the left hand side: what happens if the logical operator is false, and what happens if it is true.[25] The operation shown in Figure 120 compares the system simulation time t to 1, and so long as $t < 1$, it outputs 1000 *per year*. Once $t \geq 1$ then the output drops to zero. Therefore over the first year of the simulation, 1000 coins are created. You could add a time lag between minting and issuing if you wish, but for simplicity I've simply linked minting to issuing.

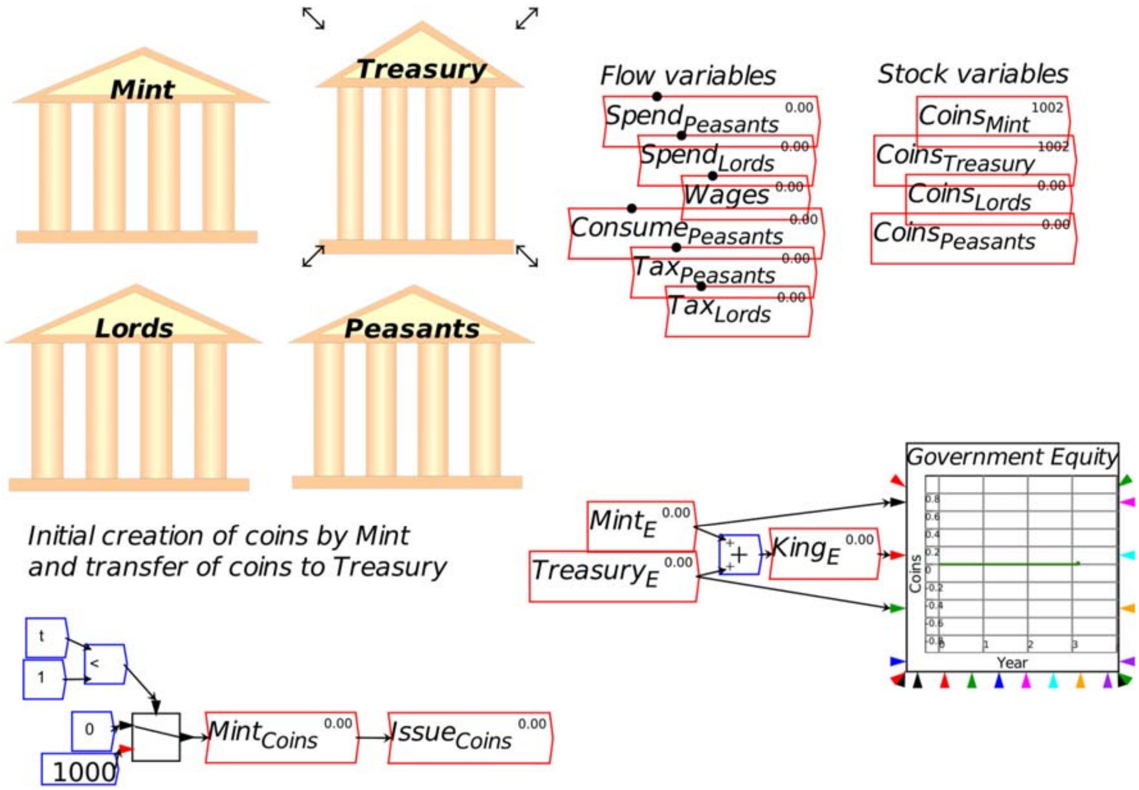
Figure 120: Using the switch to produce 1000 coins while t< 1 year



So long as the coins remain in the hands of the government, nothing of interest happens: the Equity of the Treasury, Mint and the government as a whole remain at zero: the minting of coins (which increases the Mint's equity and reduces the Treasury's) is offset by the issuing of those coins to Treasury (which increases the Treasury's equity and reduces the Mint's)—see Figure 121.

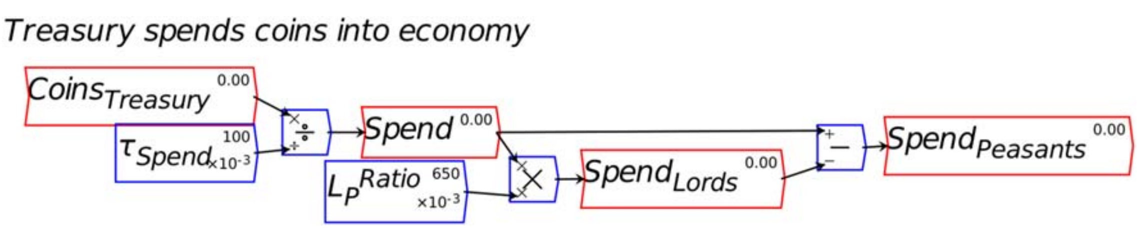
[25] We will add to the switch feature over time to enable it to handle multiple cases, but we haven't got there yet.

Figure 121



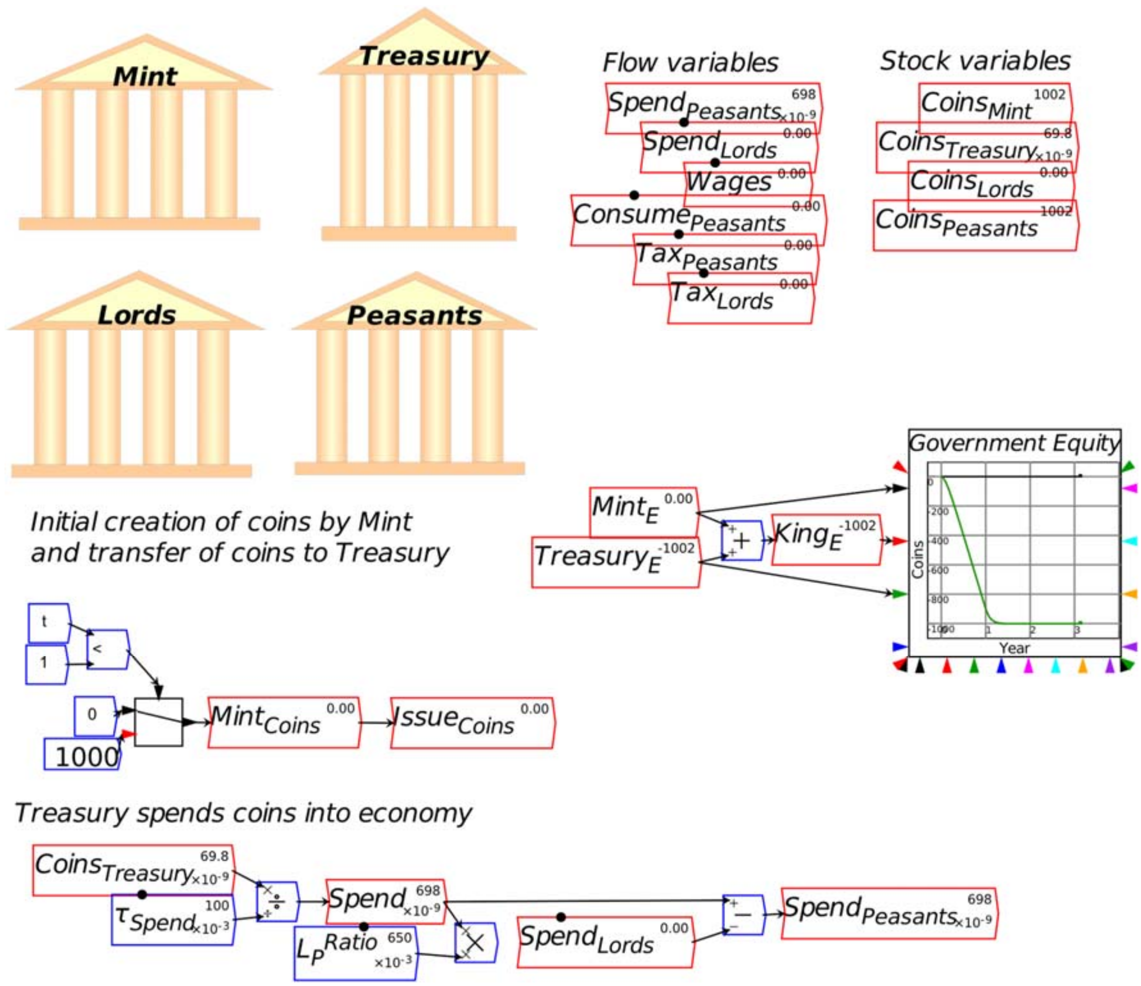
The action commences—as MMT argues—when the Treasury spends its newly created currency into circulation. As Desan emphasises, this practice replaced forced appropriation in these post-Roman and pre-Norman Kingdoms. In this simple model, I assume that the rate of spending is a function of how many coins are in the Treasury, using a time lag. One by-product of the spending is that the Treasury's equity turns negative: it still "owes" the Mint 1000 coins, but it has spent them all into the economy, where they are now in the hands of the Lords and Peasants—see Figure 122.

Figure 122: Treasury spends coins into circulation



For the Treasury, this means that it goes into negative equity: it "owes" 1000 coins to the Mint, but it has spent them into the economy, so that, with no subsequent usage of the coins, and no taxation, it has no coins to meet its "debt" to the Mint, and no coins to continue purchasing goods from the Lords and Peasants—see Figure 123.

Figure 123



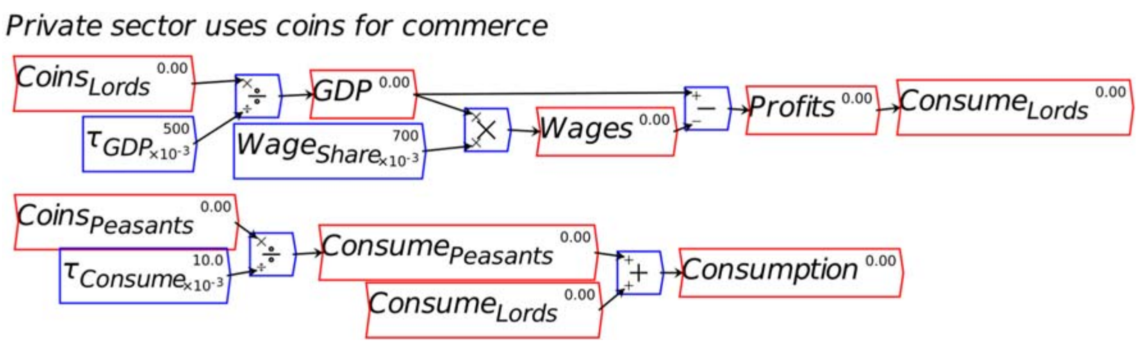
To continue purchasing goods from the private sector, it either has to produce more coins (which would undermine the value of those in existence), or tax back some of those in circulation. The latter makes far more sense, and also sets up the antagonistic relationship between the private sector, where everyone wants to hang on to the tokens they already have, given their value in exchange, and the state, which wants the tokens back, not because it needs them—it could, after all, just print more—but because taxation maintains the value of those in circulation.

As Desan emphasises, the most important impact of going from paying taxes in kind, to paying in coin, was that the first way of levying taxes simply takes resources out of the "private sector", leaving nothing behind. Paying in coin achieves the same physical outcome—resources produced in the private sector are transferred to the public—but leaves the private sector with an exchangeable token, the coins. This enables trade to expand in the private sector, *if* these coins are made valuable by being a means to pay taxes in future. Therefore, the expansion in trade that Neoclassicals attribute to using a "money commodity" in place of barter, actually occurred when an otherwise valueless token—King Offa's coin—was made valuable as a way to pay taxes in future. Taxes, which mainstream and Austrian economists rail against, is an essential aspect to maintaining the value of that commerce-enabling fiat currency.

This creates a symbiotic relationship between the public sector and the private sector, rather than the parasitic one emphasised by Austrian economists. [26] Yes, the government is taking resources from the private sector; but its manner of doing it by coins rather than payment in kind creates a token which can be—and was, as Desan explains—used to dramatically expand private sector trade.

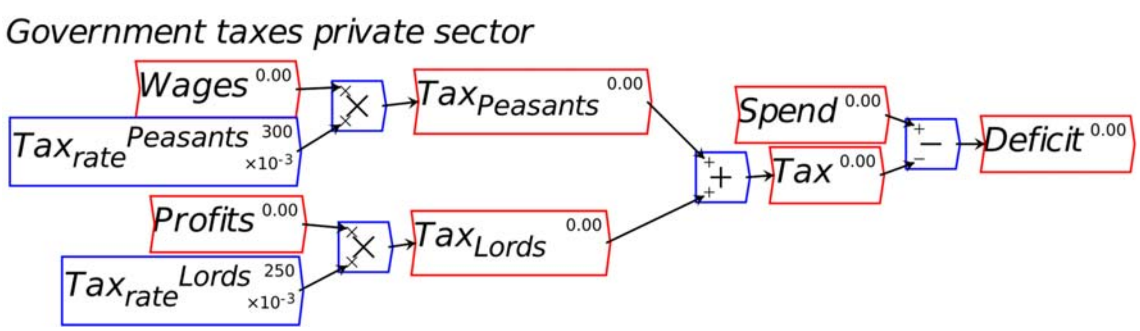
These aspects are introduced by the flows shown in Figure 124—which does a bit of historical violence by imagining that peasants are paid a wage rather than being indentured:

Figure 124: Coins are now used for private sector commerce



Finally, taxation is imposed to both get the coins back to the Treasury to finance future spending, and to give the coins a value to the public: it's worth collecting them in the course of business to be able to pay taxes when they are levied. For reasons of historical accuracy, I show the Peasants paying a higher rate of tax than the Lords. That taxation revenue, when subtracted from government spending, determines the deficit—see Figure 125.

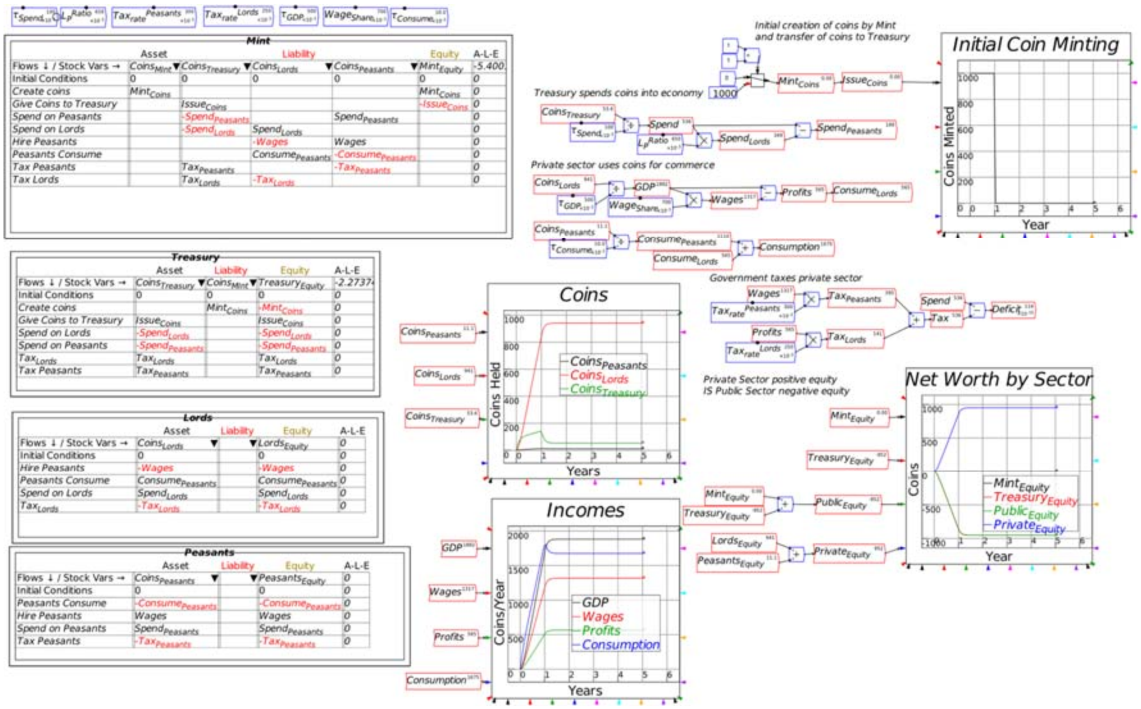
Figure 125: Taxation and the deficit



Finally, we have the full model shown in Manifesto—see Figure 126.

[26] It also inverts where the "parasitic" behaviour occurs: by spending the coin it has created, the government gets private-sector-created resources "for free" (the sword-wielding tax collector can now be assigned to other activities, such as invading the neighbouring kingdom); taxing is merely the government taking back that otherwise worthless token it created.

Figure 126: The model that produced Figures 2-3 and 2-4 in Manifesto



The Godleys Tab, with display of values turned on via the Options/Preferences menu, provides a nice overview of the flows in the model—see Figure 127. Notice that the Mint has zero equity, while the Treasury has negative equity of 952 coins—and this is precisely equal to the positive equity of the private sector, which is 11 for the Peasants and 941 for the Lords.

Figure 127: The Godley Tables in the Offa example, with display of values turned on

Mint

	Asset	Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Mint}$ = 1006	$Coins_{Treasury}$ = 53.6	$Coins_{Lords}$ = 941	$Coins_{Peasants}$ = 11.1	$Mint_{Equity}$ = 0.00	-5.40012e-13
Initial Conditions	0	0	0	0	0	0
Create coins	$Mint_{Coins}$ = 0.00				$Mint_{Coins}$ = 0.00	0
Give Coins to Treasury		$Issue_{Coins}$ = 0.00			$-Issue_{Coins}$ = -0.00	0
Spend on Peasants		$-Spend_{Peasants}$ = -188		$Spend_{Peasants}$ = 188		0
Spend on Lords		$-Spend_{Lords}$ = -349	$Spend_{Lords}$ = 349			0
Hire Peasants			-Wages = -1317	Wages = 1317		0
Peasants Consume			$Consume_{Peasants}$ = 1110	$-Consume_{Peasants}$ = -1110		0
Tax Peasants		$Tax_{Peasants}$ = 395		$-Tax_{Peasants}$ = -395		0
Tax Lords		Tax_{Lords} = 141	$-Tax_{Lords}$ = -141			0

Treasury

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Treasury}$ = 53.6	$Coins_{Mint}$ = 1006	$Treasury_{Equity}$ = -952	-2.27374e-13
Initial Conditions	0	0	0	0
Create coins		$Mint_{Coins}$ = 0.00	$-Mint_{Coins}$ = -0.00	0
Give Coins to Treasury	$Issue_{Coins}$ = 0.00		$Issue_{Coins}$ = 0.00	0
Spend on Lords	$-Spend_{Lords}$ = -349		$-Spend_{Lords}$ = -349	0
Spend on Peasants	$-Spend_{Peasants}$ = -188		$-Spend_{Peasants}$ = -188	0
Tax_{Lords}	Tax_{Lords} = 141		Tax_{Lords} = 141	0
Tax Peasants	$Tax_{Peasants}$ = 395		$Tax_{Peasants}$ = 395	0

Lords

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Lords}$ = 941		$Lords_{Equity}$ = 941	0
Initial Conditions	0		0	0
Hire Peasants	-Wages = -1317		-Wages = -1317	0
Peasants Consume	$Consume_{Peasants}$ = 1110		$Consume_{Peasants}$ = 1110	0
Spend on Lords	$Spend_{Lords}$ = 349		$Spend_{Lords}$ = 349	0
Tax_{Lords}	$-Tax_{Lords}$ = -141		$-Tax_{Lords}$ = -141	0

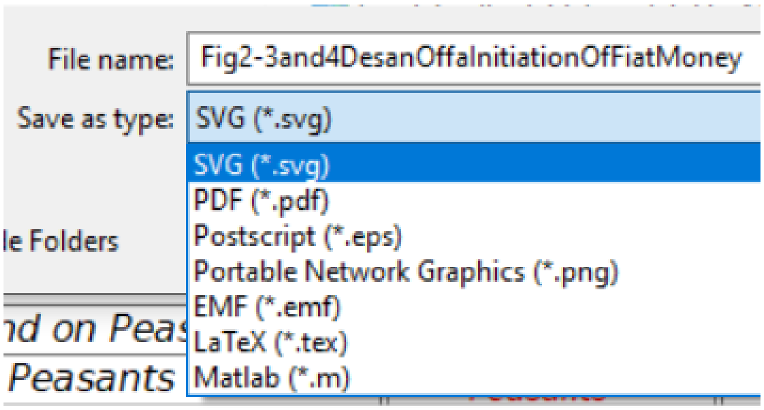
Peasants

	Asset	Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	$Coins_{Peasants}$ = 11.1		$Peasants_{Equity}$ = 11.1	0
Initial Conditions	0		0	0
Peasants Consume	$-Consume_{Peasants}$ = -1110		$-Consume_{Peasants}$ = -1110	0
Hire Peasants	Wages = 1317		Wages = 1317	0
Spend on Peasants	$Spend_{Peasants}$ = 188		$Spend_{Peasants}$ = 188	0
Tax Peasants	$-Tax_{Peasants}$ = -395		$-Tax_{Peasants}$ = -395	0

8.1.1 Check the equations

One thing Minsky does which is almost unique in the system dynamics space is generate the equations of the model in mathematical format—I'm pretty certain Mathematica's System Modeler does the same, but Vensim, Stella, etc., do not. It's an option under "Export Canvas"—see Figure 128, where LaTeX is the relevant option).

Figure 128: The options under "Export Canvas")



One of the advantages of this capacity is that things you might have missed in the flowchart model can be more obvious in the equations (if reading equations is "your thing", as it is mine). The clear flaw, in stock-flow consistency terms, in this model is that I have the Lords consuming their "Profits", but also paying taxes on their profits. I missed that in laying out the flowchart, but it was obvious when I checked the equations—see Equation (1.32), where I've highlighted the inconsistent equations in red.

Differential Equations

$$\frac{d\text{Coins}_{\text{Mint}}}{dt} = \text{Mint}_{\text{Coins}}$$

$$\frac{d\text{Coins}_{\text{Treasury}}}{dt} = \text{Issue}_{\text{Coins}} + \text{Tax}_{\text{Peasants}} + \text{Tax}_{\text{Lords}} - \left(\text{Spend}_{\text{Peasants}} + \text{Spend}_{\text{Lords}}\right)$$

$$\frac{d\text{Coins}_{\text{Lords}}}{dt} = \text{Consume}_{\text{Peasants}} + \text{Spend}_{\text{Lords}} - \left(\text{Wages} + \text{Tax}_{\text{Lords}}\right)$$

$$\frac{d\text{Coins}_{\text{Peasants}}}{dt} = \text{Wages} + \text{Spend}_{\text{Peasants}} - \left(\text{Consume}_{\text{Peasants}} + \text{Tax}_{\text{Peasants}}\right)$$

$$\frac{d\text{Lords}_{\text{Equity}}}{dt} = \text{Consume}_{\text{Peasants}} + \text{Spend}_{\text{Lords}} - \left(\text{Wages} + \text{Tax}_{\text{Lords}}\right)$$

$$\frac{d\text{Peasants}_{\text{Equity}}}{dt} = \text{Wages} + \text{Spend}_{\text{Peasants}} - \left(\text{Consume}_{\text{Peasants}} + \text{Tax}_{\text{Peasants}}\right)$$

$$\frac{d\text{Mint}_{\text{Equity}}}{dt} = \text{Mint}_{\text{Coins}} - \text{Issue}_{\text{Coins}}$$

$$\frac{d\text{Treasury}_{\text{Equity}}}{dt} = \text{Issue}_{\text{Coins}} + \text{Tax}_{\text{Lords}} + \text{Tax}_{\text{Peasants}} - \left(\text{Mint}_{\text{Coins}} + \text{Spend}_{\text{Lords}} + \text{Spend}_{\text{Peasants}}\right)$$

Equations

$$\text{Mint}_{\text{Coins}} = \theta\left(1-\theta\left(1-t\right)\right)\times 0 + \left(1-\theta\left(1-\theta\left(1-t\right)\right)\right)\times 1000$$

$$\text{Issue}_{\text{Coins}} = \text{Mint}_{\text{Coins}}$$

$$\text{Spend} = \frac{\text{Coins}_{\text{Treasury}}}{\tau_{\text{Spend}}}; \text{Spend}_{\text{Lords}} = \text{Spend}\times L_{p_{\text{Ratio}}}; \text{Spend}_{\text{Peasants}} = \text{Spend} - \text{Spend}_{\text{Lords}}$$

$$\text{GDP} = \frac{\text{Coins}_{\text{Lords}}}{\tau_{\text{GDP}}}; \text{Wages} = \text{GDP}\times\text{Wage}_{\text{Share}}; \text{Profits} = \text{GDP} - \text{Wages} \tag{1.32}$$

$$\text{Consume}_{\text{Peasants}} = \frac{\text{Coins}_{\text{Peasants}}}{\tau_{\text{Consume}}}; \text{Consume}_{\text{Lords}} = \text{Profits}; \text{Consumption} = \text{Consume}_{\text{Peasants}} + \text{Consume}_{\text{Lords}}$$

$$\text{Tax}_{\text{Lords}} = \text{Profits}\times\text{Tax}_{\text{rate}^{\text{Lords}}}; \text{Tax}_{\text{Peasants}} = \text{Wages}\times\text{Tax}_{\text{rate}^{\text{Peasants}}}; \text{Tax} = \text{Tax}_{\text{Peasants}} + \text{Tax}_{\text{Lords}}$$

$$\text{Deficit} = \text{Spend} - \text{Tax}$$

$$\text{Public}_{\text{Equity}} = \text{Mint}_{\text{Equity}} + \text{Treasury}_{\text{Equity}}$$

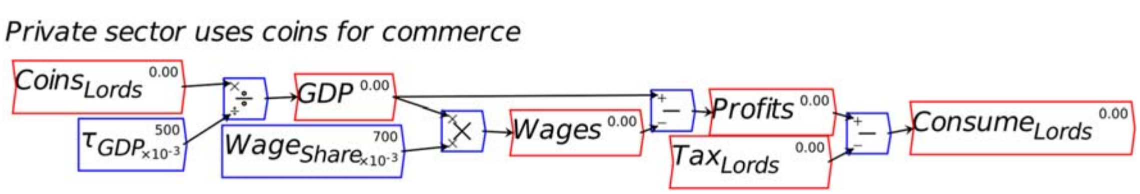
$$\text{Private}_{\text{Equity}} = \text{Lords}_{\text{Equity}} + \text{Peasants}_{\text{Equity}}$$

Parameters

$$\tau_{\text{Spend}} = 0.1; \tau_{\text{GDP}} = 0.5; \tau_{\text{Consume}} = 0.01; \text{Wage}_{\text{Share}} = 0.7; \text{Tax}_{\text{rate}^{\text{Peasants}}} = 0.3; \text{Tax}_{\text{rate}^{\text{Lords}}} = 0.25; L_{p_{\text{Ratio}}} = 0.65$$

This error was easily edited—see Figure 129—and it didn't have any impact on the model anyway, but it shows what can go wrong when you use the flowchart logic for monetary flows rather than the Godley Tables, since the flowchart paradigm doesn't automatically enforce stock-flow consistency, whereas the Godley Tables do.

Figure 129: Lords consumption now shown net of taxes



A Godley Table would have captured this error, but since I was assuming that, to consume, the Lords were buying from other Lords (there was plenty of inter-estate trade in the feudal epoch: some fiefs were almost entirely devoted to one industry, such as ship-making), this intra-class trade couldn't be entered into a Godley Table (yet).[27]

[27] One practice I follow when building serious models is to force all transactions to be monetary—including those between the same social classes (Lords) or in the same sector (Manufacturing)—otherwise, at the heart of your

One critical insight into the role of government spending in any economy—or rather, any economy where the government issues its own currency—can be garnered by adding together the equity of the Mint and the Treasury to define the equity of the government sector in this model, and the equity of the Lords and Peasants to define the change in equity of the private sector. The insight, as shown in Equation (1.33), is that *an increase in equity for one sector is necessarily a decrease in equity of the other*.[28] I've colour-coded transactions that net out within different sectors: minting of coins increases the Mint's equity and reduces the Treasury's; issuing of coins does the opposite, leaving taxation and spending the only actions that alter aggregate government equity. Conversely, Wages and consumption by peasants net out in the private sector, leaving taxation and spending the only actions that alter aggregate private equity. A deficit for the government sector (spending exceeding taxation) causes a surplus for the private sector:

$$
\begin{aligned}
&\text{Differential Equations}\\
\frac{d\text{Mint}_{\text{Equity}}}{dt} &= \text{Mint}_{\text{Coins}} - \text{Issue}_{\text{Coins}}\\
\frac{d\text{Treasury}_{\text{Equity}}}{dt} &= \text{Issue}_{\text{Coins}} + \text{Tax}_{\text{Lords}} + \text{Tax}_{\text{Peasants}} - \left(\text{Mint}_{\text{Coins}} + \text{Spend}_{\text{Lords}} + \text{Spend}_{\text{Peasants}}\right)\\
\frac{d\,Government_{\text{Equity}}}{dt} &= \text{Tax}_{\text{Lords}} + \text{Tax}_{\text{Peasants}} - \left(\text{Spend}_{\text{Lords}} + \text{Spend}_{\text{Peasants}}\right)\\
\frac{d\text{Lords}_{\text{Equity}}}{dt} &= \text{Consume}_{\text{Peasants}} + \text{Spend}_{\text{Lords}} - \left(\text{Wages} + \text{Tax}_{\text{Lords}}\right)\\
\frac{d\text{Peasants}_{\text{Equity}}}{dt} &= \text{Wages} + \text{Spend}_{\text{Peasants}} - \left(\text{Consume}_{\text{Peasants}} + \text{Tax}_{\text{Peasants}}\right)\\
\frac{d\,Private_{\text{Equity}}}{dt} &= \left(\text{Spend}_{\text{Lords}} - \text{Tax}_{\text{Lords}}\right) + \left(\text{Spend}_{\text{Peasants}} - \text{Tax}_{\text{Peasants}}\right)
\end{aligned}
\tag{1.33}
$$

This is the basic insight of MMT: the government deficit *is* the private sector surplus.

We can also integrate these rates of change to derive the result that the equity of one sector is the negative of the equity of the other:

$$
\begin{aligned}
Government_{\text{Equity}} &= \int\left(\text{Tax}_{\text{Lords}} + \text{Tax}_{\text{Peasants}} - \left(\text{Spend}_{\text{Lords}} + \text{Spend}_{\text{Peasants}}\right)\right)\\
Private_{\text{Equity}} &= \int\left(\left(\text{Spend}_{\text{Lords}} - \text{Tax}_{\text{Lords}}\right) + \left(\text{Spend}_{\text{Peasants}} - \text{Tax}_{\text{Peasants}}\right)\right)
\end{aligned}
\tag{1.34}
$$

There's no debt of any sort in this model, so I can't yet derive the other fundamental insight of MMT, that the outstanding government debt is simply the record of government money creation. But that's an obvious deduction from the next model.

8.2 Modelling Modern Fiat Money in *Minsky*: pp. 39-50 of *Manifesto*

Notice that all the accounting in the King Offa model is shown on the Mint's Godley Table (see Figure 113 on page 92), because what is circulating in the economy is its combination asset-and-

monetary model is the fiction that intra-class or intra-sectoral trade is barter. So in a 4-sector model I built for a research project for UNEP (the United Nations Environment Program), I split each of my 4 sectors (manufacturing, services, agriculture, energy) into 2 halves, and had one half buy its sector-specific inputs off the other. At some point, we'll add the same facility to the Godley Tables—so that they effectively become 3-dimensional. Then an intra-sectoral transaction would occur in the 3rd dimension, and the sum of the "slice" of the cube would be zero, rather than the sum of a row in the table.

[28] I emphasise that this is focusing on the claims one sector has on another, and not the value of physical assets which are not a liability to someone else. The sum of all claims is necessarily zero in these models.

liability, the coin, which was created by Offa's Mint. The Mint doesn't actually facilitate the exchange of coin between Peasant and Lord, or Lord and Treasury; but because the Peasants and Lords tables record these transactions—where they are exchanging assets—the exchange turns up on the Mint's Godley Table as well, as a record of the movement of its liabilities (this is a feature of *Minsky*).

When we move on to a modern monetary economy, the private banks and their liabilities of deposit accounts enter the picture as well, because private banks *do* facilitate the transfer (when the transaction is a transfer from one deposit account to another), and what is being transferred are liabilities of the private banking system, not of the Central Bank itself. They are also the conduit for government spending and tax payments

This necessitates a much more convoluted path for government spending: to actually get money to the public, it has to turn up in people's deposit accounts, which are liabilities of the private banks. So the assets of the private banks have to be increased as well, which means that *net government spending creates both deposits and reserves*.

Therefore, net government spending creates both assets and liabilities at the level of the private banking system: the assets and liabilities of the banking sector rise because of a government deficit, leaving its net position unchanged: its aggregate equity position remains at zero (in this model, at this stage of its development).

For the private sector non-bank public however (where did we develop this contradictory usage of the words "private" and "public"?), the deficit increases their assets—the Deposit accounts—without changing their liabilities. This is the key MMT point that government deficits create "net financial assets" for the public.

At the next level of the financial system, the Central Bank, there is no creation of net financial assets: instead, there is a transfer of Central Bank liabilities from one account to another. The reserves, as well as being an asset of the private banks, are a liability of the central bank. Reserves are increased by government spending and reduced by taxation. Simultaneously, government spending reduces the Treasury's deposit account and taxation increases it. At this level therefore, the deficit is a liability swap: a transfer from one Central Bank liability account—the Treasury—to another—Reserves.[29] Neither deficits nor surpluses alter the assets of the Central Bank.

At this stage of the development of the model, the Treasury only has the asset of its deposit account at the central bank, and no liabilities, so spending decreases the Treasury's assets while taxation increases them. Overall, the Treasury's capacity and willingness to go into negative equity is what drives the creation of money at the level of the private banking sector and the public.

Therefore at the minimum, four Godley Tables are needed to show the overall monetary mechanics: the Treasury, the Central Bank, the Private Banks, and the (confusingly named) Public, or non-bank private sector—see Figure 130.

[29] Of course, in the real world, there are multiple reserve accounts—one (at least) per private bank. This model focuses on the aggregate of reserves. A model covering possible liquidity default risks would use multiple reserves accounts for multiple banks.

Figure 130: Government spending and taxation in a modern monetary economy

Treasury

Flows ↓ / Stock Vars →	Treasury (Asset)	(Liability)	$Treasury_E$ (Equity)	A-L-E
Initial Conditions	0		0	0
Government Spending	-Spend		-Spend	0
Government Taxation	Tax		Tax	0

Central Bank

Flows ↓ / Stock Vars →	(Asset)	Reserves (Liability)	Treasury (Liability)	CB_E (Equity)	A-L-E
Initial Conditions		0	0	0	0
Government Spending		Spend	-Spend		0
Government Taxation		-Tax	Tax		0

Private Banks

Flows ↓ / Stock Vars →	Reserves (Asset)	Deposits (Liability)	$Bank_E$ (Equity)	A-L-E
Initial Conditions	0	0	0	0
Government Spending	Spend	Spend		0
Government Taxation	-Tax	-Tax		0

Public

Flows ↓ / Stock Vars →	Deposits (Asset)	(Liability)	$Private_E^{NB}$ (Equity)	A-L-E
Initial Conditions	0		0	0
Government Spending	Spend		Spend	0
Government Taxation	-Tax		-Tax	0

This basic situation for a modern monetary economy confirms the point made by the model of King Offa's coins, which is the fundamental insight of MMT: the government sector deficit is the private sector surplus, and vice versa. Focusing just on the Treasury, Bank Deposits and Bank Reserves, a government deficit creates both Deposits, which are an asset of the non-Bank private sector, and Reserves, which are an asset of the Banking sector—see Equation (1.35).

$$\begin{aligned} \frac{d\text{Treasury}}{dt} &= \text{Tax} - \text{Spend} \\ \frac{d\text{Deposits}}{dt} &= \text{Spend} - \text{Tax} \\ \frac{d\text{Reserves}}{dt} &= \text{Spend} - \text{Tax} \end{aligned} \tag{1.35}$$

This immediately shows that government surpluses are a bad idea, unless you actually want to reduce the amount of money in the economy; and that whatever they might do to future generations—which we'll tackle shortly—government deficits enrich the current generation, by creating net equity for it (see Equation (1.36)).

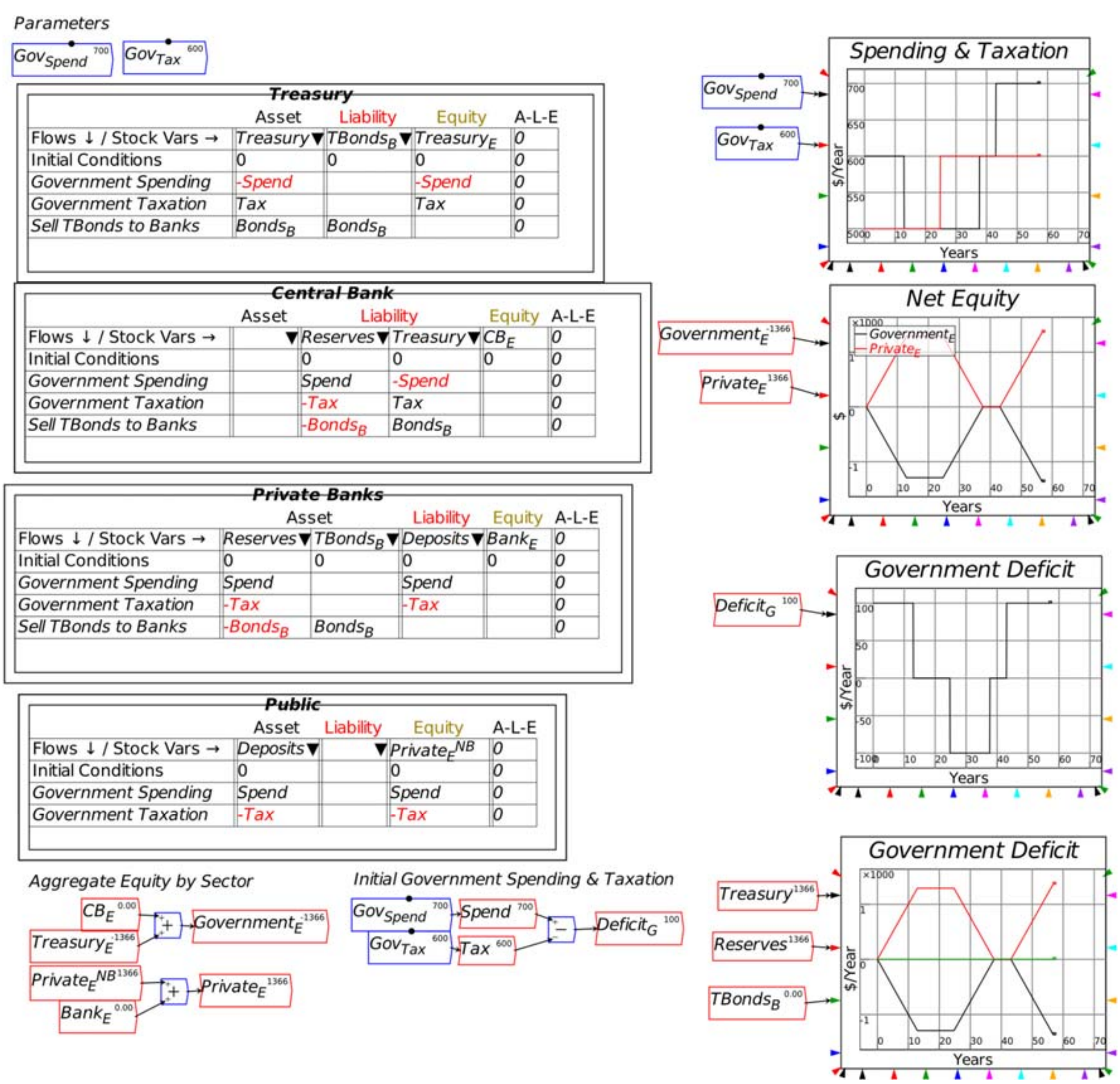
$$\frac{dBank_E}{dt}=0$$
$$\frac{d\text{Private}_{E^{NB}}}{dt}=\text{Spend}-\text{Tax} \tag{1.36}$$

Figure 131 adds the account $TBonds_B$ to the model. This records the value of Treasury Bonds that have been sold, so it is an asset for the private banking system and a liability for the Treasury.

You can see that if government spending exceeds taxation, the net equity of the private sector rises, and is identical in magnitude to the negative equity of the government, which at this stage is entirely reflected in a negative value for its account at the Central Bank—effectively, the government's Treasury runs an overdraft account with the government's Central Bank.

Also, Reserves and the Treasury's account at the Central Bank move in opposite directions: if the government runs a deficit, Reserves are created; if it runs a surplus, Reserves are destroyed. The Reserves are identical in value to the negative of the equity of the government sector.

Figure 131: Bond sales enabled but not yet simulated



I haven't defined flows for bond sales as yet however, so the simulation shown in Figure 131 is effectively of the model in Figure 130. Now let's introduce government debt by having the Treasury issue bonds, which are sold to the private banks—see Figure 132. This could be made much more complicated—bond sales could be modeled as based on forward forecasts of the deficit, extrapolating existing trends—but the simplest case will do here.

Figure 132: Bonds are sold to cover the Deficit

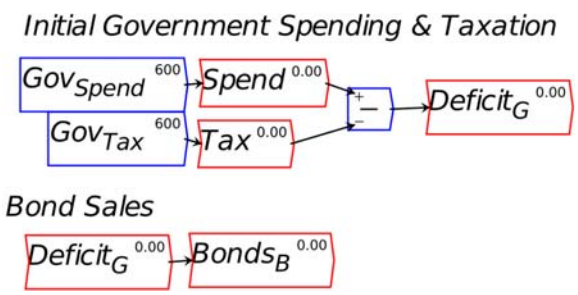


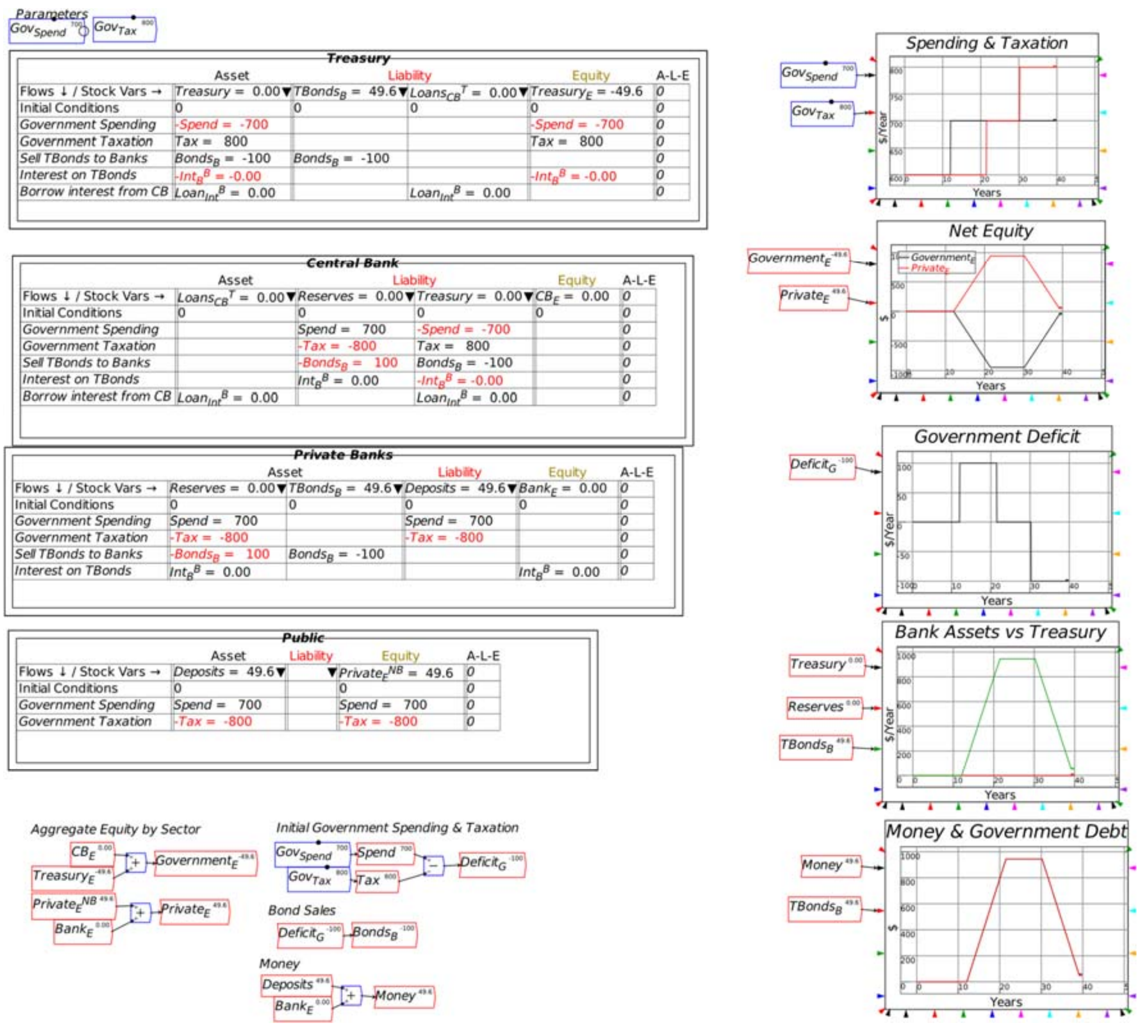
Figure 133 introduces bond sales by Treasury to the private banks. Bond sales are made equal to the deficit. The impact of this change to the model is that bank Reserves remain at zero, the Treasury's account at the Central Bank also remains at zero—whereas it went negative in the simulation without bond sales in Figure 131—and that the money in the economy is identical to the level of government debt.

It also introduces money as the sum of the amounts in deposit accounts plus (short-term) bank equity. Now we can see that—in this model—the money in existence is identical to the Treasury Bonds in existence. So the sale of bonds has had no role in the creation of money—that was done by the deficit itself—but it has enabled the banking sector to exchange a non-tradeable, non-income-earning asset (Reserves) for a tradeable, income-earning asset (Treasury Bonds).

This confirms the crucial points made by MMT, that the "debt" itself doesn't create the money, nor does the government need to sell the bonds in order to finance its deficit. If the Treasury didn't sell the bonds, then it would be in the same situation as the Treasury in the King Offa model: it would be in debt to the Central Bank (as King Offa's Treasury was to his Mint), with an overdraft taking the place of a deficiency of coins in its possession. The bond sales let the Treasury maintain its account at the Central Bank at zero (in this model—in the real world, a positive level could be maintained as well), because the bond sales bring in revenue equivalent to the deficit.

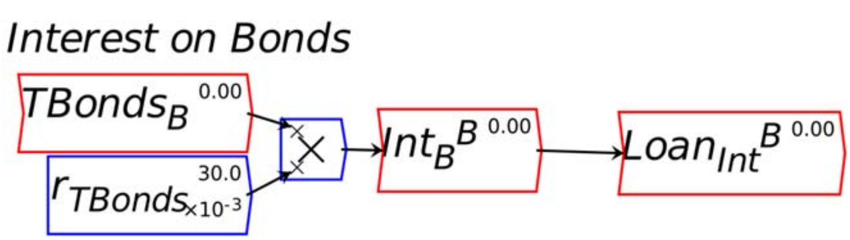
To illustrate an important feature of this model, I've enabled display of numbers on the Godley Tables, via the Options/Preferences dialog box.

Figure 133: Bond sales, but no interest on bonds



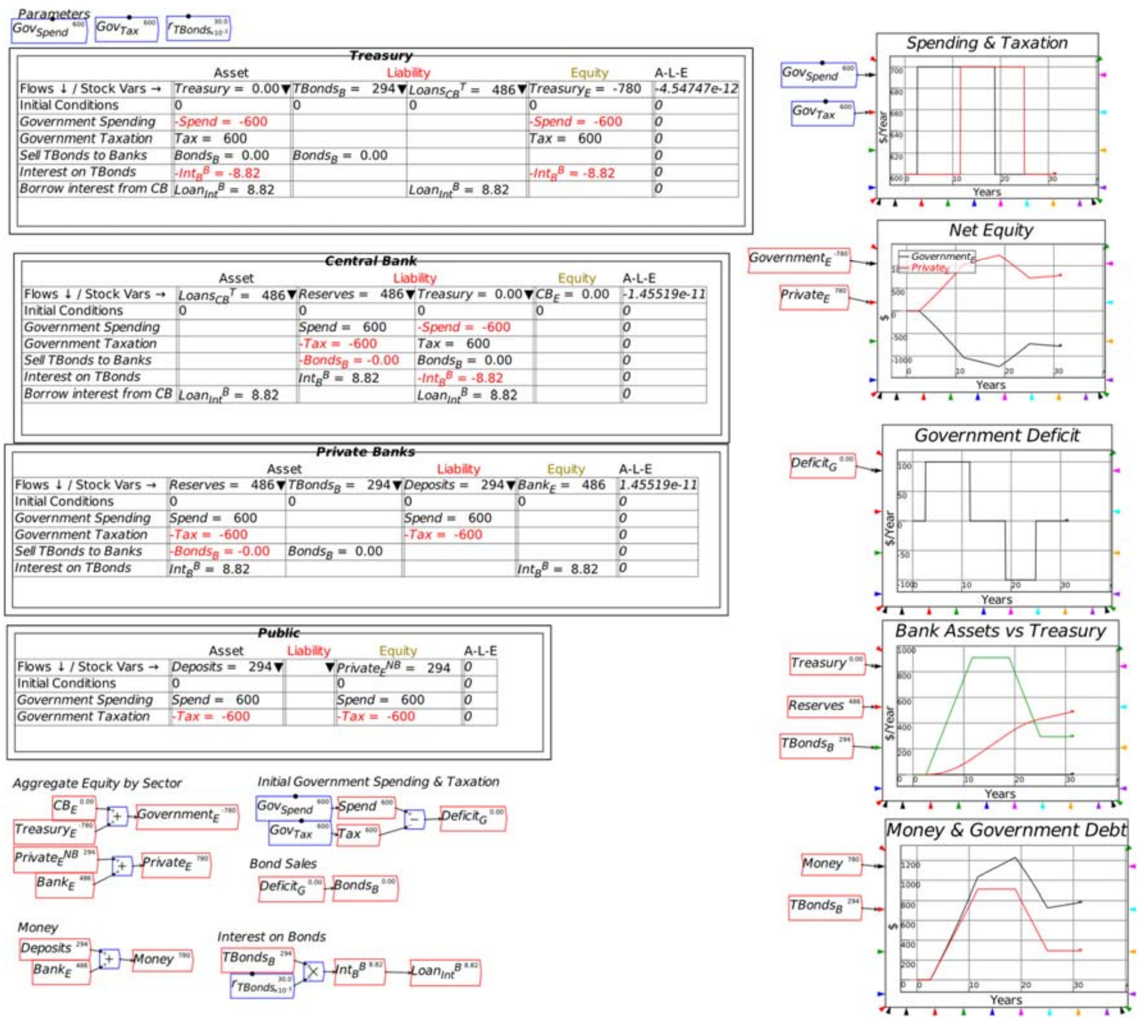
The government does, of course, pay interest on Treasury Bonds. I assume that it then borrows the funds needed to pay the interest from the Central Bank—see Figure 134.

Figure 134: Interest on Bonds



Introducing this fact has a dramatic effect on the model: compare Figure 135 to Figure 133, and you will see that private bank equity, which remained at zero with no interest on bonds, turns positive when interest is paid, because the interest on bonds adds to bank reserves, without also adding to liabilities. So it increases bank equity, which has been zero through all the previous models. Notice also that the amount of money in existence exceeds the amount of bonds—otherwise known as government debt to the private sector. The difference is made up by the Treasury's debt to the Central Bank, which is equivalent to the total interest paid on Treasury bonds.

Figure 135: Interest on bonds creates positive equity for the banking sector



What this shows is that a government deficit can actually "kick-start" a private banking system, by creating positive net equity for the banking sector, which is necessary for real-world lending: a bank with negative equity is bankrupt, while to start operation as a bank, a private corporation needs to raise share capital so that it can start with its activities as a bank with positive equity.

Before writing *Manifesto*, I had primarily used Minsky to model the dynamics of private credit—largely because that's the topic that mainstreamers like Krugman get so badly wrong. One puzzle, when working with models of a pure credit economy, was how did banks accumulate the positive equity needed to operate: in a pure credit system, positive equity for the banking sector means identical negative equity for the non-banking sectors.

This model shows that, arguably, interest on government bonds enables the banking sector to have positive equity, without driving the non-banking sector into negative equity, because the bonds create positive equity for the non-banking sector (notice that the equity of the public in Figure 135 is 294, which is identical to the value of bonds on issue), while the interest on those bonds creates positive equity for the banking sector (the positive equity of the banking sector, of 486, is identical to the debt of the Treasury to the Central Bank, which is identical to the sum of interest paid on bonds).

One personal opportunity cost of all the time I waste reading Neoclassical literature is that I am not up to date on the literature of MMT. That said, I am not aware of any MMT authors making this same case—that interest paid on Treasury bonds creates positive equity for the banking sector. If any reader knows of papers making that case, please let me know and I'll read them and cite them here. That said, this may be a novel discovery—and a good reason for the rate of interest on Treasury Bonds to be positive.

This model can obviously be extended to include private bank lending, and financial transactions between subsectors of the non-bank public.

8.3 An integrated view of deficits and credit: pp. 59-65 of *Manifesto*

This section of the *Manifesto* covers a vital episode in America and the world's economic history: the boom of the 1920s and the bust of the 1930s. These events should be the focus on economic analysis, but instead, Neoclassical economists prefer to ignore them as "outlier events". The Great Depression, according to them, shouldn't have happened; and nor should the Stock Market reach such ridiculous heights and then crashed. But of course they did happen.

Before I use *Minsky* to explain why this boom and bust happened, I'm going to take the opportunity here to go more deeply into the data than I had space for in *Manifesto*. The first thing to note is that this was a time of extreme volatility, compared to the relative stability of the post-WWII economy—a point that was fundamental to Hyman Minsky's analysis of capitalism:

> The most significant economic event of the era since World War II is something that has not happened: there has not been a deep and long-lasting depression.
>
> As measured by the record of history, to go more than thirty-five years without a severe and protracted depression is a striking success. Before World War II, serious depressions occurred regularly. The Great Depression of the 1930s was just a "bigger and better" example of the hard times that occurred so frequently. This postwar success indicates that something is right about the institutional structure and the policy interventions that were largely created by the reforms of the 1930s.
>
> Can "It"—a Great Depression—happen again? And if "It" can happen, why didn't "It" occur in the years since World War II? These are questions that naturally follow from both the historical record and the comparative success of the past thirty-five years. To answer these questions it is necessary to have an economic theory which makes great depressions one of the possible states in which our type of capitalist economy can find itself. We need a theory which will enable us to identify which of the many differences between the economy of 1980 and that of 1930 are responsible for the success of the postwar era. (Minsky 1982, p. xix)

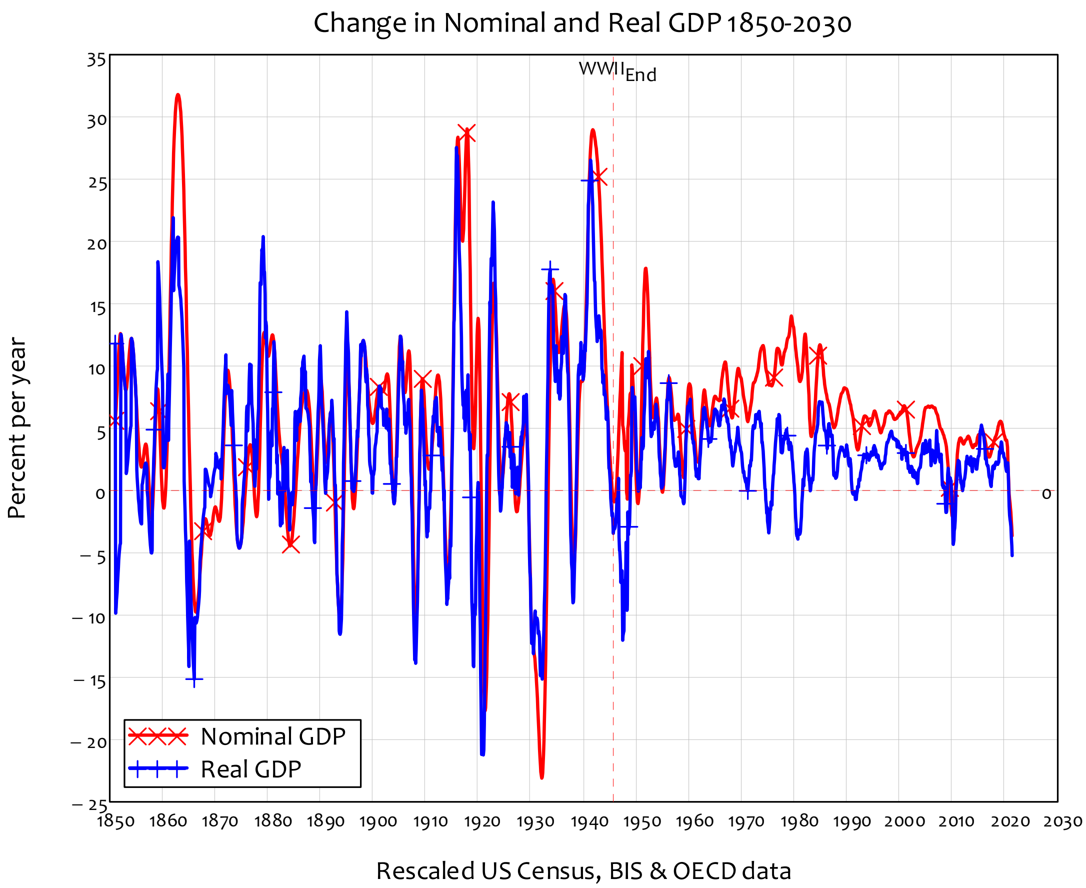
The volatility of the pre-WWII period is striking, which we, embedded in out post-WWII "Baby Boomer"/Gen X/ Gen Y reality, can fail to fully appreciate. Table 2 and Figure 141 show that not only was real growth higher in the pre-WWII period (averaging 3.7% per year versus 2.5% since 1945), it was also much more volatile: the ratio of the standard deviation of growth to the rate of growth was 2.2 for 1850-1945 versus 1.17 for 1945-2021.

Table 2:Growth and Volatility 1850-2030

	Nominal Growth			Real Growth		
Period	Mean	Standard Deviation	Ratio	Mean	Standard Deviation	Ratio

1850-1945	5.3%	9.6%	1.83	3.7%	8.2%	2.21
1945-2021	6.2%	3.2%	0.52	2.5%	3.0%	1.17

Figure 136: Pre-WWII Volatility & Post-WWII Stability of growth

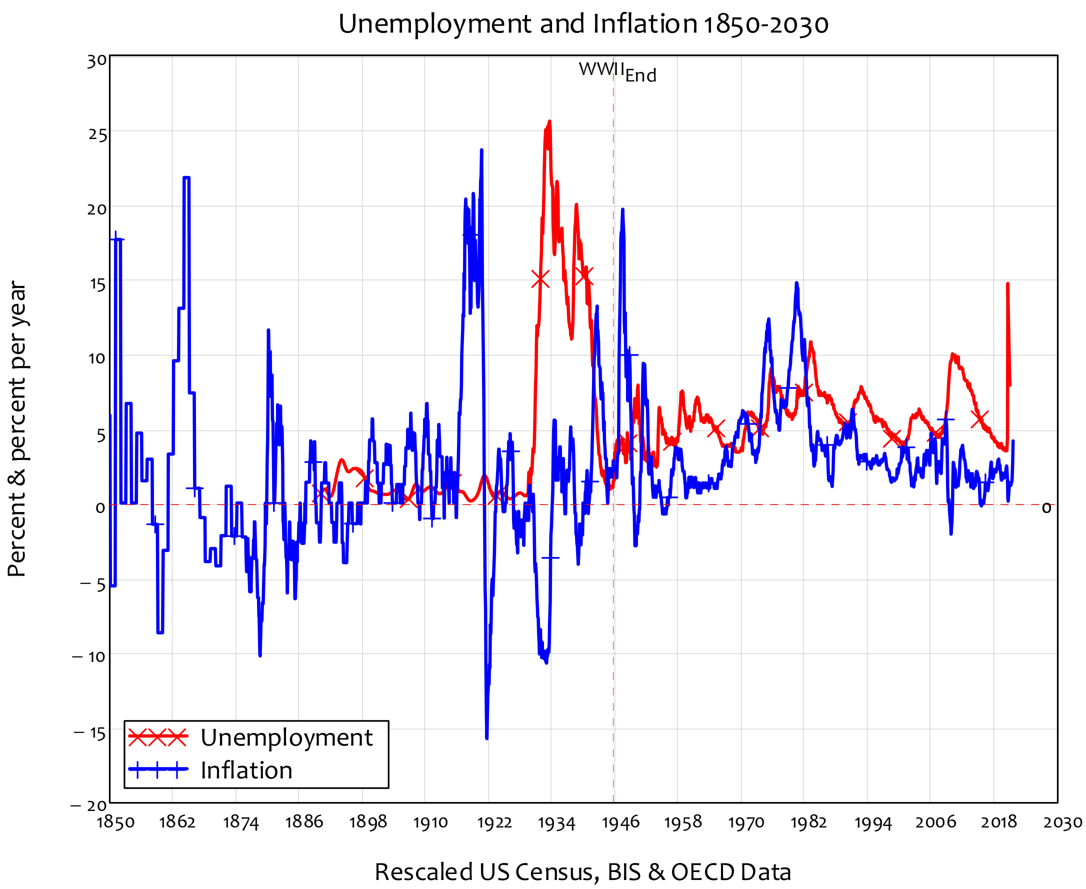


The same applies to the unemployment and inflation rates: both were lower on average before the end of WWII than in the post-War period, but the volatility was far higher in the pre-War period than after

Table 3: Unemployment and Inflation Volatility 1850-2030

	Unemployment			Inflation		
Period	Mean	Standard Deviation	Ratio	Mean	Standard Deviation	Ratio
1850-1945	4.2%	6.5%	1.5	1.0%	6.4%	6.1
1945-2021	5.7%	1.7%	0.3	3.7%	3.3%	0.9

Figure 137: Unemployment and Inflation Volatility 1850-2030



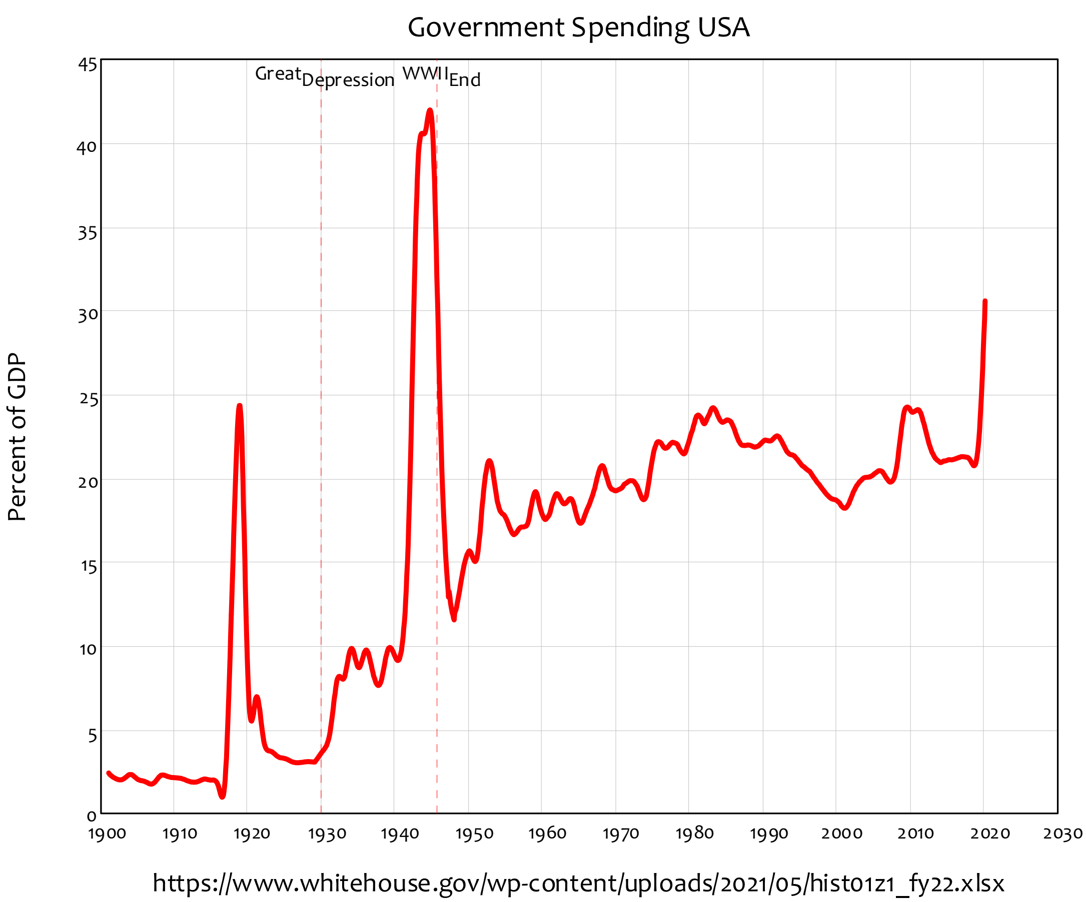
Minsky was obviously right that there was a significant change in the nature of American capitalism after WWII, which he identified as the evolution of "Big Government":

> Whereas in the small government economy of the 1920s profits were well nigh exclusively dependent on the pace of investment, the increase in direct and indirect state employment along with the explosion of transfer payments since World War II means that the dependence of profits on investment has been greatly reduced.
>
> *With the rise of **big government**, the reaction of tax receipts and transfer payments to income changes implies that any decline in income will lead to an explosion of the government deficit.* Since it can be shown that profits are equal to investment plus the government's deficit, profit flows are sustained whenever a fall in investment leads to a rise in the government's deficit.
>
> *A cumulative debt deflation process that depends on a fall of profits for its realization is quickly halted when government is so big that the deficit explodes when income falls*. The combination of refinancing by lender-of-last-resort interventions and the stabilizing effect of deficits upon profits explain why we have not had a deep depression since World War II. The downside vulnerability of the

> economy is significantly reduced by the combination of these types of "interventions." (Minsky 1982, p. xxviii. Emphasis added)

As Figure 138 shows, the transition from small to Big Government is emphatically a product of the Great Depression and World War II. World War I caused a sharp spike in government spending as a percentage of GDP, but the post-War period saw a rapid return to small government—the pre-WWI period had, from today's perspective, an unthinkably low level of government spending of just 2% of GDP. World War I saw that rise to almost 25%, but it fell rapidly back to below 5% of GDP in the early 1920s. It then continued to fall during that decade, as Coolidge used the prosperity of the era to reduce government debt by running a surplus that, across much of the decade, was equivalent to 1% of GDP.

Figure 138: From small to Big Government between the Great Depression and the end of WWII



This is what Calvin Coolidge thought was responsible for the apparent prosperity of The Roaring Twenties. To repeat, at much greater length than in *Manifesto*, the quote from Coolidge's State of the Union address, he attributed the prosperity of the 1920s to his policy of running a sustained government surplus, and using it to pay down government debt:

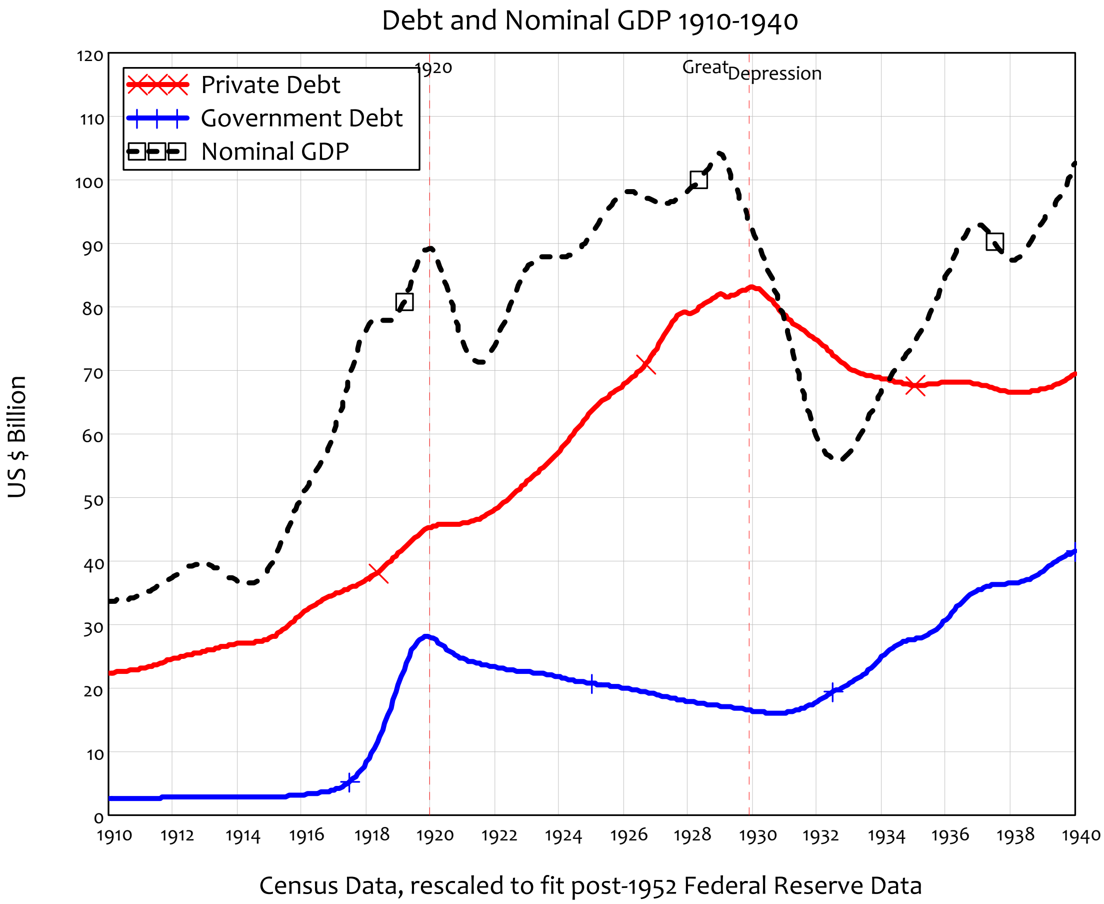
> No Congress of the United States ever assembled, on surveying the state of the Union, has met with a more pleasing prospect than that which appears at the present time. In the domestic field there is tranquility and contentment, harmonious relations between management and wage earner, freedom from industrial strife, and the highest record of years of prosperity… The country can regard the present with satisfaction and anticipate the future with optimism.

> The main source of these unexampled blessings lies in the integrity and character of the American people… Yet these remarkable powers would have been exerted almost in vain without the constant cooperation and careful administration of the Federal Government…
>
> Wastefulness in public business and private enterprise has been displaced by constructive economy… We have substituted for the vicious circle of increasing expenditures, increasing tax rates, and diminishing profits the charmed circle of diminishing expenditures, diminishing tax rates, and increasing profits…
>
> One-third of the national debt has been paid … the national income has increased nearly 50 per cent, until it is estimated to stand well over $90,000,000,000. It has been a method which has performed the seeming miracle of leaving a much greater percentage of earnings in the hands of the taxpayers with scarcely any diminution of the Government revenue. That is constructive economy in the highest degree. It is the corner stone of prosperity. It should not fail to be continued…
>
> Last June the estimates showed a threatened deficit for the current fiscal year of $94,000,000. Under my direction the departments began saving all they could out of their present appropriations… The combination of economy and good times now indicates a surplus of about $37,000,000. This is a margin of less than 1 percent on our expenditures and makes it obvious that the Treasury is in no condition to undertake increases in expenditures to be made before June 30. It is necessary therefore during the present session to refrain from new appropriations for immediate outlay, or if such are absolutely required to provide for them by new revenue; *otherwise, we shall reach the end of the year with the unthinkable result of an unbalanced budget*. (Coolidge 1928, December 4 1928. Emphasis added)

The June to which Coolidge referred was June, 1929. The “unthinkable result” at the end of that year was not “an unbalanced budget”, but America’s second Great Depression (the first, as I note in *Manifesto*, was “The Panic of 1837”).

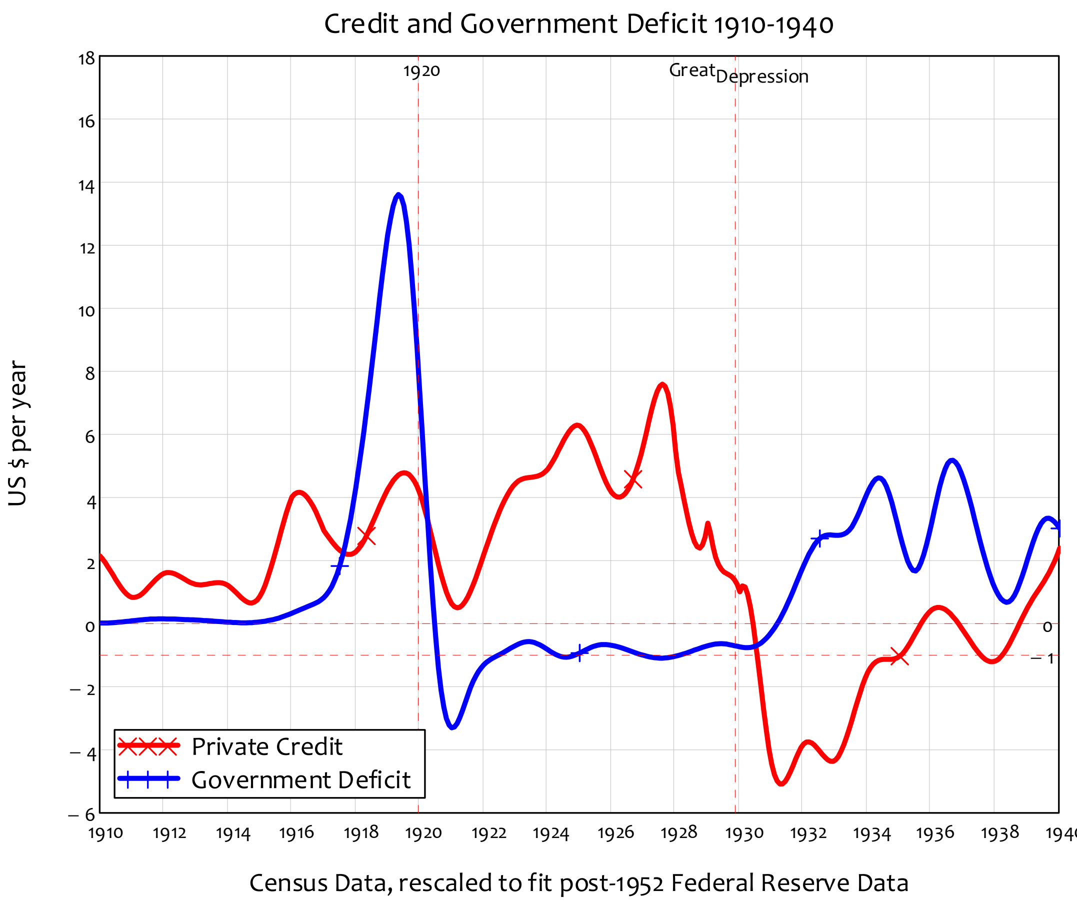
Coolidge was acutely aware of the declining government debt of his time. However, he, like his successors one century later, had no idea of what was happening with private debt. As he applauded halving government debt, from $30 to roughly $15 billion, private debt rose from $45 to $80 billion—see Figure 139. He was, as Clinton and Bush did in the 1990s and early 2000s, conducting an unwitting experiment into how long credit-based money creation could counter the destruction of fiat-based money, without causing a crisis.

Figure 139: GDP, Private and Government Debt 1910-1940



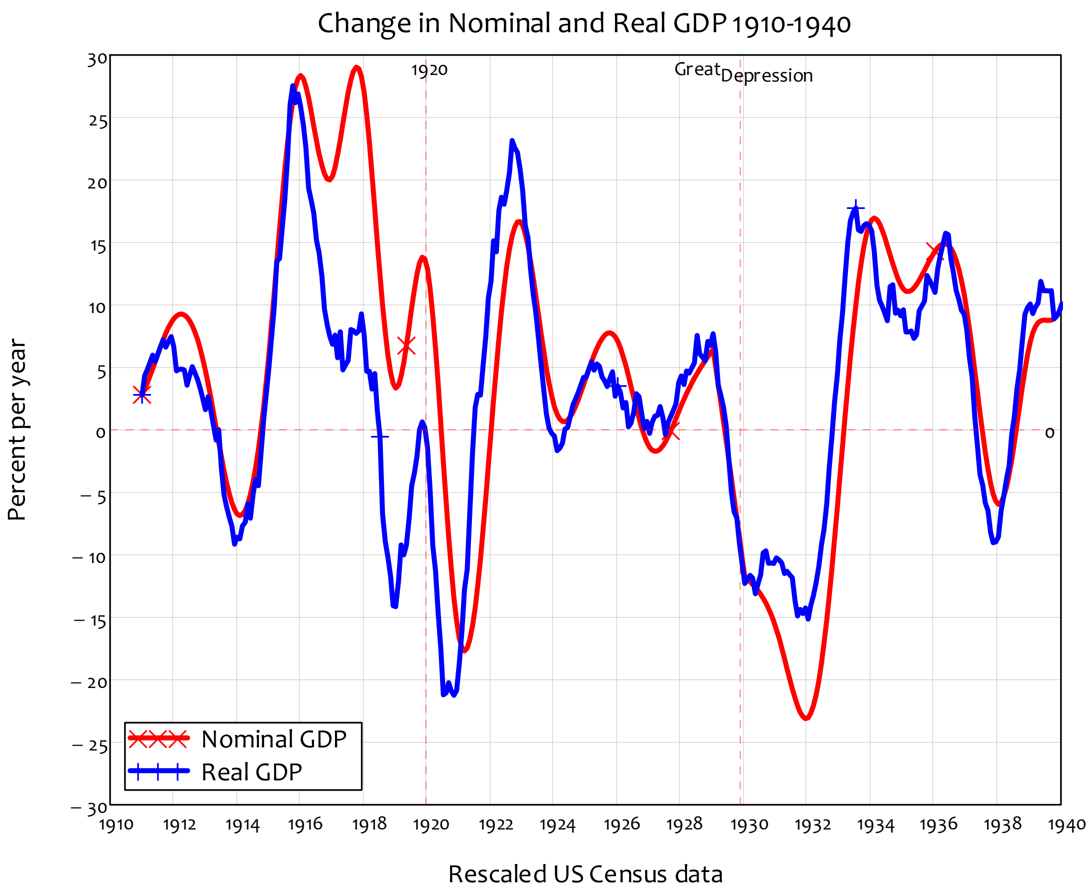
In Coolidge's time, the answer turned out to be "about 8 years". Between 1921 and 1929, the government surplus of roughly $1 billion a year was more than counterbalanced by credit of between $1 billion and $8 billion per year—see Figure 140.

Figure 140: Private Credit and the Government Deficit 1910-1940



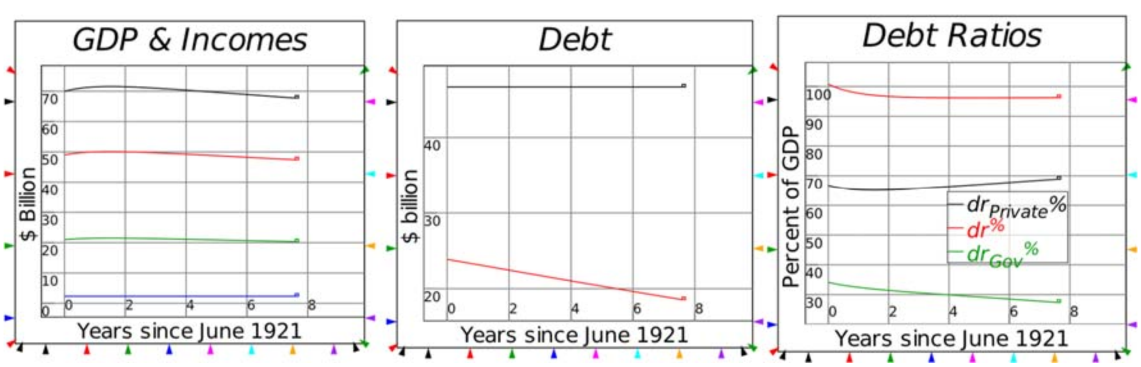
While that situation endured, GDP rose from a low of $71 billion in 1921 to a maximum of $104 billion at the start of 1929—see the dotted plot in Figure 139. It was a volatile performance, as Figure 141 indicates—inflation-adjusted growth ranged from as low as minus 21% in 1921 to a high of 23% in 1922—but the average was still a very high 5.8% real growth per year. Coolidge's rhetoric extrapolated this rate of growth forward thanks to his good budget management, but that's not what transpired: the average real growth rate from the high of 1929 till the 1930s low in 1933 was *minus* 9% per year. Nominal GDP in 1933 was $16 billion lower than in 1921.

Figure 141: Volatile GDP



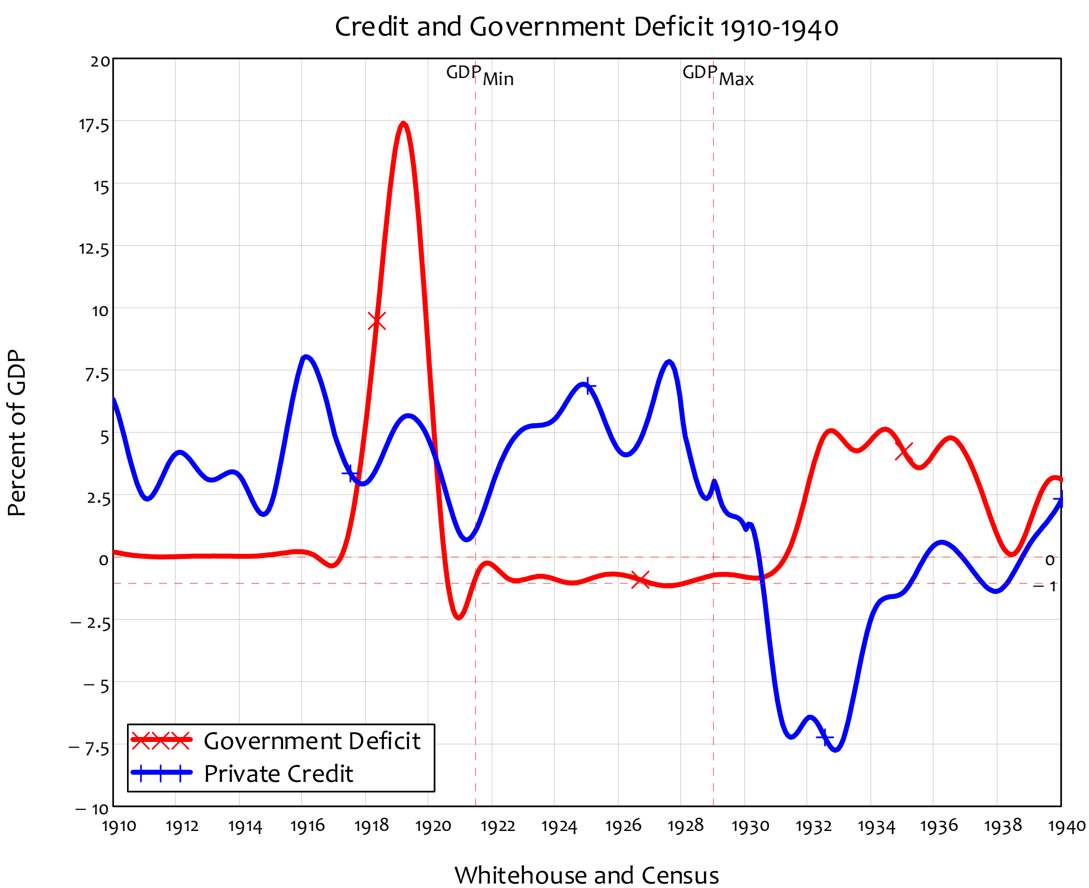
The core feature of the Great Depression that, even today, is seared into people's minds, is the huge increase and then sustained level of unemployment. Unemployment data wasn't as systematic back then—much of it was recorded by trade unions—but nor was it as corrupted as is has become in the last fifty years: back then you were recorded as unemployed if you had registered as unemployed, either with your union or an employment office. The boom of the 1920s was so extreme at its end that the percentage rate of unemployment in October 1929 was zero. Three years later, it was an unprecedented 25%, and it remained at elevated levels throughout the 1930s.

Figure 142: Unemployment and Inflation 1910-1940



There is much more to the story of the 1920s and 1930s than just government money destruction and private money creation. But since that part of the story has never been properly told—because the mainstream, as well as misunderstanding the role of fiat money creation in a well-functioning capitalist economy, also continues to deny the role of credit in aggregate demand—I'm going to attempt to use Minsky to reproduce the effects of government surpluses and private credit across the boom period of the 1920s. I'll define this as starting at the nadir for nominal GDP in the 1920s—mid 1921, when the USA's nominal GDP was US$72.25 billion—and ending at its apogee in 1929, when it peaked at $104.6 billion. Across almost all of that time, the government surplus was 1% of GDP, while credit began at 1% of GDP and peaked at 8% in mid-1927—see Figure 143. I'm showing the deficit rather than the surplus, since a deficit has the same impact on the economy as positive credit. So, for most of the 1920s, the government deficit was minus 1% of GDP. Though it fluctuated much more than the government surplus, the average value of credit between 1921 and 1929 was 5% of GDP.

Figure 143: Government deficits and private credit 1910-1940



The data that I wish to emulate in a *Minsky* model are the following:

Table 4: Historical data to emulate in the Minsky model

Parameters	% of GDP	Date	GDP	Government Debt	Private Debt
Deficit	-1%	1921.5	$ 71.26	$ 23.87	$ 46.66
Credit	5%	1929	$ 104.60	$ 17.20	$ 82.07

I have created a very simplified model here, because I want as few complications as possible to get in the way of the three basic questions that I want to pose: what would have happened to the economy had the private sector not gone on a borrowing binge; and what would have happened had Coolidge run either a deficit, or a balanced budget during that private sector borrowing binge; and what would have happened had Coolidge run a deficit while the private sector's debt remained constant?

Table 5: Godley Table for the banking sector in the model

Banking Sector	Asset		Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	$Loans_F$	Reserves	Firms	Workers	$Bank_E$	-0.028
Initial Conditions	46.658	23.874	63	1.96	5.6	-0.028
Government Deficit		Deficit	Deficit			0
Bank lending	Credit		Credit			0
Bank interest			$-Int_F$		Int_F	0
Hire workers			-Wages	Wages		0
Workers consume			$Cons_W$	$-Cons_W$		0
Bankers consume			$Cons_B$		$-Cons_B$	0

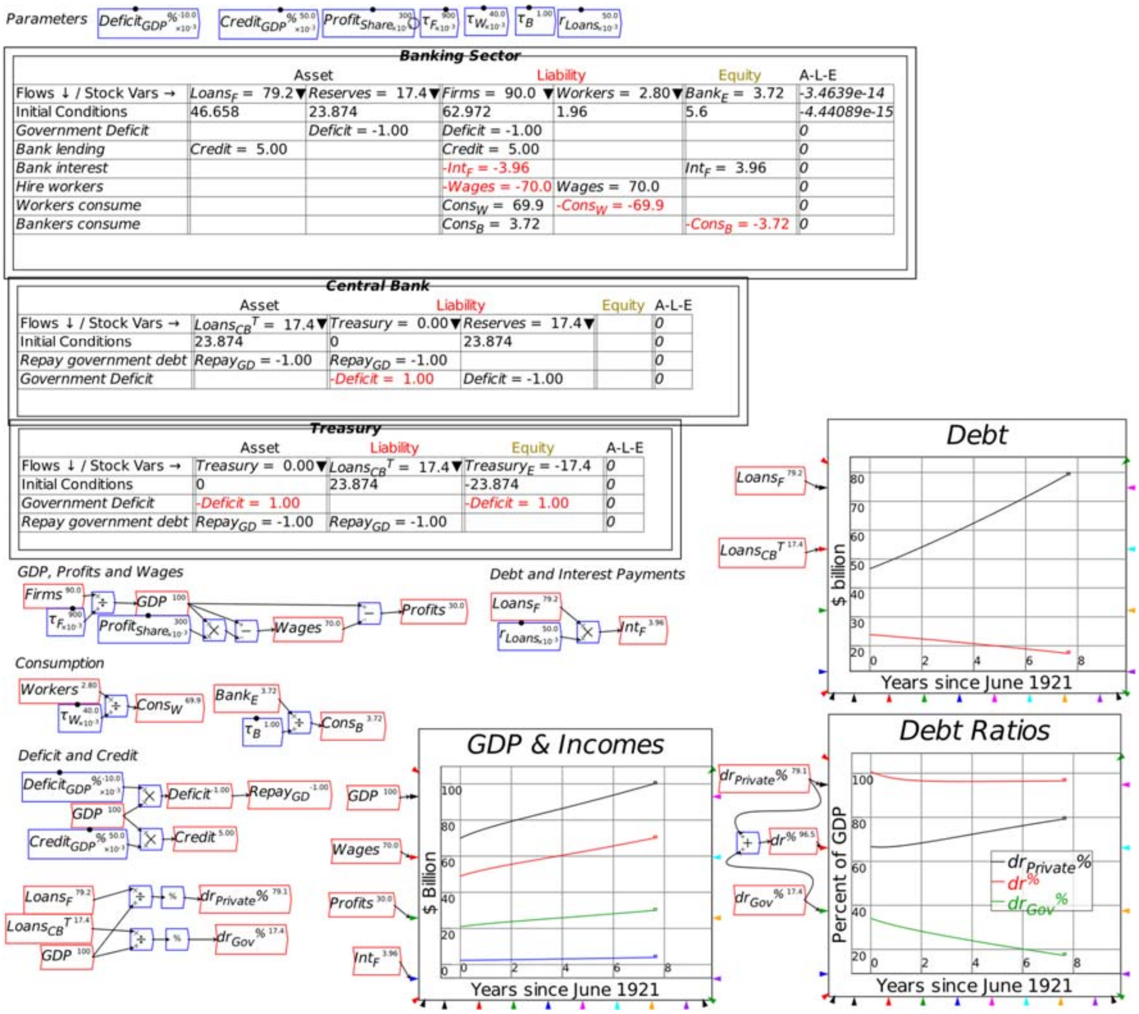
The model came pretty close to fitting the data (see Table 6), even though the time path of credit was much simpler—a constant 5% of GDP throughout, rather than the range from 1 to 7.5%—than the actual data.

Table 6: Simulation results

Simulation	% of GDP	Simulation Years	GDP	Government Debt	Private Debt
Deficit	-1%	0	$ 70.00	$ 23.87	$ 46.66
Credit	5%	7.5	$ 100.00	$ 17.40	$ 79.20

The full model is shown in Figure 144. It is, as noted, extremely simple: there are no government bonds for example, all the government debt is owed by the Treasury to the Central Bank. But as the previous model showed, bonds don't finance government spending: instead, they are a way of creating reserves for the private banks. They could be added here, and quite possibly could improve the accuracy of the simulation; but this is a sufficient structure to disentangle the causal factors behind the boom of the 1920s.

Figure 144: Simulating Coolidge's Golden Years

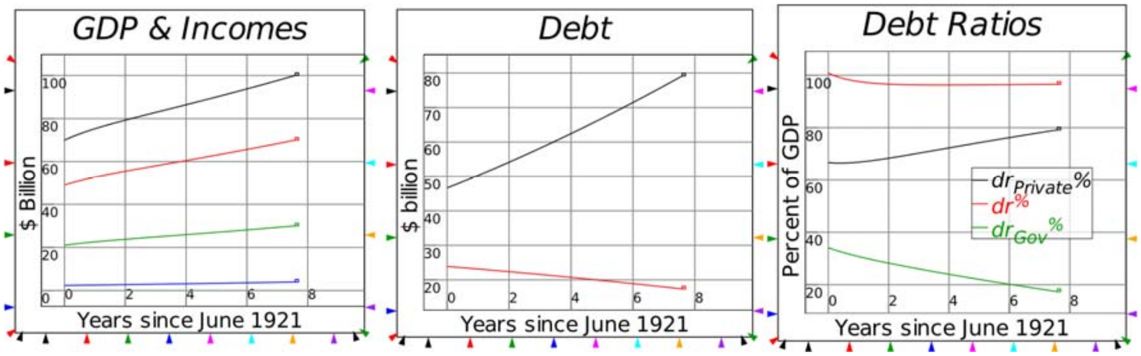


With the model constructed, we can now use it to answer those "what if?" questions:

- What if the private sector had not increased its debt?;
- What if Coolidge had run a balanced budget or deficit, while the private sector borrowed?;
- What if Coolidge had run a deficit while the private sector's debt remained constant?;
- What if Coolidge had run a deficit while the private sector borrowed; and
- How large a deficit would have been needed to reproduce the prosperity of the 1920s without increasing private debt?

All the subsequent plots of this model export the Canvas from the Plots tab.

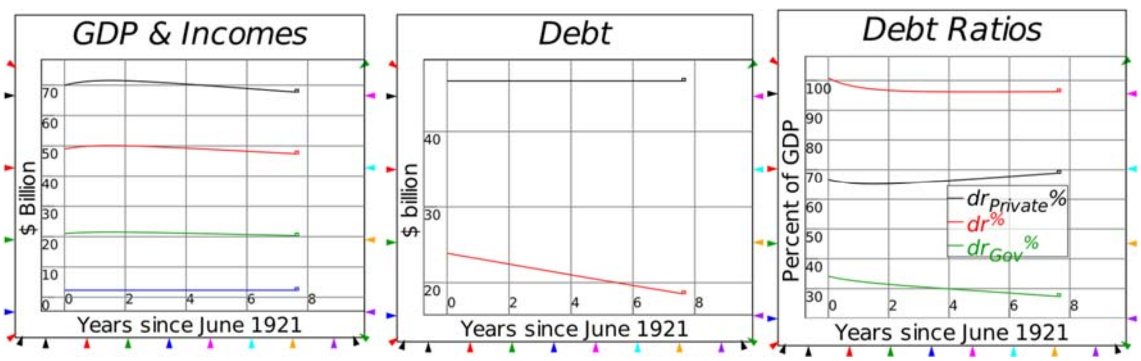
8.3.1 The actual event: Coolidge Surplus and private sector credit binge



8.3.2 Coolidge Surplus with no private sector borrowing

This first simulation indicates that, without the rise in private debt, there would have been no boom—no growth even—during the 1920s.

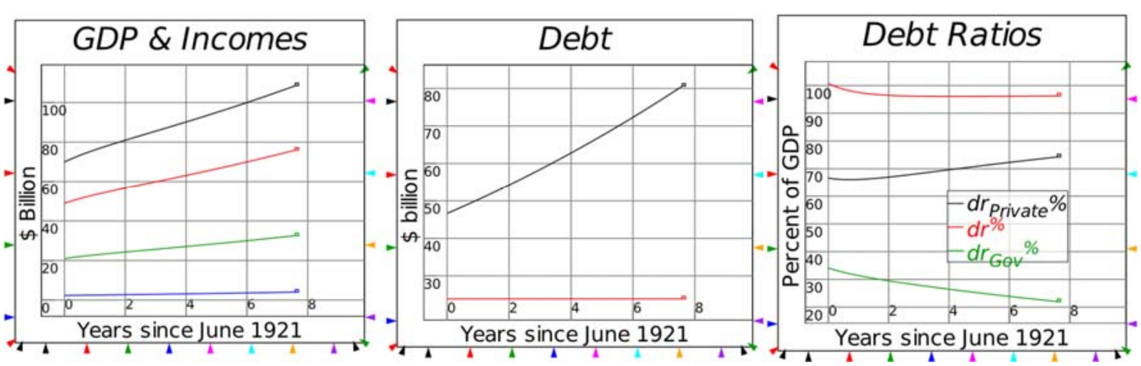
Figure 145: No credit: the Roaring Twenties loses its Roar



8.3.3 Coolidge Balanced Budget with the credit binge

This simulation indicates that the impact of Coolidge's surplus was to *reduce* the level of economic growth, compared to what it would have been with the credit binge alone.

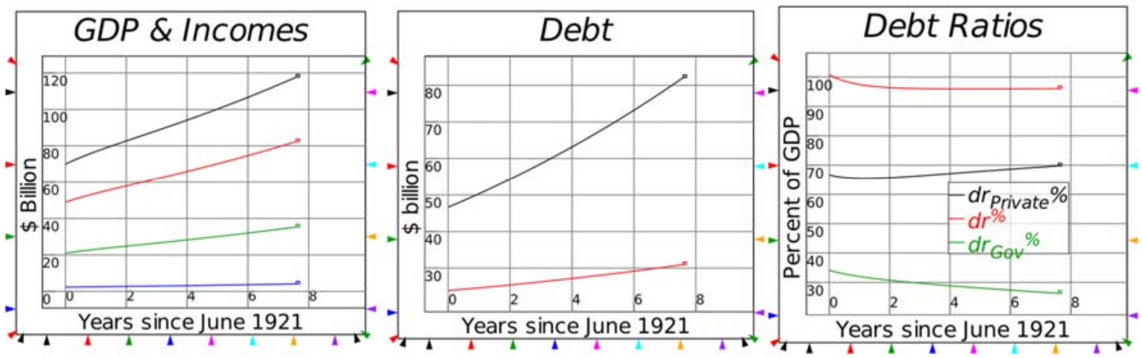
Figure 146



8.3.4 Coolidge runs a Deficit with the credit binge

This generates higher growth and a lower private sector debt ratio.

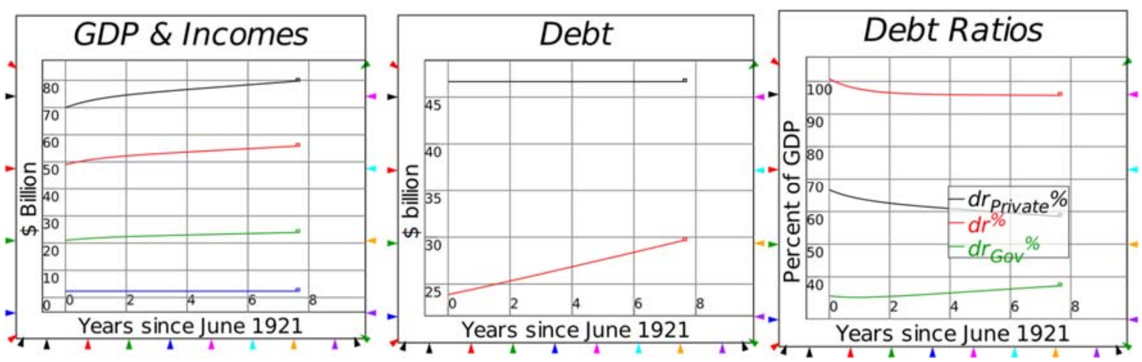
Figure 147



8.3.5 Coolidge runs a Deficit with no credit

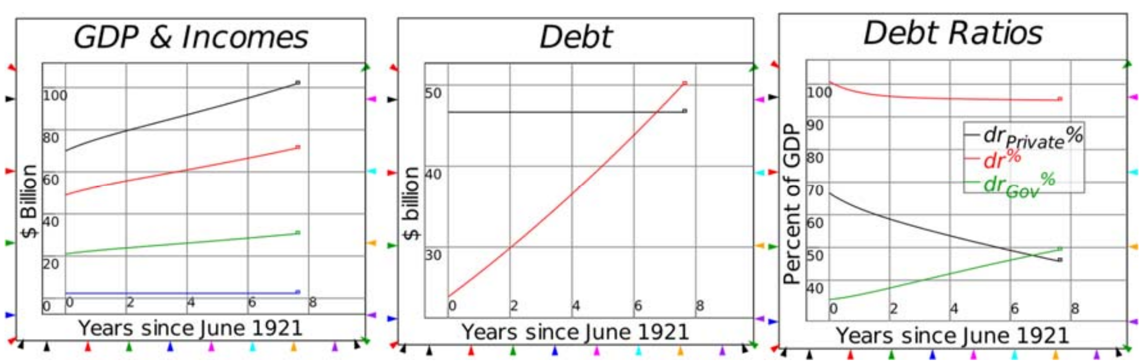
This is with the 1% of GDP surplus converted into a deficit: the economy grows rather than shrinks.

Figure 148



8.3.6 Coolidge runs a Deficit to reproduce the 1920s boom without credit

A deficit of 4% of GDP is sufficient to reproduce the boom of the 1920s, without any growth in private debt.



8.3.7 Disentangling cause and effect

This simple model shows the advantages of the monetary, system dynamics approach to modelling. But it would be nothing without including the key causal factors, which Neoclassical economics omit by assumption: private debt and credit. In the next chapter I'll explain my "Keen" model of Minsky's Financial Instability Hypothesis, which shows that capitalism can be overwhelmed by excessive private debt, and plunged into a deep and lasting downturn as credit turns substantially negative.

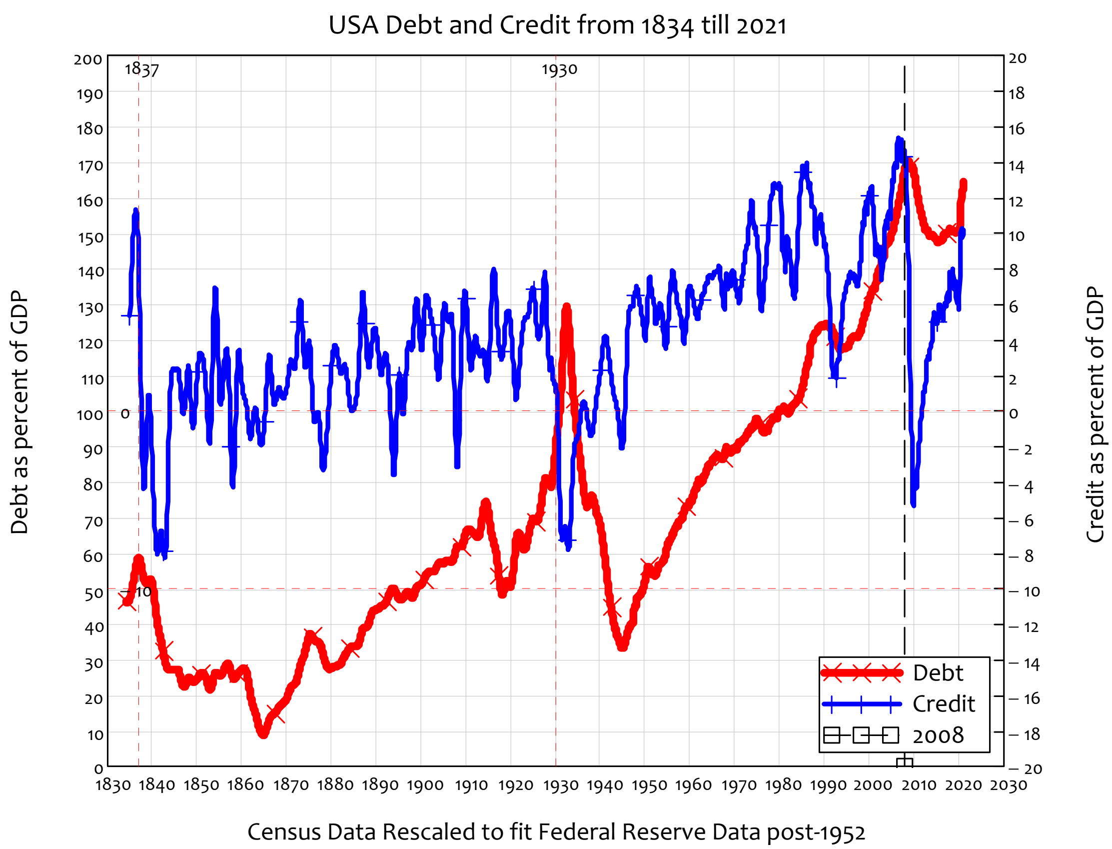
This has now happened three times in the history of American capitalism: during "The Panic of 1837", the Great Depression, and the Great Recession. The unifying factor in each of these crises has been a prolonged period of negative credit, as noted in Table 2.5 in *Manifesto* (reproduced here as Table 7).

Table 7: Magnitude of Credit and duration of negative credit in the USA's major economic crises

	Credit (annual change in private debt)			
	Credit as percent of GDP			Years
Crisis	Maximum	Minimum	Change	Negative Duration[30]
Panic of 1837	12.2	-8.9	21.1	6.2
Great Depression	9.1	-9.1	18.2	8.2
Great Recession	15.4	-5.3	20.7	2.6

The next four figures illustrate this, along with the rising level of private debt associated with each boom and bust, and the correlation of credit to the rate of economic growth (nominal in these plots, since I'm focusing here on the impact of credit on the monetary level of output: changes in the price level—but do not eliminate—the impact).

Figure 149: USA Debt and Credit since 1834



The major crises in America's economic history were all negative credit events, and Richard Vague's magisterial survey of global credit crises, *A Brief History of Doom* (Vague 2019), shows that this rule applies to all of global capitalism's roughly 150 crises in the last 150 years. The next three charts focus on America and its three major crises: the "Panic of 1837, the Great Depression, and the Great

[30] This is measured from the first negative month to the last, but includes some periods of positive credit (most of 1839, and late 1935 till late 1936).

Recession. Though the levels of private debt were substantially different, the scale of the negative credit events were quite similar: as shown by Table 7, each crisis was preceded by a credit boom, with credit-based demand reaching between 9% and 15% of GDP, while the plunge from this peak was roughly 20% of GDP in each case.

Figure 150: The Panic of 1837

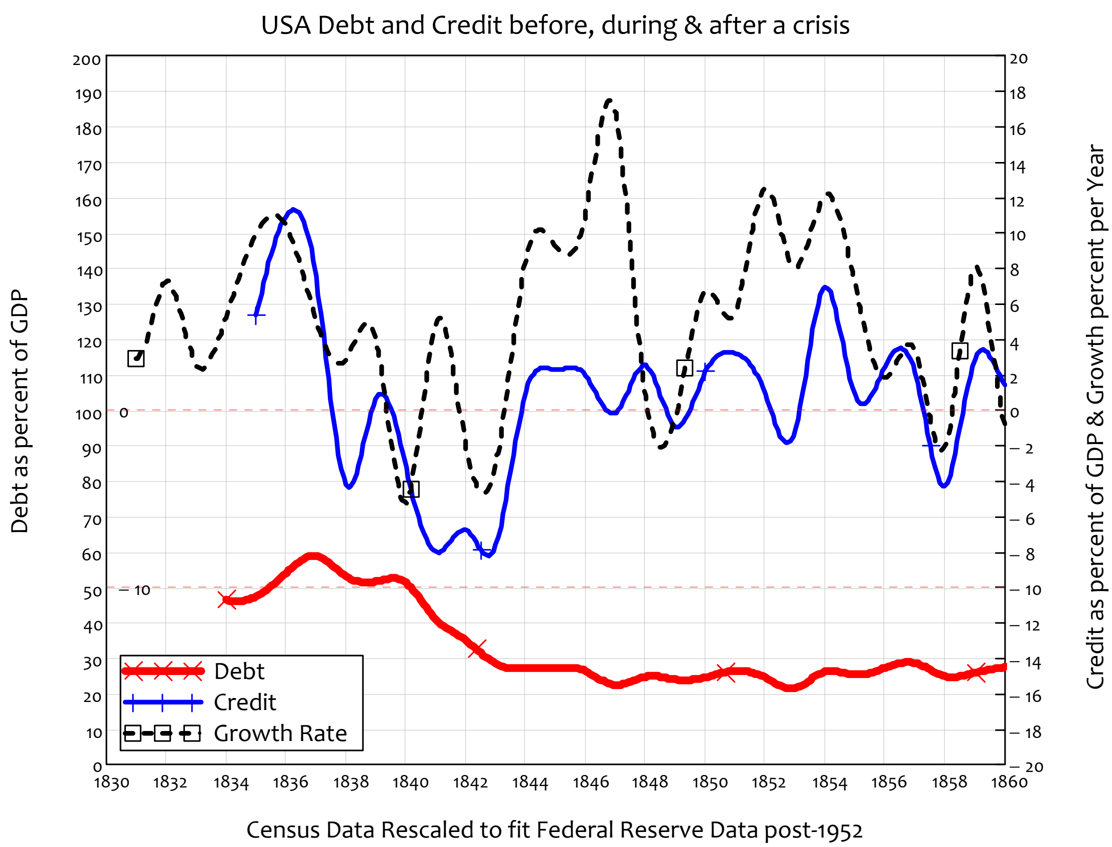


Figure 151: The Great Depression

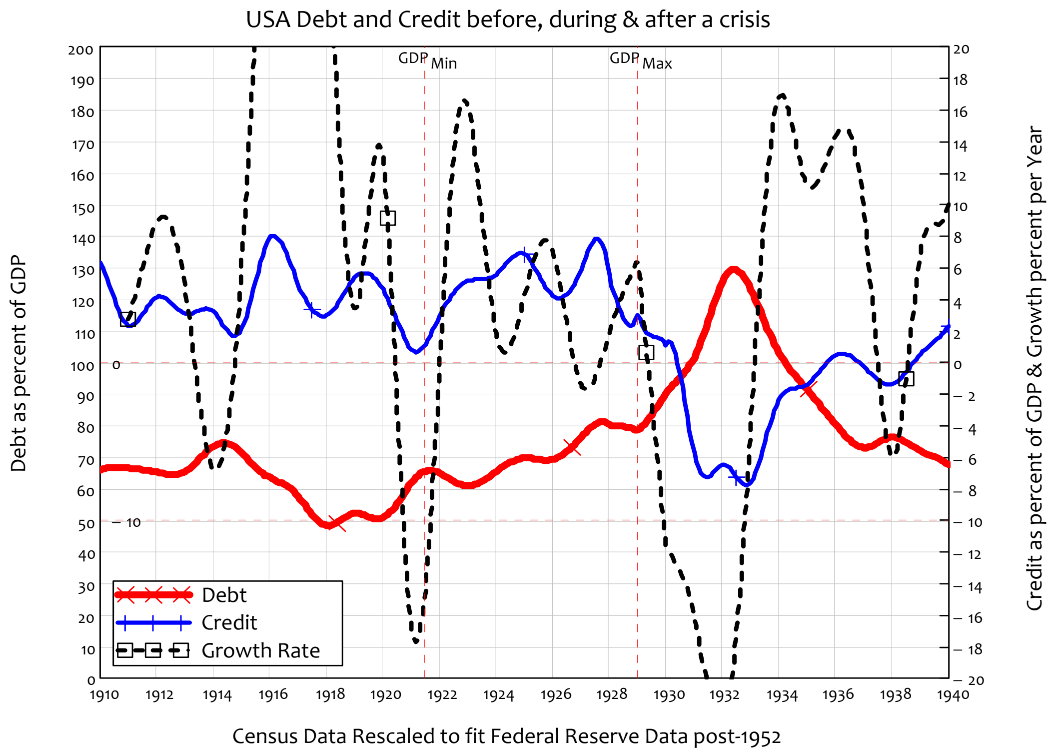
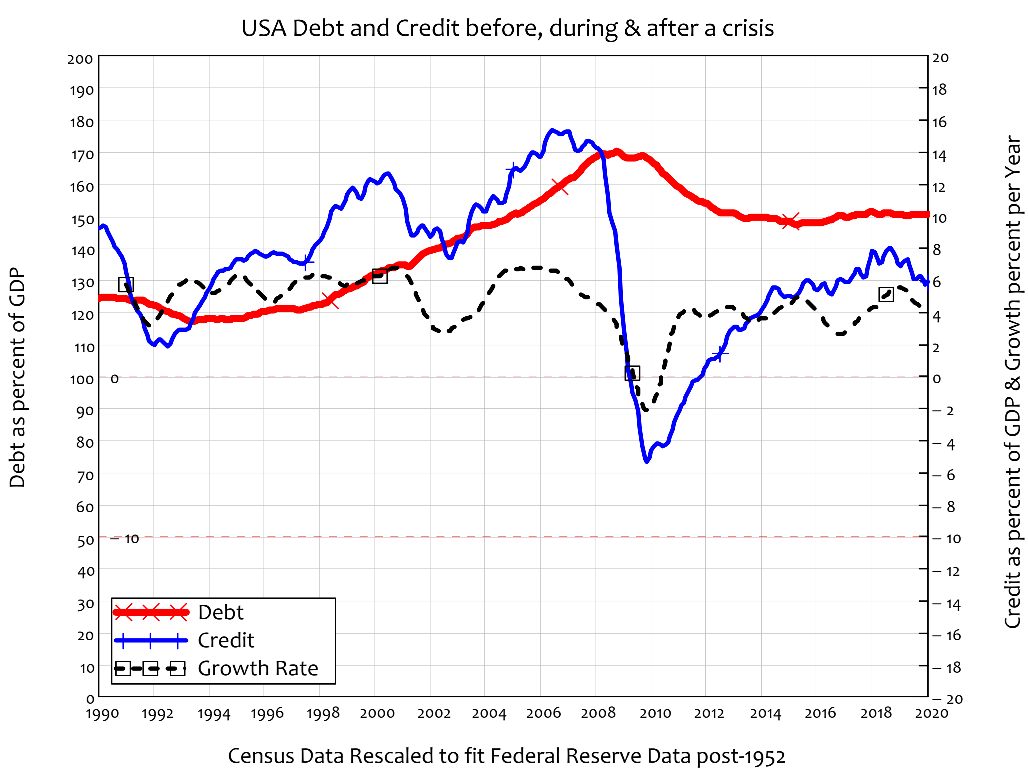


Figure 152: The Great Recession



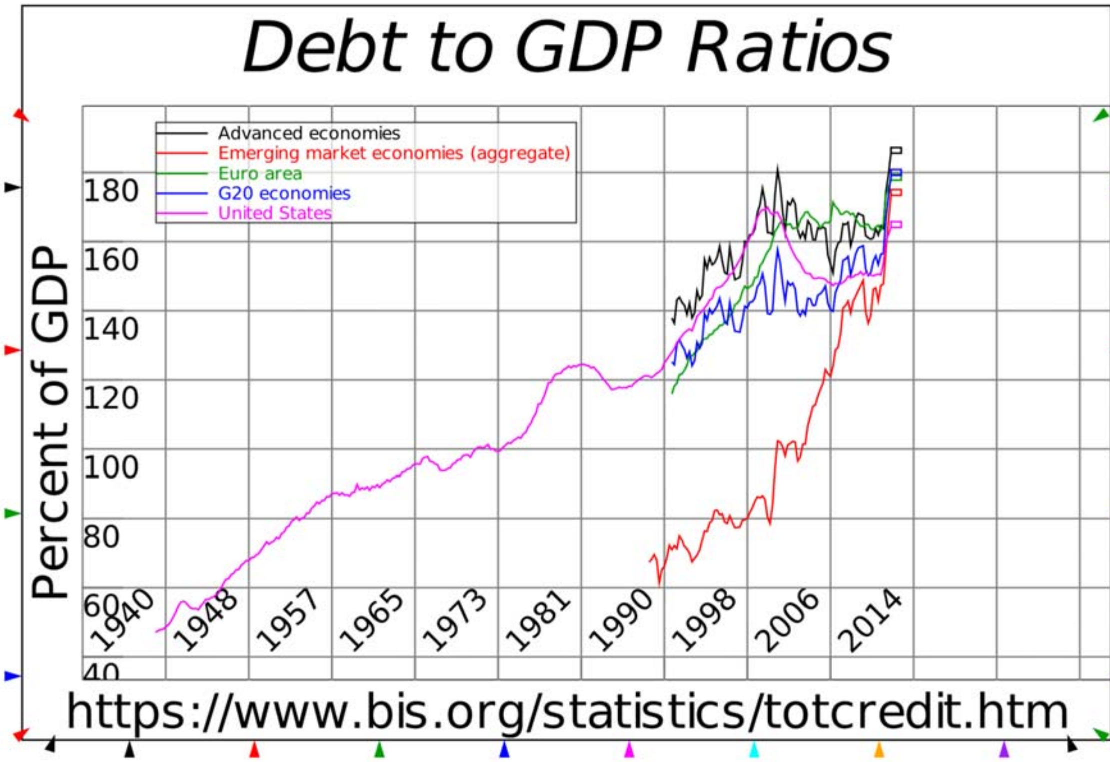
8.4 A Modern Debt Jubilee: pp. 65-68 of *Manifesto*

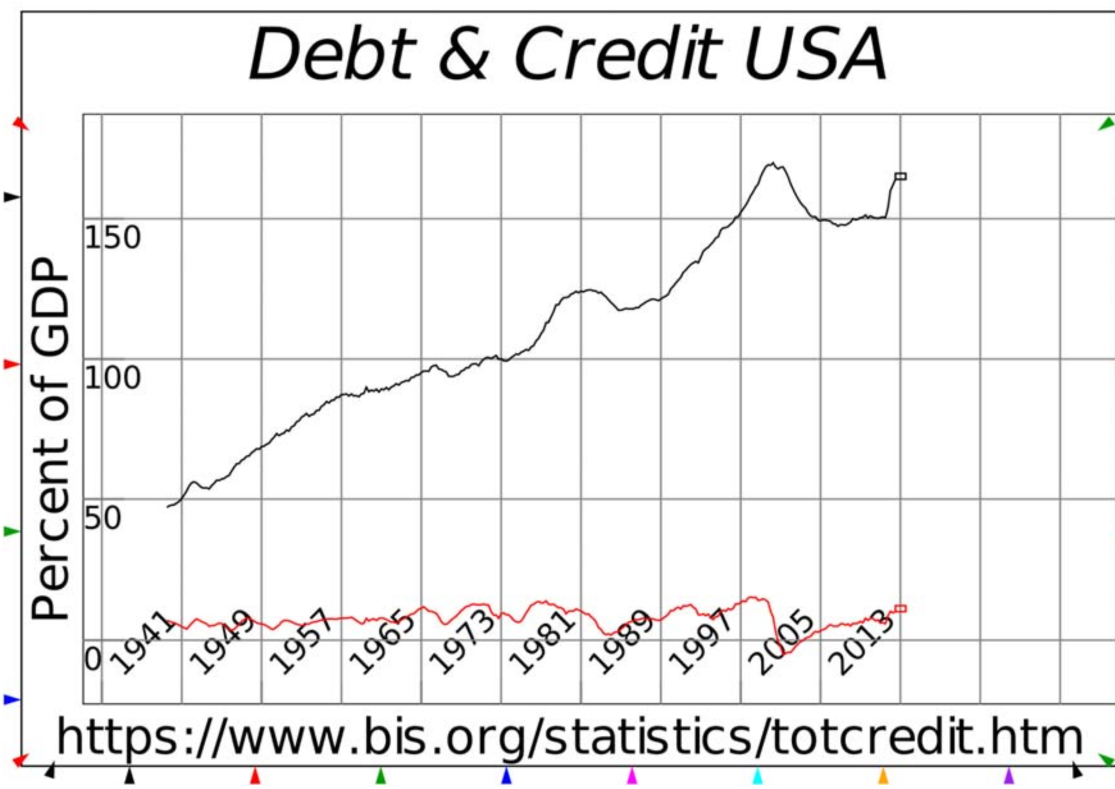
The end result of almost two centuries of widespread misunderstandings about how money is created, developed and maintained by the false beliefs of mainstream economists, is a global economy terminally indebted to the private creators of money. The previous charts show this phenomenon in the home of modern capitalism, America. The next chart--Figure 153—shows that this is a global phenomenon.

Here I have to thank the Bank of International Settlements for assembling an excellent database on debt across over 40 countries. When I started warning that a global financial crisis was imminent back in December 2005, the only data I could get easily was on America from the Federal Reserve Flow of Funds, and Australia from the Reserve Bank of Australia's statistical tables. Today, thanks to Bill White—who, as Research Director for the Bank of International Settlements, was the only person in an official position to warn that a financial crisis was likely (Borio and White 2004)—the Bank of International Settlements publishes a database with standardized measures of private and public debt from over 40 countries. That data shows unequivocally that the level of private debt, relative to GDP, is the highest it has been in the post-WWII period, which, by reference to long term data series for the USA and UK, is also the highest it has been in the history of capitalism—see Figure 153.

This has numerous deleterious effects on the economy, which I discuss with respect to my model of Minsky's Financial Instability Hypothesis in the next chapter. Here, I want to model something that I first proposed in 2012 (on January 1st, as it happens): a "Modern Debt Jubilee", as a means of escaping from this debt trap, by effectively replacing credit-based money with fiat-based money in a way that does not discriminate between those who had joined the 2000s speculative bubble and those who did not.

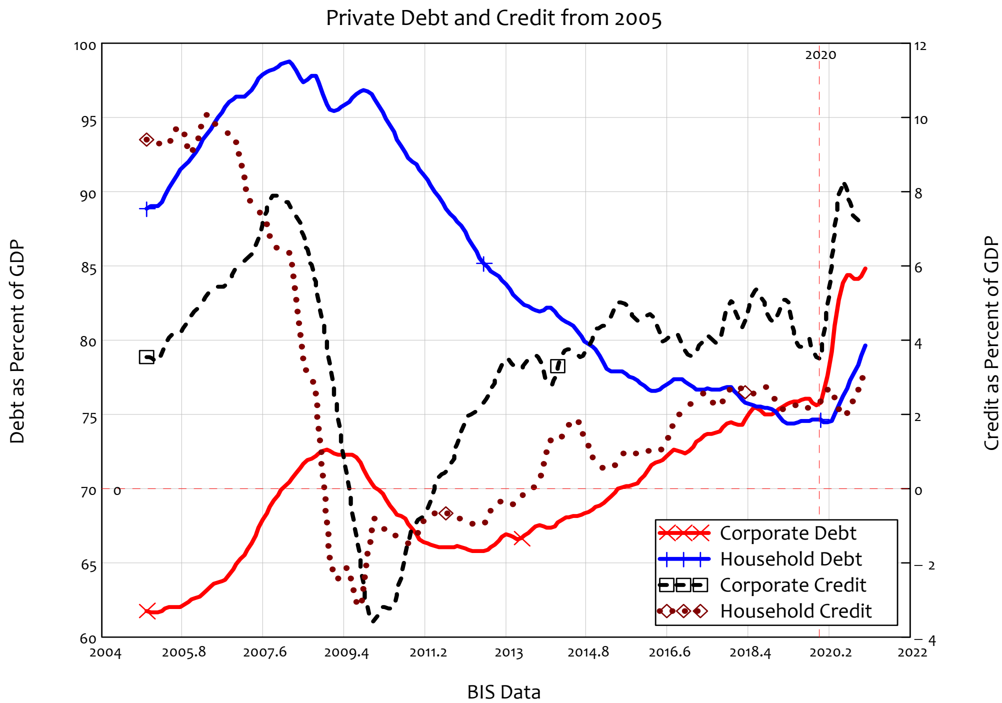
Figure 153: Record private debt levels afflict almost all economies





Though I thought of the idea a decade ago, I didn't subsequently develop it, because I believed that it had a snowflake's chance in Hell of actually being implemented. And then along came Hell, in the form of Covid. In 2020, private debt in the USA rose faster than it had even done.

Figure 154: Covid and its impact upon private debt and credit



All previous instances of rapidly rising private debt have occurred during a speculative binge. This one is occurring because the corporate sector in particular can't meet its financial obligations during Covid, and so has rolled over existing debt that would otherwise have been retired, and taken on new debt in order to meet financial commitments that were ordinarily covered from cash flow. So there will not be the typical economic boom from private sector borrowing—but there could well be the typical bust after Covid, if there is ever an after, especially if the welcome if insufficient government supports are removed too quickly.

This meant that it was time to actually model how a Modern Debt Jubilee could be undertaken, and the results surprised even me. The model in *Manifesto* was extremely simple—I just covered the accounting involved and showed that it was consistent: see Figure 155. I'll include the simulation model for this at the end of this chapter, just for the sake of completeness, but what I want to do now is develop a much more comprehensive model of a Modern Debt Jubilee, to show how it might be used to reduce America's private *and* government debt levels.

Figure 155: The basic mechanics of a Modern Debt Jubilee

	Asset					Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	*Reserves*	*Bonds$_J$*	*Loans$_D$*	*Loans$_F$*	*Savers*	*Debtors*	*Firms*	*Loans$_{CB}$^{PB}*	*Banks$_E$*	*0*
Initial Conditions	100	0	400	600	400	100	400	100	100	*0*
Jubilee for Debtors	*Jubilee$_D$*					*Jubilee$_D$*				*0*
Jubilee for Savers	*Jubilee$_S$*				*Jubilee$_S$*					*0*
Debtors pay off Debt			*-Jubilee$_R$*			*-Jubilee$_R$*				*0*
Savers buy shares					*-Jubilee$_I$*		*Jubilee$_I$*			*0*
Firms pay off Debt				*-Jubilee$_F$*			*-Jubilee$_F$*			*0*
Treasury sells Jubilee Bonds	*-Jubilee$_B$*	*Jubilee$_B$*								*0*
Interest on Jubilee Bonds	*Interest$_J$*								*Interest$_J$*	*0*

This is the first moderately large Minsky model that I've developed here, so it will take quite a while to explain its structure and dynamics. The basic structure of the Jubilee, and outcomes of the model, are as follows:

(1) The Jubilee is used to convert Jubilee % of existing private sector debt into government debt, thus converting credit-backed money into fiat-backed. Fiat-money increases, credit-money decreases.
(2) The Jubilee is distributed on a per capita basis, so every adult (person over 18) receives the same amount. If the per capita amount exceeds a person's debt, the excess is used to buy newly-issued corporate debt, which must be used to pay down corporate debt.
(3) The Jubilee creates money, but the allocation of it to debt repayment cancels precisely as much, so there is no net creation of money by the Jubilee itself.
(4) Treasury issues Jubilee Bonds, which are sold to the Banking sector. The Banking sector gets the funds to buy these bonds from the Jubilee itself, which creates excess Reserves equal in magnitude to the fiat money created by the Jubilee.
(5) Interest payments by Treasury on the Jubilee bonds then compensate the Banking sector for the fall in its income from interest on private debt
(6) Side-effects of the Jubilee include a fall in inequality and an increase in GDP from dramatic rise in the velocity of money. These occur because the Jubilee increases the money held by workers, whose higher propensity to spend also boosts the economy.
(7) If interest is paid on Jubilee Bonds, this creates money over time, thus expanding GDP.

Though the model is the most complicated to date, the monetary model is essentially a combination of the endogenous money model of Figure 97 with the Jubilee components of Figure 155. I have divided the non-bank private sector into three sectors—Firms, where output is produced, Capitalists, who own the Firms, and Workers, who work in the Firms. The first eight rows of Figure 156 are the basic financial operations of the private sector: interest and dividend payments, wages, and consumption. I have omitted bank lending and debt repayment here, just to simplify the model—they could easily be added.

The remaining rows implement the mechanics of the Jubilee:

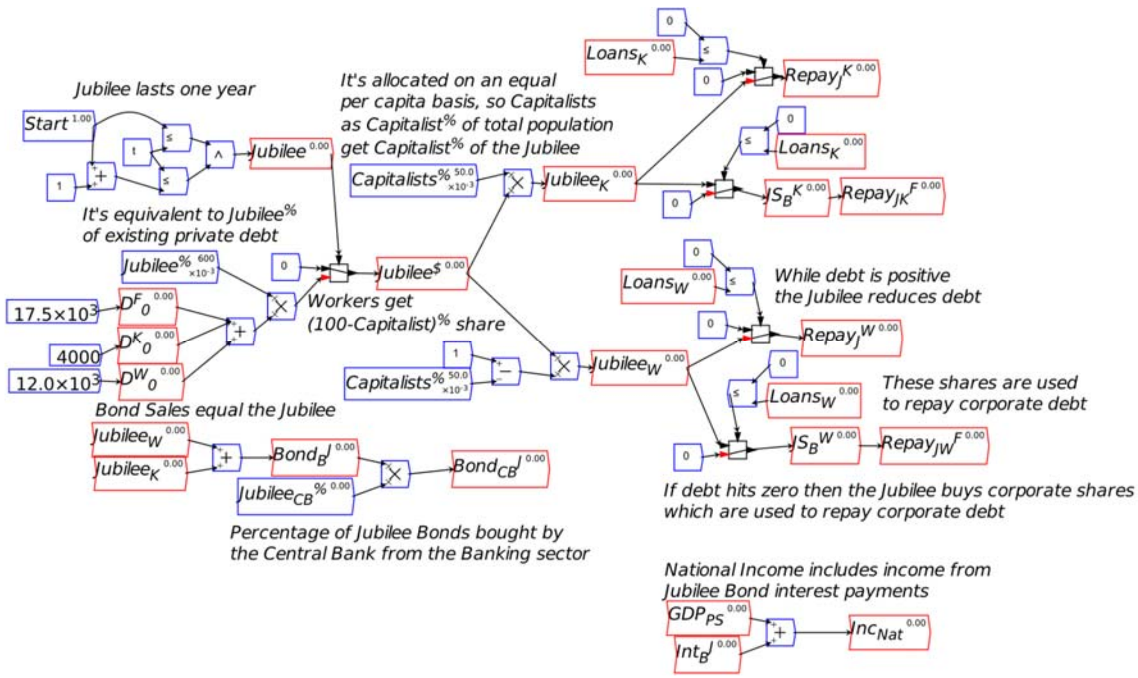
- Jubilee payments to workers and capitalists;
- Debt repayment by workers and capitalists;
- Share purchases by those workers and capitalists whose debts were less than the value of the per-capita Jubilee payment (set at 60% of aggregate private debt in this simulation, which is roughly equal to 100% of GDP in this simulation, and equal to the debt level of the US private sector);
- Firms using these share sales to pay down corporate debt;
- Bond sales by the Treasury, of a value equal to the Jubilee itself—which has created the excess Reserves that banks will use to buy the bonds; and
- Interest payments on those bonds by Treasury to the Banks.

Figure 156: Banking sector Godley Table for a Modern Debt Jubilee

	Asset							Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	$Reserves$	$Bonds_B$	$Bonds_B^J$	$Loans_F$	$Loans_W$	$Loans_K$	$Loans_{CB}^B$	$Firms$	$Workers$	$Capitalists$	$Banks$	0
Initial Conditions	4000	4000	0	17500	12000	4000	0	6500	450	16050	18500	0
Firms dividend payments								$-Div_{NS}$		Div_{NS}		0
Firms interest payments								$-Int_F$			Int_F	0
Capitalist consumption								$Cons_K$		$-Cons_K$		0
Capitalist interest payments										$-Int_K$	Int_K	0
Workers wages								$-Wages$	$Wages$			0
Workers interest payments									$-Int_W$		Int_W	0
Workers consumption								$Cons_W$	$-Cons_W$			0
Bank purchases from Firms								Buy_B			$-Buy_B$	0
Jubilee Workers	$Jubilee_W$								$Jubilee_W$			0
Debt repayment Workers					$-Repay_J^W$				$-Repay_J^W$			0
Jubilee Capitalists	$Jubilee_K$									$Jubilee_K$		0
Debt repayment Capitalists						$-Repay_J^K$				$-Repay_J^K$		0
Buy Jubilee Shares Workers								JS_B^W	$-JS_B^W$			0
Jubilee dividends Workers								$-Div_{JS}^W$	Div_{JS}^W			0
Buy Jubilee Shares Capitalists								JS_B^K		$-JS_B^K$		0
Jubilee dividends Capitalists								$-Div_{JS}^K$		Div_{JS}^K		0
Firms repay debt Worker Jubilee				$-Repay_{JW}^F$				$-Repay_{JW}^F$				0
Firms repay debt Capitalist Jubilee				$-Repay_{JK}^F$				$-Repay_{JK}^F$				0
Sell Jubilee Bonds to Banks	$-Bond_B^J$		$Bond_B^J$									0
CB buys Jubilee Bonds from Banks	$Bond_{CB}^J$		$-Bond_{CB}^J$									0
Bond interest to Banks	Int_B^J										Int_B^J	0

The Jubilee component of the model is shown in Figure 157. The top left-hand corner determines the Jubilee itself. The switch means that the Jubilee doesn't commence until *Start* year, after which it lasts for one year.[31] The other logic switches determine that, if debt is paid down to zero, the payments are used to purchase "Jubilee shares" instead—shares newly issued by companies (so that they receive the revenue, rather than a trader), the revenue from which must be used to pay down corporate debt.

Figure 157: Mechanics of the Jubilee



The scale of the Jubilee is based on current US private debt data, which totalled US$29.5 trillion in 2021. This is broken into corporate debt of $17.5 trillion and household debt of $12 trillion. I made an arbitrary division of initial household debt into ¼ as debt of capitalists D_0^K and ¾ as debt of workers D_0^W, since the Flow of Funds doesn't provide that information.[32] The Jubilee equals 60% of this outstanding debt, or $17.7 trillion.

The division of the population into workers and capitalists is somewhat arbitrary as well: I assume that 5% of the population earns its income primarily from ownership, and 95% primarily from wages. Since the Jubilee is on a per capita basis (which works out to US$100,000 per adult American in this simulation), 95% of the Jubilee goes to workers and 5% to capitalists. This is hardly unfair to capitalists as individuals—everyone gets the same amount, regardless of social class—and it goes some way to

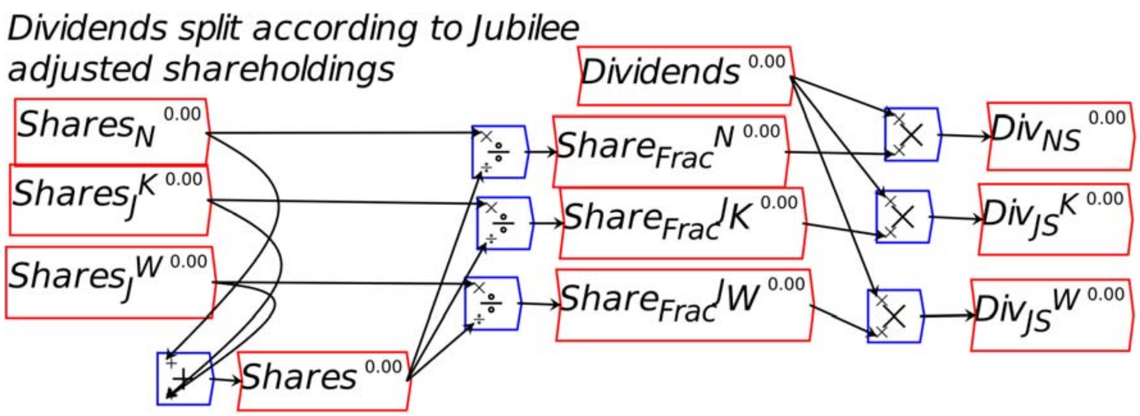
[31] At the moment, it's not possible to select a set of icons from Minsky and export those as an SVG file, so I had to copy these elements to a blank canvas, which reset the parameters and variable values. Hopefully by the time this book is published, we'll add support for exporting a selection of icons on the canvas. This is the sort of thing that continued funding of Minsky enables, so please consider signing up to its Patreon page https://www.patreon.com/HPCODER.

32 The variables D_0^K and D_0^W and the constants that determine them replicate details of the Godley Table in Figure 156, because at the moment Godley Tables only take numbers as the initial conditions (the first row in the table). At some point we'll add the capability to take parameters as the inputs, which would remove the need for this separate definition of D_0^K and D_0^W.

redressing the impact of Quantitative Easing, which had the express objective of increasing share prices, and therefore overwhelmingly favoured capitalists over workers.[33]

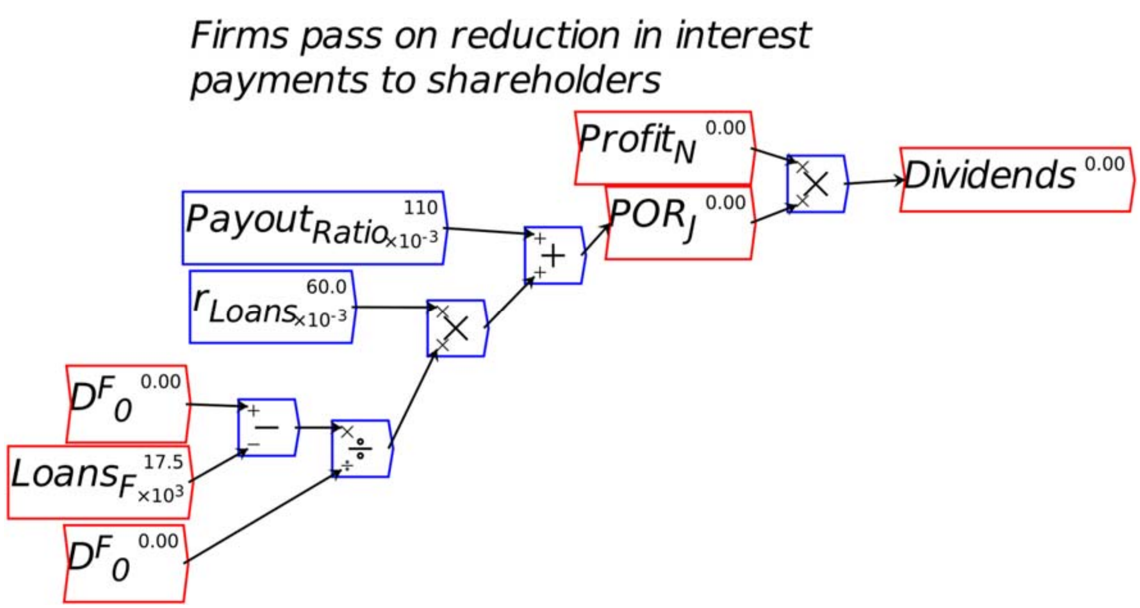
There are some complicated issues as a result of the change in share ownership, which are handled by the component shown in Figure 158: the new shares dilute existing shareholdings, so there has to be a change in where the dividends go.

Figure 158: Share ownership and dividend effects of the Jubilee



This doesn't make existing shareholders worse off however, because the fall in interest payments by firms is partly passed on to all shareholders via a rise in dividend payments—see Figure 159, and Equation (1.37), which I think is easier to read than the flowchart.[34]

Figure 159: Firms pass on the fall in interest payments in the form of dividends



[33] The Atlantic Council asserts that cumulatively QE in America has totalled \$7.6 trillion as of 2021: see https://www.atlanticcouncil.org/blogs/econographics/global-qe-tracker/.

[34] This is another planned enhancement of Minsky: make it possible to show a flowchart as an equation, where an equation is easier to read.

$$r_{Loans} \times D^F{}_0 \times \left(1 - \frac{Loans_F}{D^F{}_0}\right) \tag{1.37}$$

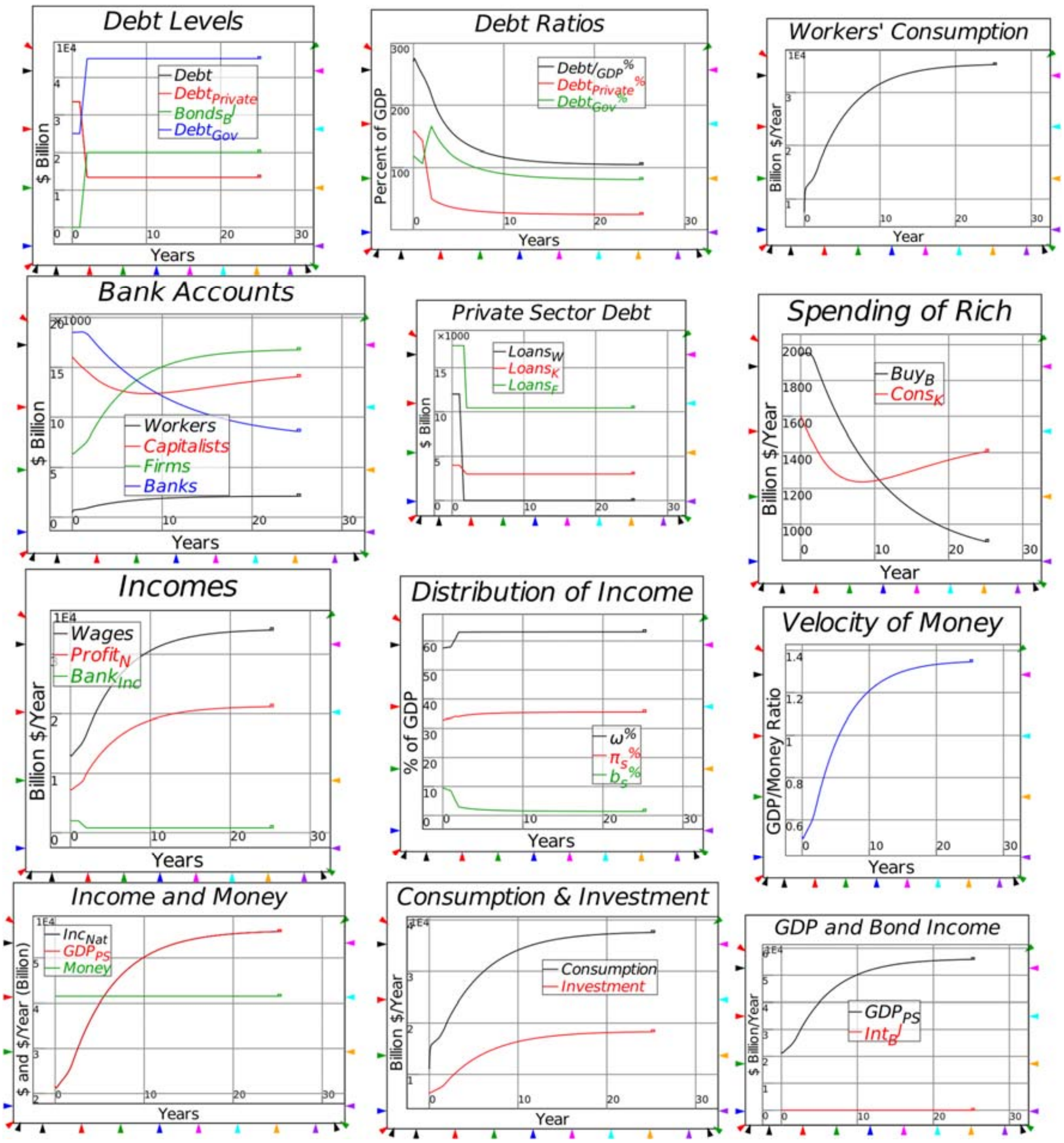
Equation (1.37) cover the variable part of dividend payments by firms because of the Jubilee. When the simulation starts and before the Jubilee, Equation (1.37) equals zero, and the dividend payout is 11% of profits, as specified by $Payout_{Ratio}$. When the debt level of firms $Loans_F$ falls because of the Jubilee, this becomes positive, and is added to the dividend payout ratio. In this way, the reduction in firm interest payments is passed back to the owners of shares—who also change in social composition, because, with 95% of the Jubilee going to workers, their debt of $12 trillion is extinguished, and they buy $7 trillion of Jubilee Shares.

In the first simulation shown below in Figure 160, the interest rate on Jubilee Bonds is set to zero, to illustrate what the Jubilee does if the amount of money in the economy remains constant—with interest payments, the amount of money increases. The effects include a large increase in GDP—which surprised even me when I first ran the model.

The reason for this is the impact of the Jubilee on the distribution of money, with initially more of it turning up in workers' bank accounts, but then—because workers have a much higher rate of spending than capitalists or bankers—most of that money ends up in the firm sector, rather than in the bank accounts of capitalists and bankers. The firm sector's turnover of money determines private sector GDP (I have omitted normal government spending and taxation from this model), and its rate of turnover is lower than that of workers, but much higher than bankers and capitalists. So the impact of the redistribution of money via the Jubilee is a much higher level of GDP via an increased rate at which money turns over in the economy.

This modelling phenomenon is the obverse of a real-world phenomenon that I have long observed and attributed to the impact of higher private debt levels on people's willingness to spend: the fall in the velocity of money since the peak inflationary period of the early 1980s—see Figure 161. This explanation still has legs as an inadvertent macro effect of a micro phenomenon: the higher average debt to income ratio today makes people "hoard" money to be able to pay their interest and principal commitments, but at the aggregate level hoarding merely reduces the rate of turnover of money. This leads to lower incomes, defeating the micro objective people have of saving more.

Figure 160: Simulation with no interest on Jubilee Bonds



However, this model has constant turnover rates for each social class (workers, capitalists and bankers) and the firm sector, so the rise in velocity it generates comes from the redistribution of existing money (and the fall in indebtedness, which reduces interest payments on existing debt, thus enabling that money to be used for commodity purchases instead).

Figure 161: Velocity of money of zero maturity (https://fred.stlouisfed.org/series/MZMV)

The next simulation has interest paid on Jubilee bonds at the same rate as private debt, of 6% per year. The outcome is that bank income does not fall because of the Jubilee, while the payment of interest also creates new money. The banks don't lose income out of the Jubilee—the interest they used to receive from private debtors is now provided by the government. As with MMT's insight in general, the negative equity of the Treasury enables the positive equity of the private sector—see Figure 162.

Figure 162: The Treasury's Godley Table for the Modern Debt Jubilee

	Asset	Liability				Equity	A-L-E
Flows ↓ / Stock Vars →	$Treasury$	$Bonds_{CB}$	$Bonds_B$	$Bonds_{CB}^J$	$Bonds_B^J$	$Loans_{CB}^T$	$Treasury_E$	7.27596e-12
Initial Conditions	22000	5000	4000	0	0	21000	-8000	0
Sell Jubilee Bonds to Banks	$Bond_B^J$				$Bond_B^J$			0
Bond interest to Banks	$-Int_B^J$						$-Int_B^J$	0
Jubilee Capitalists	$-Jubilee_K$						$-Jubilee_K$	0
Jubilee Workers	$-Jubilee_W$						$-Jubilee_W$	0
Loan for Bond Interest payments	$Lend_{CB}^T$					$Lend_{CB}^T$		0
CB buys Jubilee Bonds from Banks				$Bond_{CB}^J$	$-Bond_{CB}^J$			0

Rather than this leading to an increase in the government's debt to GDP ratio however, over time, it leads to a fall—see the second plot in Figure 163, which shows the debt to GDP ratios for the private sector, public sector, and the sum of the two. The government debt to GDP ratio rises as a direct consequence of the Jubilee initially, as government debt replaces private debt; but the growth in the economy triggered by the Jubilee means that the government debt ratio falls over time. After a decade, the government debt ratio is lower than it was before the Jubilee. The aggregate debt ratio also falls: the economy transitions from a private sector based on debt to one based on share equity.

This is because the stimulatory effect of the Jubilee on private sector activity more than outweighs the increased debt the government takes on in Jubilee Bonds. Direct attempts to reduce the government debt to GDP ratio by austerity have the opposite effect on the real economy—depressing GDP and counteracting the attempt to reduce the debt ratio by reducing government debt.

Figure 163: A Modern Debt Jubilee with interest on Jubilee Bonds

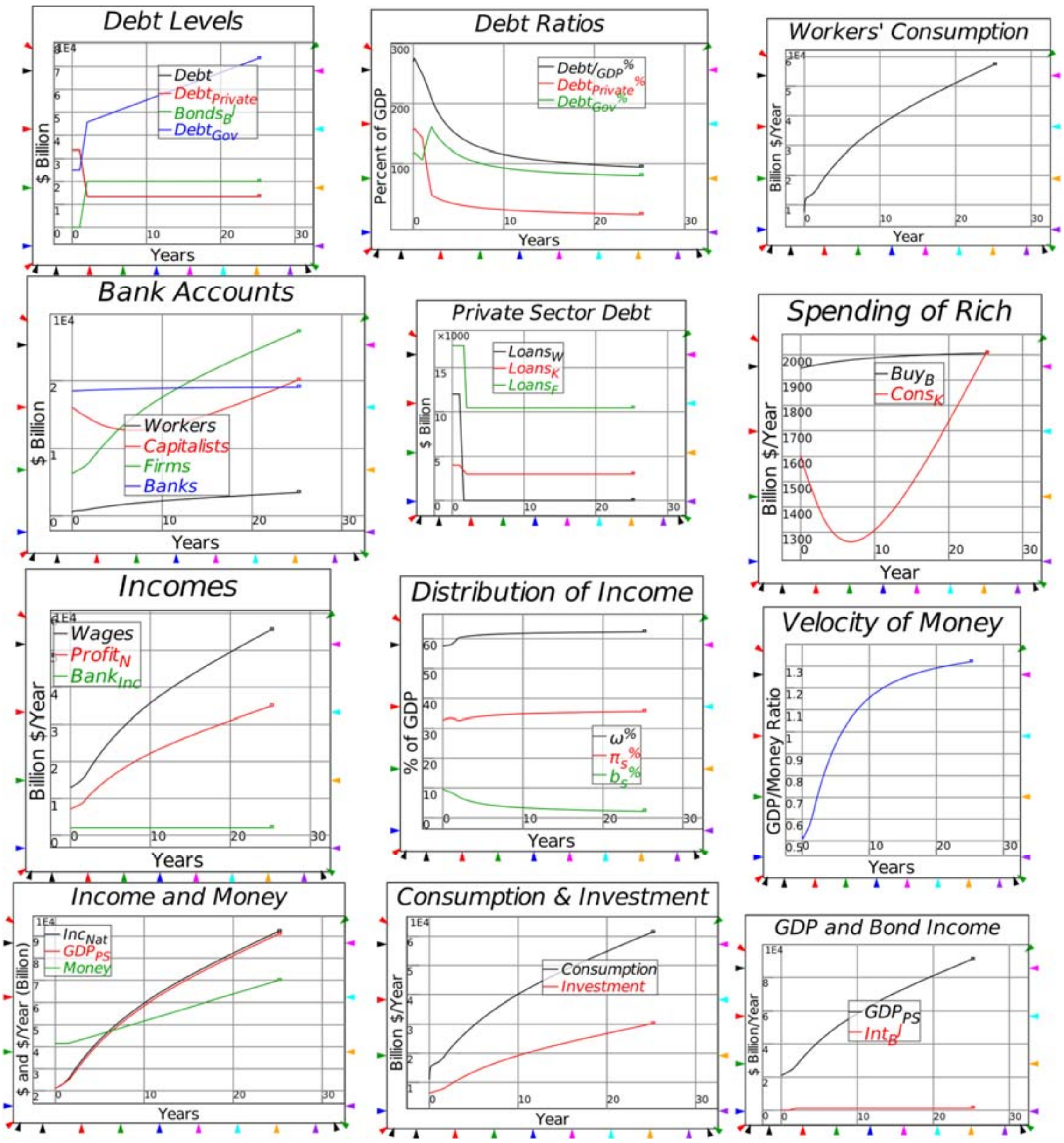


Figure 164: Modern Debt Jubilee Godley Tables

Banks

	Asset							Liability			Equity	A-L-E
Flows ↓ / Stock Vars →	Reserves	$Bonds_B$	$Bonds_B^J$	$Loans_F$	$Loans_W$	$Loans_K$	$Loans_{CB}^B$	Firms	Workers	Capitalists	Banks	0
Initial Conditions	4000	4000	0	17500	12000	4000	0	6500	450	16050	18500	0
Firms dividend payments								$-Div_{NS}$		Div_{NS}		0
Firms interest payments								$-Int_F$			Int_F	0
Capitalist consumption								$Cons_K$		$-Cons_K$		0
Capitalist interest payments										$-Int_K$	Int_K	0
Workers wages								$-Wages$	$Wages$			0
Workers interest payments									$-Int_W$		Int_W	0
Workers consumption								$Cons_W$	$-Cons_W$			0
Bank purchases from Firms								Buy_B			$-Buy_B$	0
Jubilee Workers	$Jubilee_W$								$Jubilee_W$			0
Debt repayment Workers					$-Repay_J^W$				$-Repay_J^W$			0
Jubilee Capitalists	$Jubilee_K$									$Jubilee_K$		0
Debt repayment Capitalists						$-Repay_J^K$				$-Repay_J^K$		0
Buy Jubilee Shares Workers								JS_B^W	$-JS_B^W$			0
Jubilee dividends Workers								$-Div_{JS}^W$	Div_{JS}^W			0
Buy Jubilee Shares Capitalists								JS_B^K		$-JS_B^K$		0
Jubilee dividends Capitalists								$-Div_{JS}^K$		Div_{JS}^K		0
Firms repay debt Worker Jubilee				$-Repay_{JW}^F$				$-Repay_{JW}^F$				0
Firms repay debt Capitalist Jubilee				$-Repay_{JK}^F$				$-Repay_{JK}^F$				0
Sell Jubilee Bonds to Banks	$-Bond_B^J$		$Bond_B^J$									0
CB buys Jubilee Bonds from Banks	$Bond_{CB}^J$		$-Bond_{CB}^J$									0
Bond interest to Banks	Int_B^J										Int_B^J	0

Capitalists

	Asset			Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Capitalists	$Shares_N$	$Shares_J^K$	$Loans_K$	$Capitalists_E$	0
Initial Conditions	16050	10000	0	4000	22050	0
Capitalist consumption	$-Cons_K$				$-Cons_K$	0
Jubilee dividends Capitalists	Div_{JS}^K				Div_{JS}^K	0
Firms dividend payments	Div_{NS}				Div_{NS}	0
Capitalist interest payments	$-Int_K$				$-Int_K$	0
Jubilee Capitalists	$Jubilee_K$				$Jubilee_K$	0
Buy Jubilee Shares Capitalists	$-JS_B^K$		JS_B^K			0
Debt repayment Capitalists	$-Repay_J^K$			$-Repay_J^K$		0

Workers

	Asset		Liability	Equity	A-L-E
Flows ↓ / Stock Vars →	Workers	$Shares_J^W$	$Loans_W$	$Workers_E$	0
Initial Conditions	450	0	12000	-11550	0
Workers consumption	$-Cons_W$			$-Cons_W$	0
Jubilee dividends Workers	Div_{JS}^W			Div_{JS}^W	0
Workers interest payments	$-Int_W$			$-Int_W$	0
Jubilee Workers	$Jubilee_W$			$Jubilee_W$	0
Buy Jubilee Shares Workers	$-JS_B^W$	JS_B^W			0
Debt repayment Workers	$-Repay_J^W$		$-Repay_J^W$		0
Workers wages	$Wages$			$Wages$	0

Treasury

	Asset	Liability					Equity	A-L-E
Flows ↓ / Stock Vars →	Treasury	$Bonds_{CB}$	$Bonds_B$	$Bonds_{CB}^J$	$Bonds_B^J$	$Loans_{CB}^T$	$Treasury_E$	0
Initial Conditions	22000	5000	4000	0	0	21000	-8000	0
Sell Jubilee Bonds to Banks	$Bond_B^J$				$Bond_B^J$			0
Bond interest to Banks	$-Int_B^J$						$-Int_B^J$	0
Jubilee Capitalists	$-Jubilee_K$						$-Jubilee_K$	0
Jubilee Workers	$-Jubilee_W$						$-Jubilee_W$	0
Loan for Bond Interest payments	$Lend_{CB}^T$					$Lend_{CB}^T$		0
CB buys Jubilee Bonds from Banks				$Bond_{CB}^J$	$-Bond_{CB}^J$			0

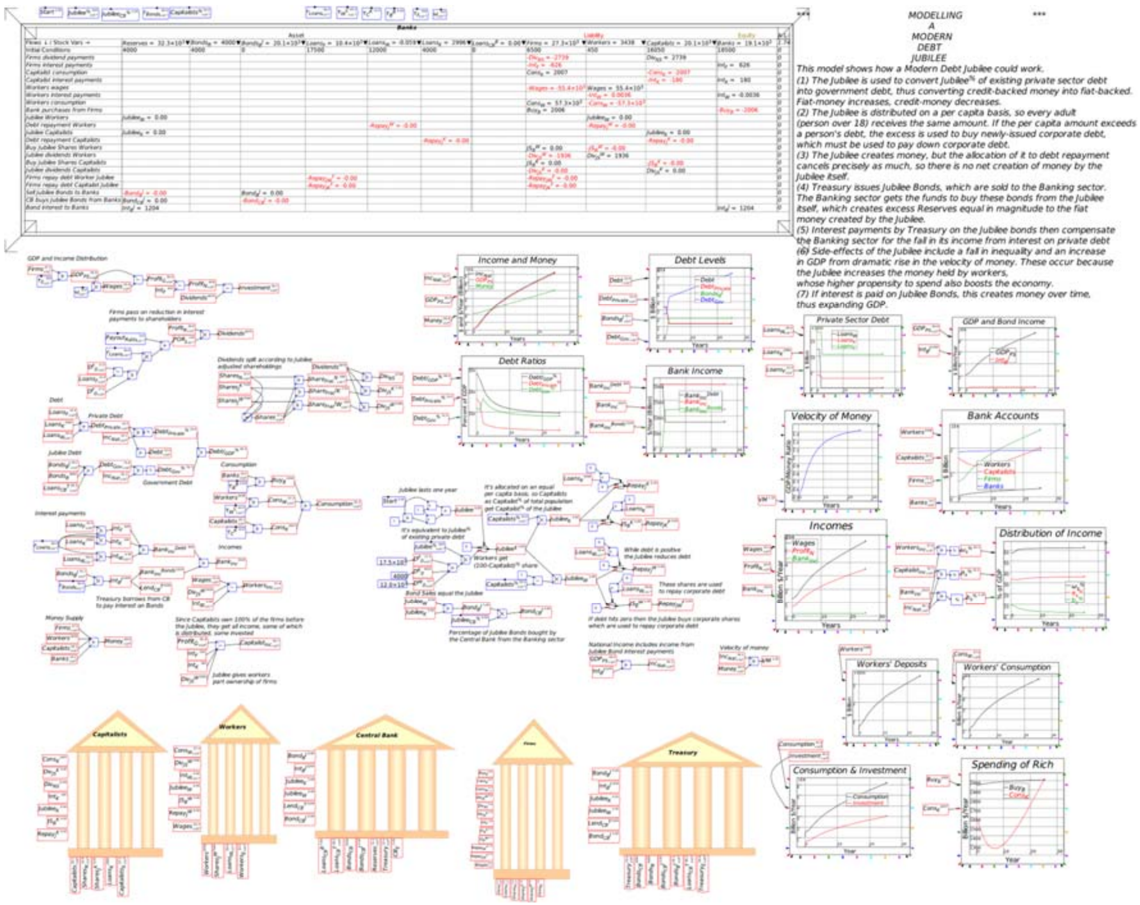
Central Bank

	Asset				Liability		Equity	A-L-E
Flows ↓ / Stock Vars →	$Loans_{CB}^B$	$Loans_{CB}^T$	$Bonds_{CB}$	$Bonds_{CB}^J$	Reserves	Treasury	CB_E	0
Initial Conditions	0	21000	5000	0	4000	22000	0	0
Sell Jubilee Bonds to Banks					$-Bond_B^J$	$Bond_B^J$		0
Bond interest to Banks					Int_B^J	$-Int_B^J$		0
Jubilee Capitalists					$Jubilee_K$	$-Jubilee_K$		0
Jubilee Workers					$Jubilee_W$	$-Jubilee_W$		0
Loan for Bond Interest payments		$Lend_{CB}^T$				$Lend_{CB}^T$		0
CB buys Jubilee Bonds from Banks				$Bond_{CB}^J$	$Bond_{CB}^J$			0

Firms

	Asset	Liability				Equity	A-L-E
Flows ↓ / Stock Vars →	Firms	$Loans_F$	$Shares_N$	$Shares_J^K$	$Shares_J^W$	$Firms_E$	0
Initial Conditions	6500	17500	10000	0	0	-21000	0
Bank purchases from Firms	Buy_B					Buy_B	0
Capitalist consumption	$Cons_K$					$Cons_K$	0
Workers consumption	$Cons_W$					$Cons_W$	0
Jubilee dividends Workers	$-Div_{JS}^W$					$-Div_{JS}^W$	0
Firms dividend payments	$-Div_{NS}$					$-Div_{NS}$	0
Jubilee dividends Capitalists	$-Div_{JS}^K$					$-Div_{JS}^K$	0
Firms interest payments	$-Int_F$					$-Int_F$	0
Buy Jubilee Shares Capitalists	JS_B^K			JS_B^K			0
Buy Jubilee Shares Workers	JS_B^W				JS_B^W		0
Firms repay debt Capitalist Jubilee	$-Repay_{JK}^F$	$-Repay_{JK}^F$					0
Firms repay debt Worker Jubilee	$-Repay_{JW}^F$	$-Repay_{JW}^F$					0
Workers wages	$-Wages$					$-Wages$	0

Figure 165: The full Modern Debt Jubilee model



This completes the models showcased in *Manifesto*. As time goes on, I'll add new models here, developed by myself and others, to show what Minsky can do.

9 Complexity

9.1 Predator-Prey model

As I explain in *Manifesto*, Lotka's predator-prey model (Lotka 1920) was the first mathematical model to demonstrate the hallmarks of complexity: nonlinear interactions in a system leading to sustained non-equilibrium behavior. Its foundations are extremely simple: two population models, a prey species with an assumed limitless supply of food, and a predator population whose survival depends on the availability of prey.

We can start from the equation for population growth—or rather population change. Call the prey species F (for Fish) and the predator population S (for Sharks).[35] Then using the same "hat" notation as in Chapter 6, we start from:

$$\begin{aligned} \hat{F} &\equiv \frac{1}{F}\frac{d}{dt}F = a \\ \hat{S} &\equiv \frac{1}{S}\frac{d}{dt}S = -c \end{aligned} \tag{1.38}$$

Where a is the annual rate of growth of Fish (in the absence of Sharks) and c is the annual death rate of Sharks (in the absence of Fish). Expanding this out into differential equation form gives us:

$$\begin{aligned} \frac{d}{dt}F &= a \cdot F \\ \frac{d}{dt}S &= -c \cdot S \end{aligned} \tag{1.39}$$

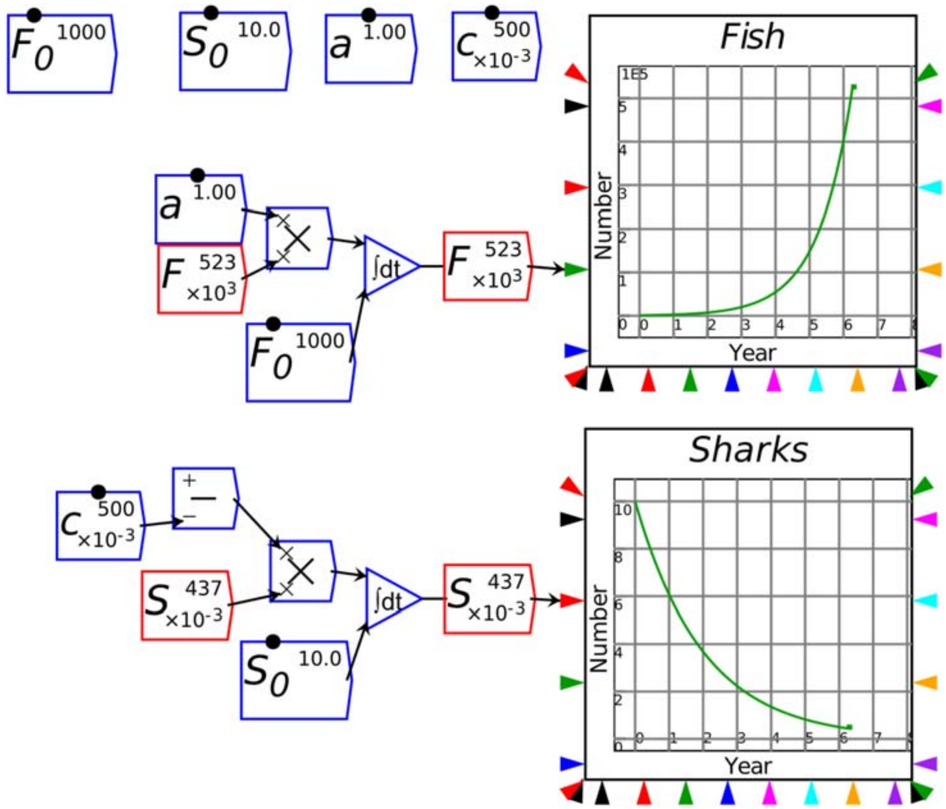
In integral equation form, this is:

$$\begin{aligned} F(t) &= \int_0^t a \cdot F(s) \cdot ds \\ S(t) &= \int_0^t -c \cdot S(s) \cdot ds \end{aligned} \tag{1.40}$$

This is precisely how *Minsky* models it: see Figure 166, where the equation (1.40) can be seen by reading *Minsky*'s symbols from right to left:

[35] I'm showing my Australian roots here: most European models use Rabbits and Foxes.

Figure 166: Predator and Prey without interaction



The model in Figure 166 also demonstrates one of *Minsky*'s shortcuts: to negate a number, simply feed it into the bottom port of a minus operator . With no input to the top port, Minsky interprets this as $(0 - c) = -c$.

Now we need to include the interaction between the species: predation by sharks reducing fish numbers, and increasing shark numbers. Lotka made the simplest possible assumption, that sharks reduce the growth rate of fish by a constant, and decrease the death rate of sharks by another constant. This is most easily shown using the hat notation used in equation (1.38):

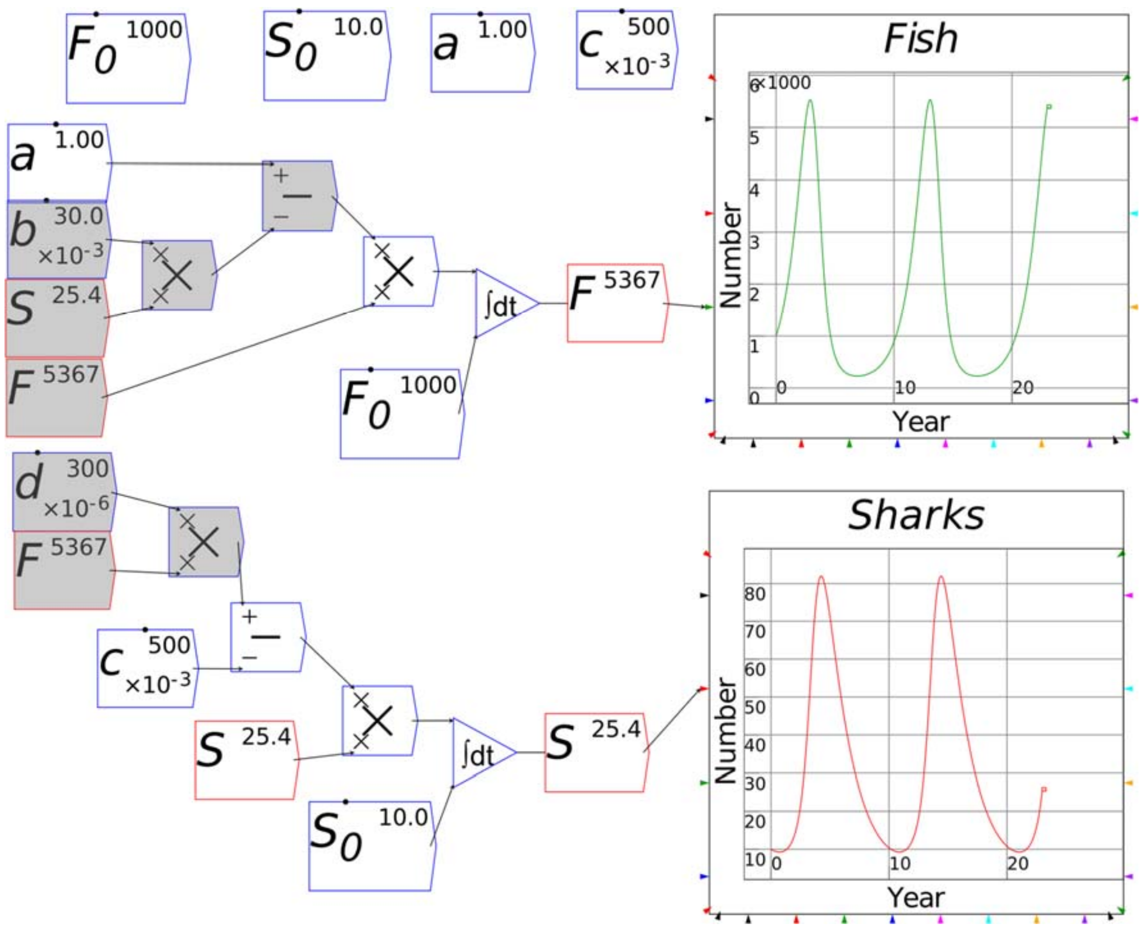
$$\begin{aligned} \hat{F} &= a - b \cdot S \\ \hat{S} &= -c + d \cdot F \end{aligned} \tag{1.41}$$

In integral form, this is

$$\begin{aligned} F(t) &= \int_0^t (a - b \cdot S) \cdot F(s) \cdot ds \\ S(t) &= \int_0^t (-c + d) \cdot S(s) \cdot ds \end{aligned} \tag{1.42}$$

This can be put into *Minsky* by adding the widgets shown in grey in Figure 167, and the characteristic cycles of the predator-prey model emerge.

Figure 167: Predator and Prey with interaction



I was actually lucky here: I simply used “suck it and see” values for the parameters and initial conditions, and they worked out OK: the ranges for the numbers of fish and sharks were reasonable. But if you do the same, you may well get “crazy” cycles, because the combination of your initial values and your parameters may have numbers of both species cycling wildly. This is because, in one of the neatest illustrations of how complex systems behave, the equilibrium value for the number of fish depends on the parameters for sharks, and the equilibrium for the number of sharks depends on the parameter for fish.

This is easiest to see by setting the equations in (1.41) to zero—since this shows you the point at which the rates of change of the number of fish and the number of sharks are both zero:

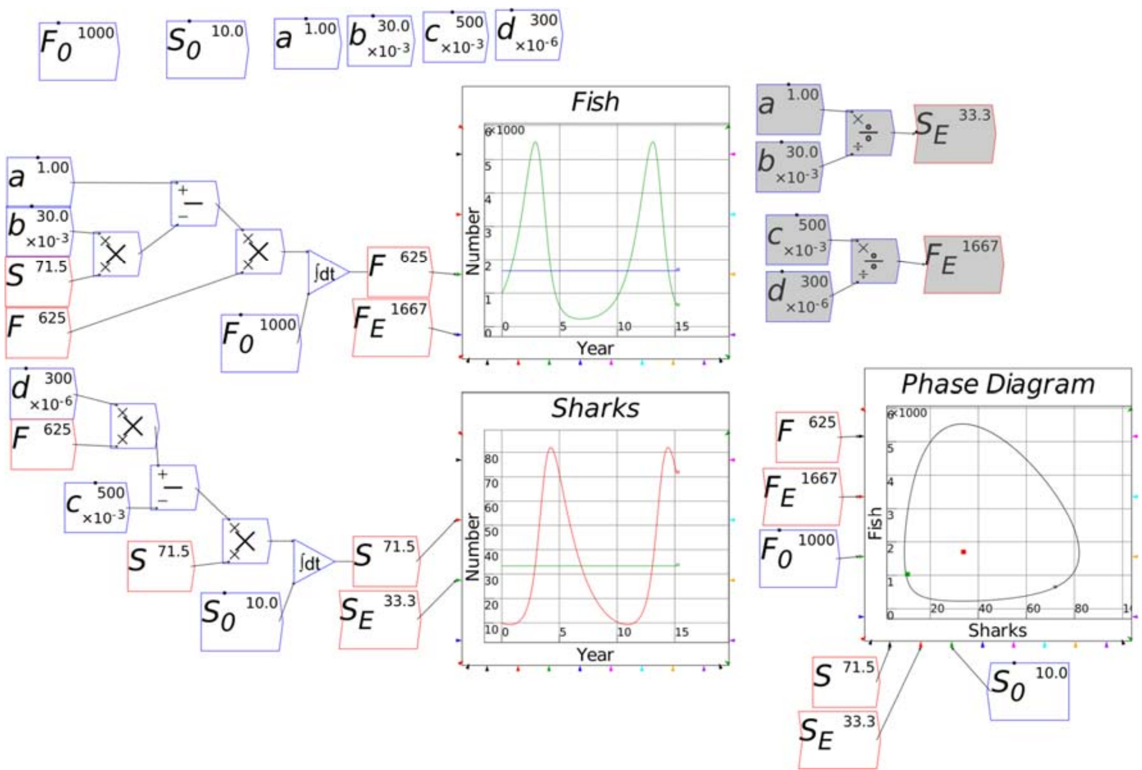
$$\begin{aligned}\hat{F} &= a - b \cdot S = 0 \\ \hat{S} &= -c + d \cdot F = 0\end{aligned} \tag{1.43}$$

This is only true for specific—equilibrium—values of F and S, which I denote by F_E and S_E respectively:

$$\begin{aligned}S_E &= \frac{a}{b} \\ F_E &= \frac{c}{d}\end{aligned} \tag{1.44}$$

Figure 168 adds the equilibrium calculations with the greyed widgets, as well as a phase diagram showing the repeating cycles over time, the equilibrium here (the other equilibrium—which is unstable—zero sharks and zero fish), and the initial values on the phase plot.

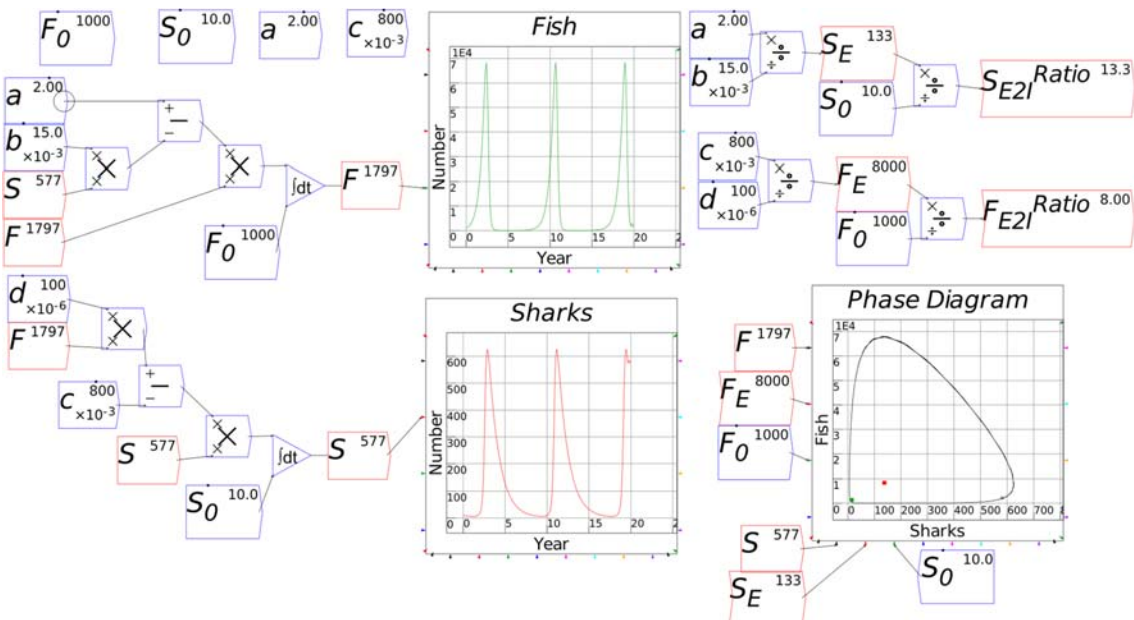
Figure 168: Predator and Prey with phase diagram and equilibria



I was lucky that my choice for the initial number of fish and sharks—1000 and 10 respectively—weren't too far from the equilibrium values for fish and sharks—1667 and 33.3 respectively—given the values I used for the parameters. But if you give initial conditions that differ substantially from the equilibrium determined by the parameters, you will get wild cycles where each species "almost disappears" before spiking up dramatically and then collapsing once more—as illustrated by Figure 169.

Your best bet, when designing a model, is to either (a) check the equilibrium conditions of your model, and choose initial conditions that aren't too far removed from (one of) the equilibria; (b) if you're working from data for the initial conditions, choose parameter values that generate equilibria that aren't radically different; or (c) if you're working on a large-scale empirically based model, follow the parameter estimation techniques outlined in Chapter 11.

Figure 169: The same model with badly chosen initial conditions



The final things needed to reproduce the figure in *Manifesto* is to replace the androgynous a, b, c, d parameters with more meaningful labels, and to put in the plot with the two Y-axes. We can do the former quickly using the right-mouse button menu item "Rename all instances"—see Figure 170.

Figure 170: Using "Rename all instances"

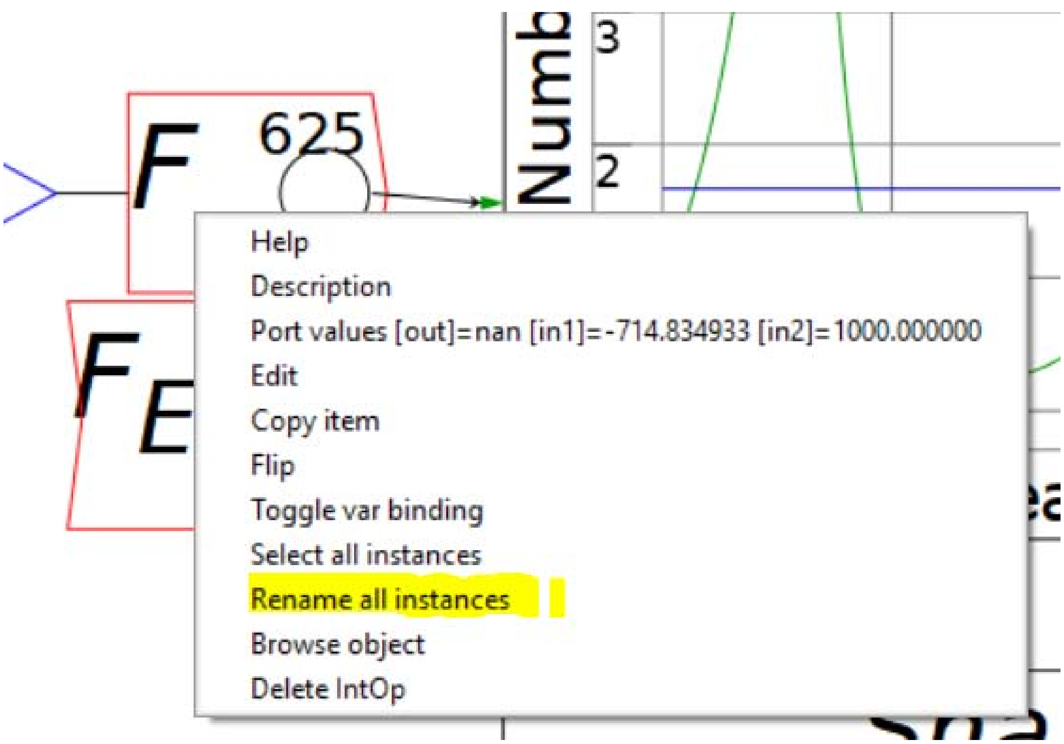
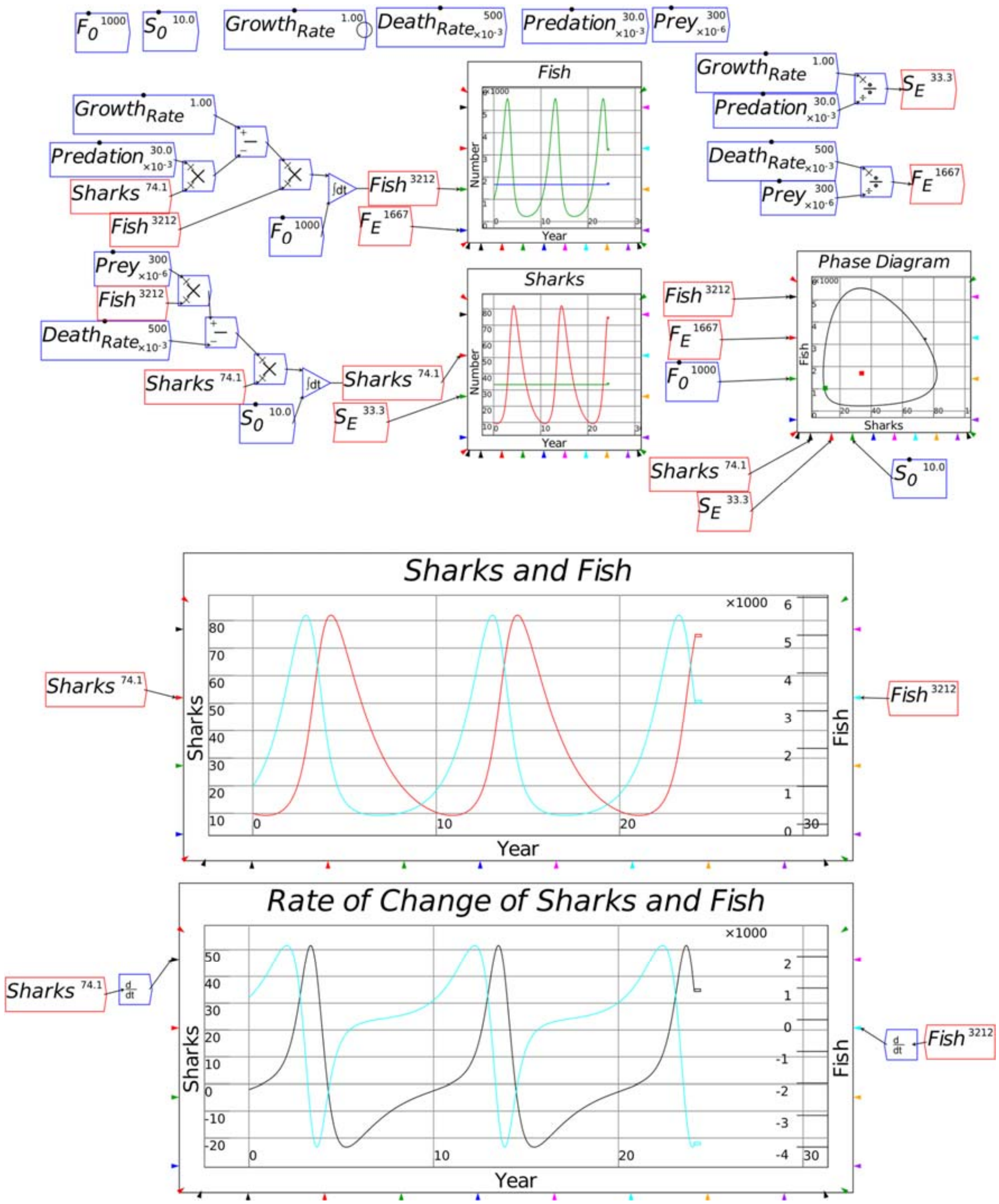


Figure 171 shows the final model including two plots with 2 y-axes—one showing the numbers of Sharks and Fish, and the other showing the rates of change of the two populations. This is partly to show off *Minsky*'s rate of change operator $\frac{d}{dt}$—which, unlike similar operators in most other system dynamics programs, actually performs a symbolic differentiation rather than a numerical one—and partly to make the point that, no matter how often you "first/second/third difference" these variables, they will always be out of phase. This is despite the complete lack of time lags in this model: the instantaneous value of (*the rate of change of*) Fish depends on the instantaneous value of Sharks, but in a nonlinear way. So no matter how often they are "differenced", they will remain "not cointegrated" in the jargon of econometrics.

Figure 171: The final model with rates of change shown as well



Lotka's model was easily derived, simply by acknowledging that sharks eat fish, and by using the simplest possible mathematical operation to link the two species together.[36] It's also easily analyzed, since with just two dimensions, its dynamic properties depend on a simple quadratic, as I'll explain later in Chapter XX. The next model, which is the first simulated model in the history of complex systems analysis, is an entirely different … kettle of fish.

[36] Things get far more complicated when 3 or more species are considered: with 3 dimensions, as I explain in Manifesto, you enter the realm of chaotic dynamics—which we'll explore using Lorenz's model.

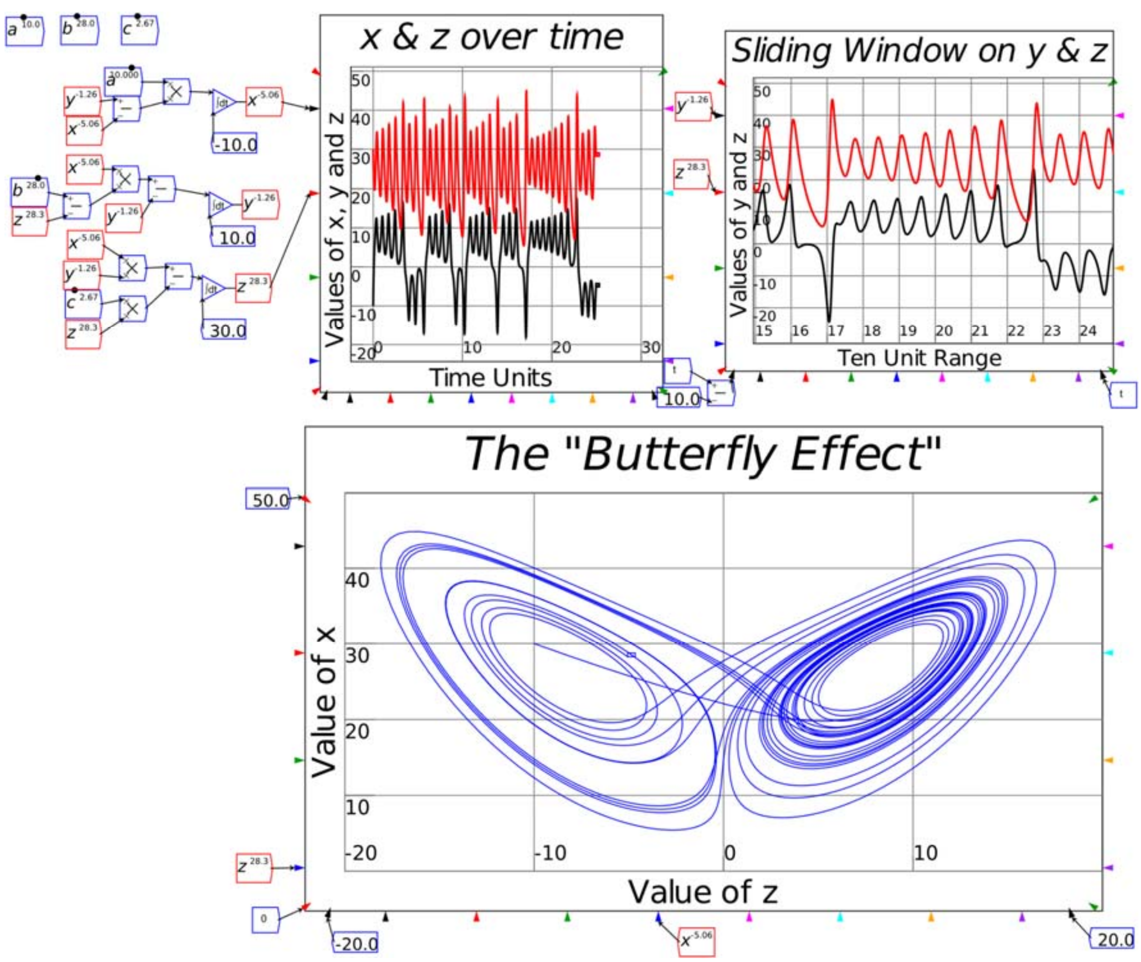
9.2 Lorenz model

The derivation of Lorenz's model of turbulent flow required mathematical skills well in advance of those possessed by the vast majority of economists—including me—so don't let the simplicity of the equations in (1.45) fool you. Superficially, they are only slightly more complicated than the Lotka predator prey model: rather than 2 variables and 4 parameters (Lotka), there are 3 variables and 3 parameters:

$$\begin{aligned}\frac{dx}{dt} &= a \times (y - x) \\ \frac{dy}{dt} &= x \times (b - z) - y \\ \frac{dz}{dt} &= x \times y - c \times z \\ a &= 10, b = 28, c = 2.67\end{aligned} \tag{1.45}$$

However, the behavior of the model is from another planet: Planet Complexity—see Figure 172. This happens to be the planet on which we actually live.

Figure 172: Lorenz's model with its chaotic behavior and "strange attractors"



I won't repeat the step-by-step explanation of deriving this model that I gave for Lokta's model—there is no simple "step by step" process to explain, and you should be able to pretty rapidly design this model in Minsky yourself by converting the differential equations in equation (1.45) to integral equations, and then coding them in *Minsky*. But one feature of this model is worth noting: the "sliding window" plot that shows a ten time-unit slice of the plots for *y* and *z*. This uses the "range" inputs for the plots—the angled inputs on each of the X, Y1 and Y2 axes. Normally these are constants, but they can take variable inputs, and the variable inputs for this plot are the input for the system's simulation time 

at the far right of the plot, and $t - 10$ on the far left.

What is worth repeating is the exercise of deriving the equilibrium of the model, by setting all the differential equations in (1.45) to zero:

$$\begin{aligned}\frac{dx}{dt} &= a \times (y - x) = 0 \\ \frac{dy}{dt} &= x \times (b - z) - y = 0 \\ \frac{dz}{dt} &= x \times y - c \times z = 0\end{aligned} \tag{1.46}$$

One obvious solution here is where $x = y = z = 0$. The non-zero solutions to (1.46) give us these three conditions for the equilibrium values, which I identify using the subscript *E*:

$$\begin{aligned}y_E &= x_E \\ y_E &= x_E \times (b - z_E) \\ z_E &= \frac{x_E \times y_E}{c}\end{aligned} \tag{1.47}$$

A bit of algebraic manipulation yields:

$$\begin{aligned}z_E &= (b - 1) \\ y_E &= x_E = \pm\sqrt{c \cdot (b - 1)}\end{aligned} \tag{1.48}$$

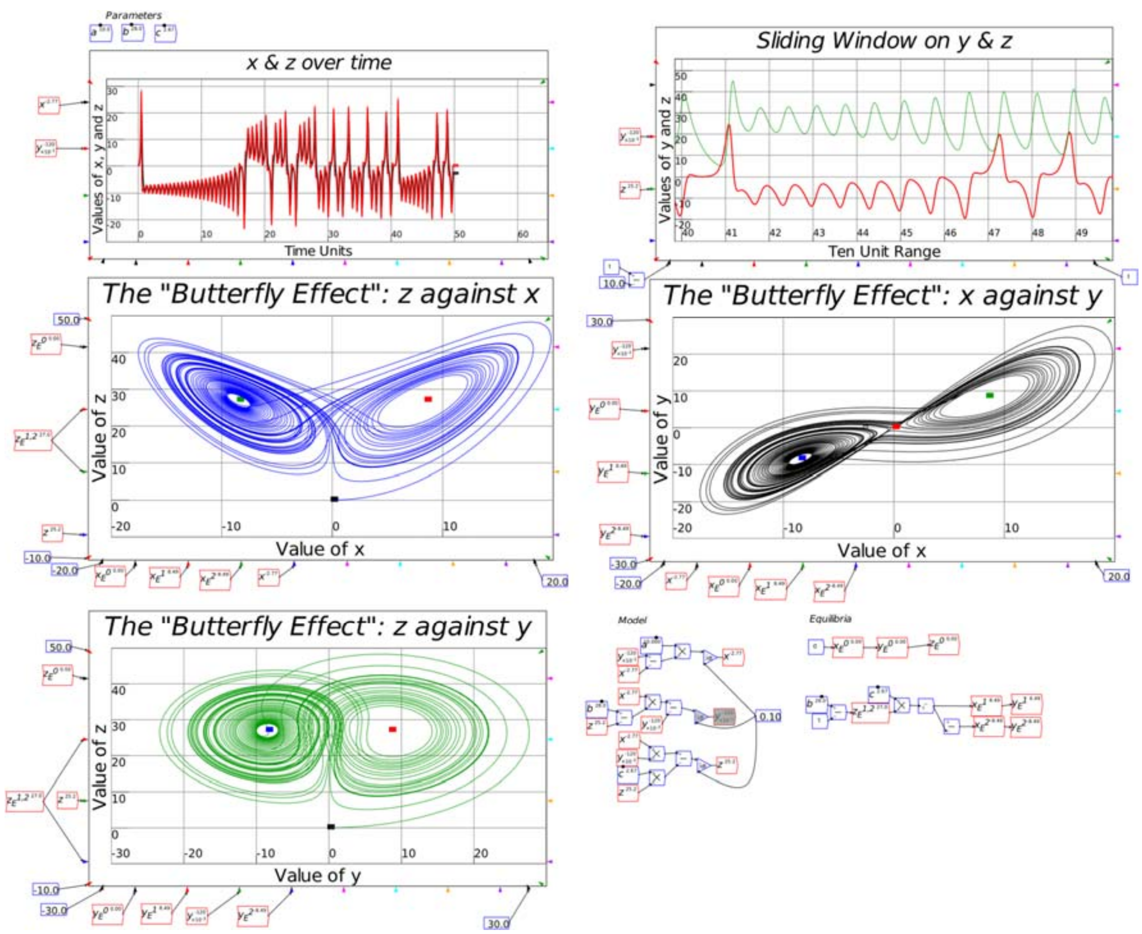
There are thus 3 equilibria for this model:

$$\begin{bmatrix} x_E \\ y_E \\ z_E \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} +\sqrt{c \cdot (b-1)} \\ +\sqrt{c \cdot (b-1)} \\ b - 1 \end{bmatrix}, \begin{bmatrix} -\sqrt{c \cdot (b-1)} \\ -\sqrt{c \cdot (b-1)} \\ b - 1 \end{bmatrix} \tag{1.49}$$

One reason I love this model, as a non-mainstream economist, is that it makes a mockery of the Neoclassical obsession with equilibrium modelling, because it has three equilibria, *all three of which are unstable*. The equilibria are the colored dots on the phase plots of z against x & y, and y against x. The simulation starts at the values $(0.1, 0.1, 0.1)$, just a slight displacement from the (0,0,0) equilibrium. Because the simulation starts so close to this equilibrium, the system is rapidly pushed away from it: this equilibrium is stable on two of its three eigenvalues, but unstable on one.

The system is then attracted towards one of the other two equilibria, but they are "strange attractors": they attract the system from a distance but repel it—in a cyclical fashion—when it gets closer to them. We'll get into the detail of how to analyze this instability in Chapter XX, but for now it's primary characteristics are that the system will *never* converge to any of its equilibria, and yet the system will also never return values that are unrealistic. Its dynamics are therefore necessarily far-from-equilibrium dynamics: the very idea of "equilibrium dynamics"—as ensconced in Neoclassical "*Dynamic* Stochastic General *Equilibrium*" modeling—is an oxymoron.

Figure 173: Lorenz model with equilibria. Simulation starting from (0.1,0.1,0.1)



My model of Minsky's Financial Instability Hypothesis, which we'll develop in the next section, has related, though not quite so complex, far-from-equilibrium dynamics.

9.3 A complex systems model of economic instability

On page 50, I introduced the Goodwin model, which reduced to the following two equations for the employment rate λ and the wages share of output ω:

$$\begin{aligned}\frac{d}{dt}\lambda &= \lambda\cdot\left(\left(\frac{1-\omega}{v}-\delta_K\right)-\alpha-\beta\right)\\ \frac{d}{dt}\omega &= \omega\cdot\left(S_\lambda\cdot\left(\lambda-Z_\lambda\right)-\alpha\right)\end{aligned} \tag{1.50}$$

When I constructed my model of Minsky's Financial Instability Hypothesis in 1992 (Keen 1995), I added the private debt to GDP ratio $d_r \equiv D/Y$ to Goodwin's growth cycle model as a third system state:

$$d_r \equiv \frac{D}{Y} \tag{1.51}$$

As a simplification—one that is actually favourable to capitalism, and far too kind to the finance sector, since it omits the modern finance sector's main business model of Ponzi finance[37]—I assumed that all borrowing finances investment, so that the rate of change of private debt D was equal to gross investment minus profits:

$$\frac{d}{dt}D = I_G - \Pi \tag{1.52}$$

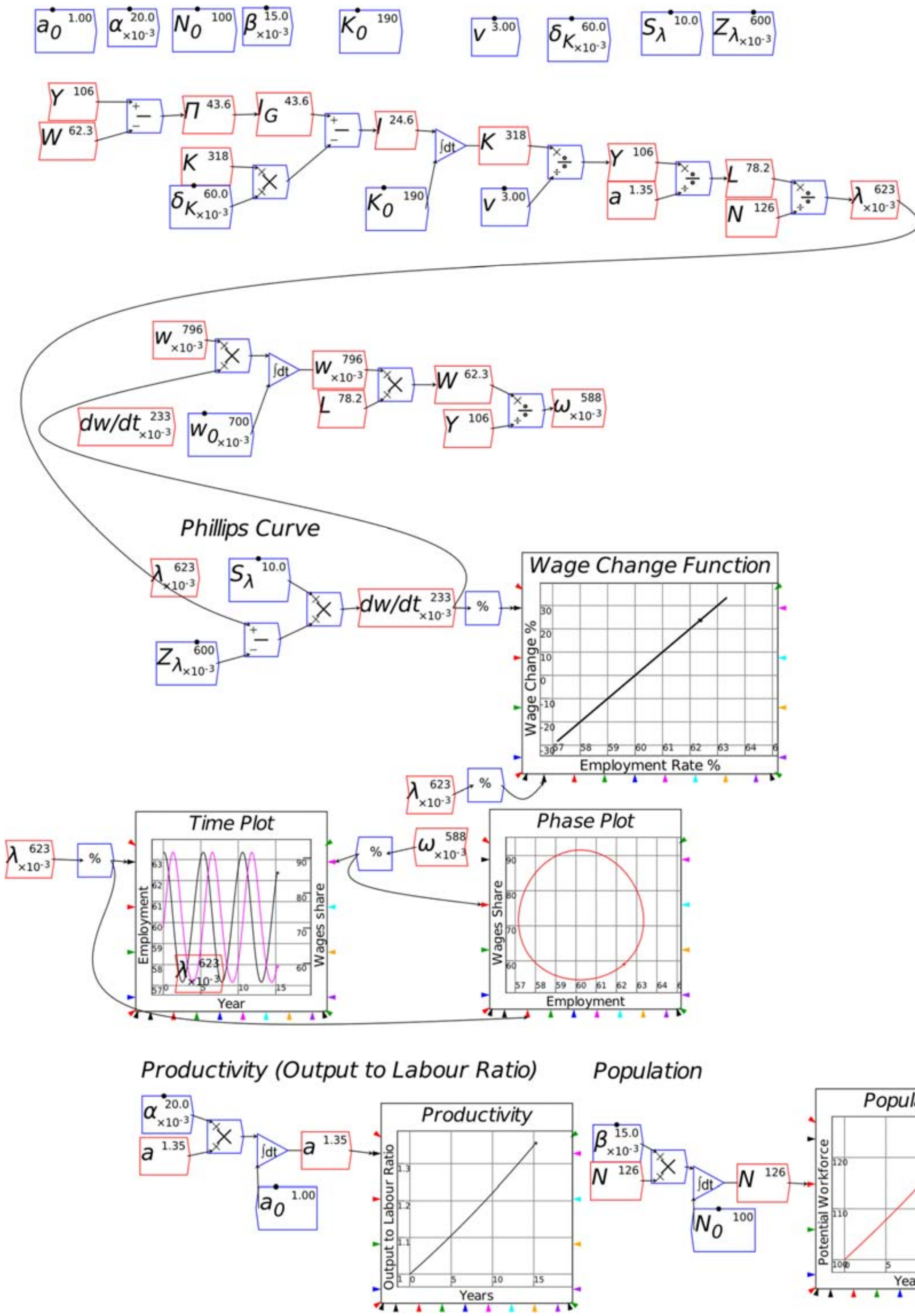
This then required a redefinition of profit: it became output minus wages, minus also the interest rate r times outstanding debt D. The profit share of GDP π_s and the rate of profit π_r also had to be redefined:

$$\begin{aligned} \Pi &= Y - W - r \cdot D \\ \pi_s &= \frac{\Pi}{Y} = 1 - \omega - r \cdot d_r \\ \pi_r &= \frac{\Pi}{K} = \frac{1 - \omega - r \cdot d_r}{v} \end{aligned} \tag{1.53}$$

These definitions are easily added to the flowchart rendition of Goodwin's model developed in Figure 72 to Figure 74. I'll start from a slightly modified form of the incomplete model in Figure 72, to help illustrate a few additional features of Minsky—see Figure 174. Notice the curved wires connecting the output from the employment rate λ to the Phillips Curve function (which I copied from Figure 68), and from the output of the Phillips function (which I've renamed dw/dt) to the input of the integral block that determines the wage rate? Do you like them?

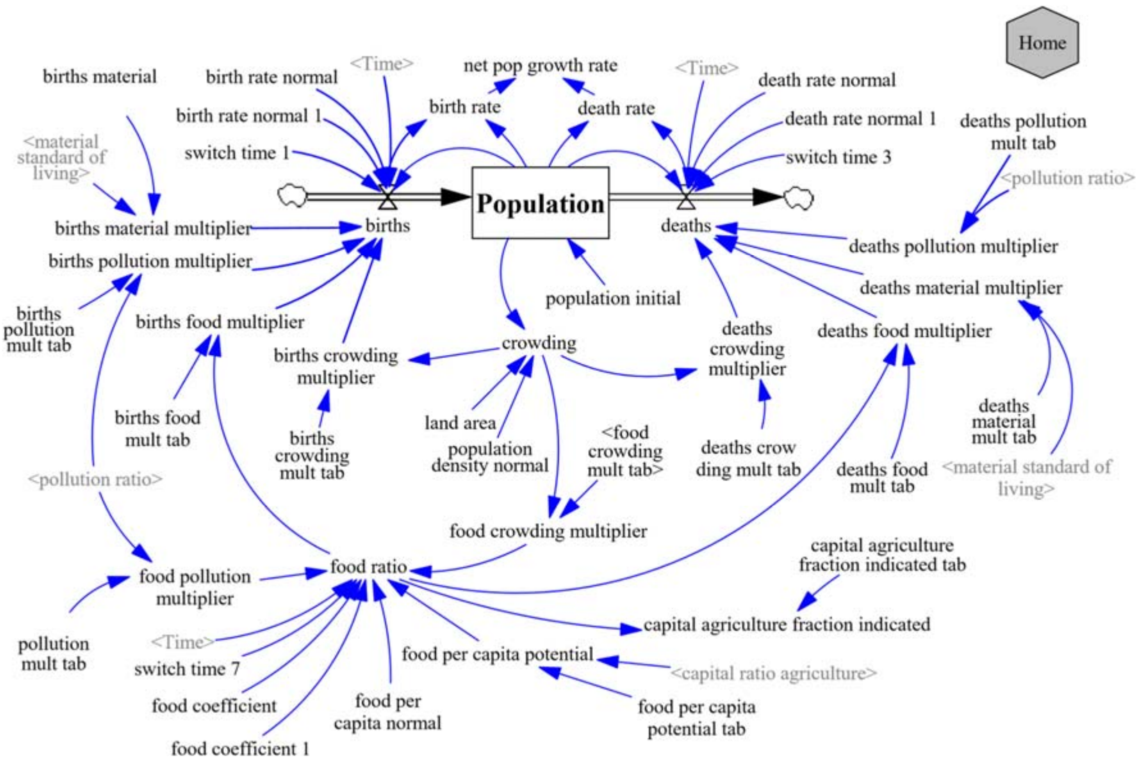
[37] Ponzi finance can easily be added by including debt that doesn't create new productive capacity. See (Giraud and Grasselli 2019).

Figure 174: Modified layout of a Goodwin model



Me neither! Well, they might look OK here, but the plethora of wires in standard system dynamics models make some of them—most of them—unreadable by anyone apart from the original designer. But in those programs, the wires are necessary to link one part of the model to any other: there's no other way to pass values from one variable to another. For example, Figure 175 shows the population component of the famous *Limits to Growth* (Meadows et al. 1972) model in Vensim:

Figure 175: Limits to Growth Population components in Vensim

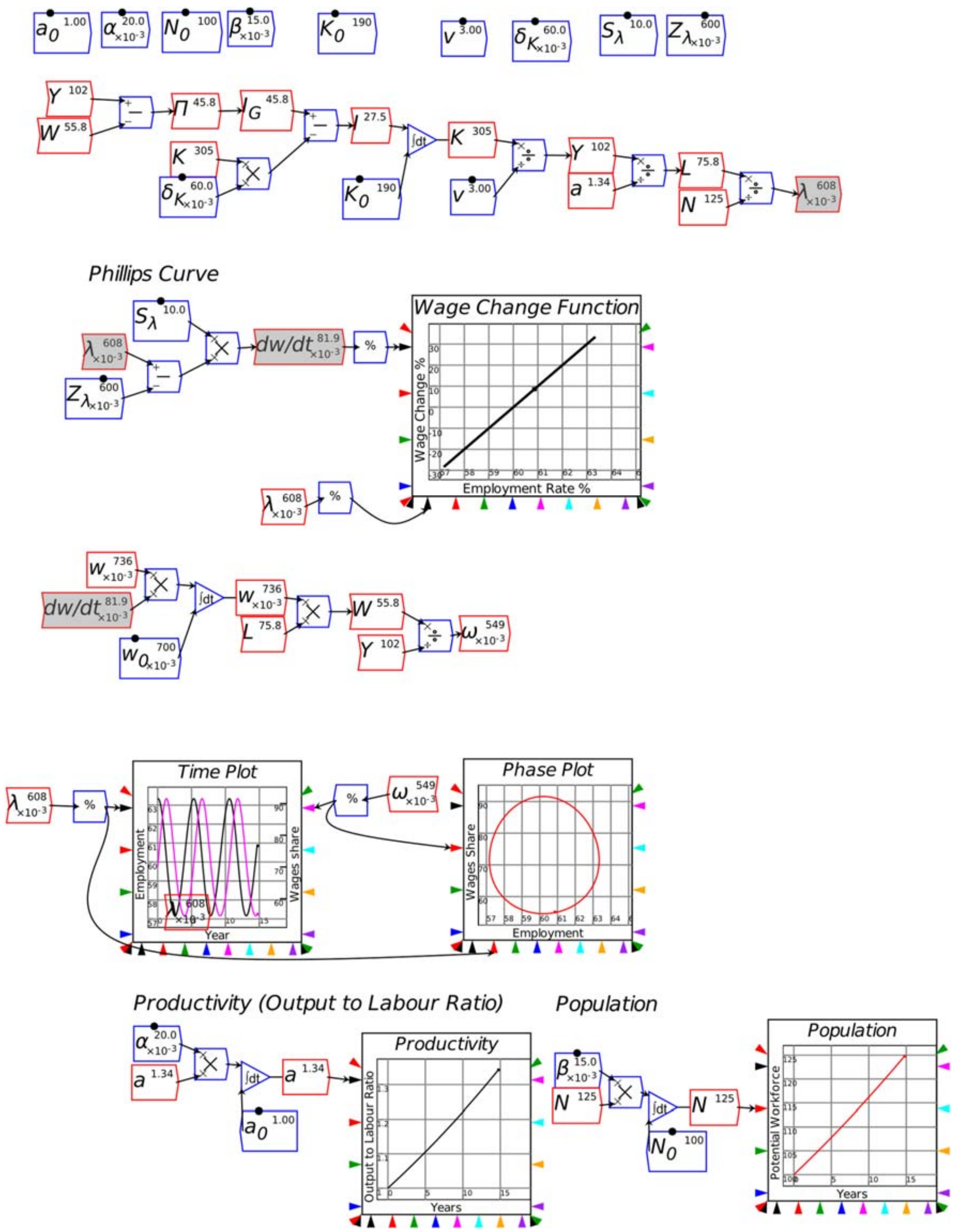


Even experts in Vensim, Stella, etc., describe these as “spaghetti diagrams”: interpreting them, for anyone other than the author, is like trying to interpret a bowl of spaghetti. There are several reasons for this—including that the equations are not visible on screen, but only inside forms sitting behind the on-screen text—but the plethora of wires alone obscures rather than illuminates the causal processes in the model.

Minsky has two simple design features that get around this weakness of standard system dynamics programs: you can simply copy a variable (or parameter) and place it wherever you like; and equations are directly visible on the canvas.[38] Figure 176 shows the same model without the curvy wires, and highlights the locations in which the variables λ and dw/dt are repeated on the canvas.

[38] Of course, this may make models harder to understand for readers without training in mathematics, so we’re supplementing this with a range of Tabs that enable a model builder to document a model in stages. In the current release we have Tabs for Parameters, Variables, Godley Tables and Plots, and a “Publication” Tab will come in a future release of Minsky.

Figure 176: Figure 174 without the curvy wires



OK, back to the modeling. To convert this into a model of Minsky rather than Goodwin, we need to:

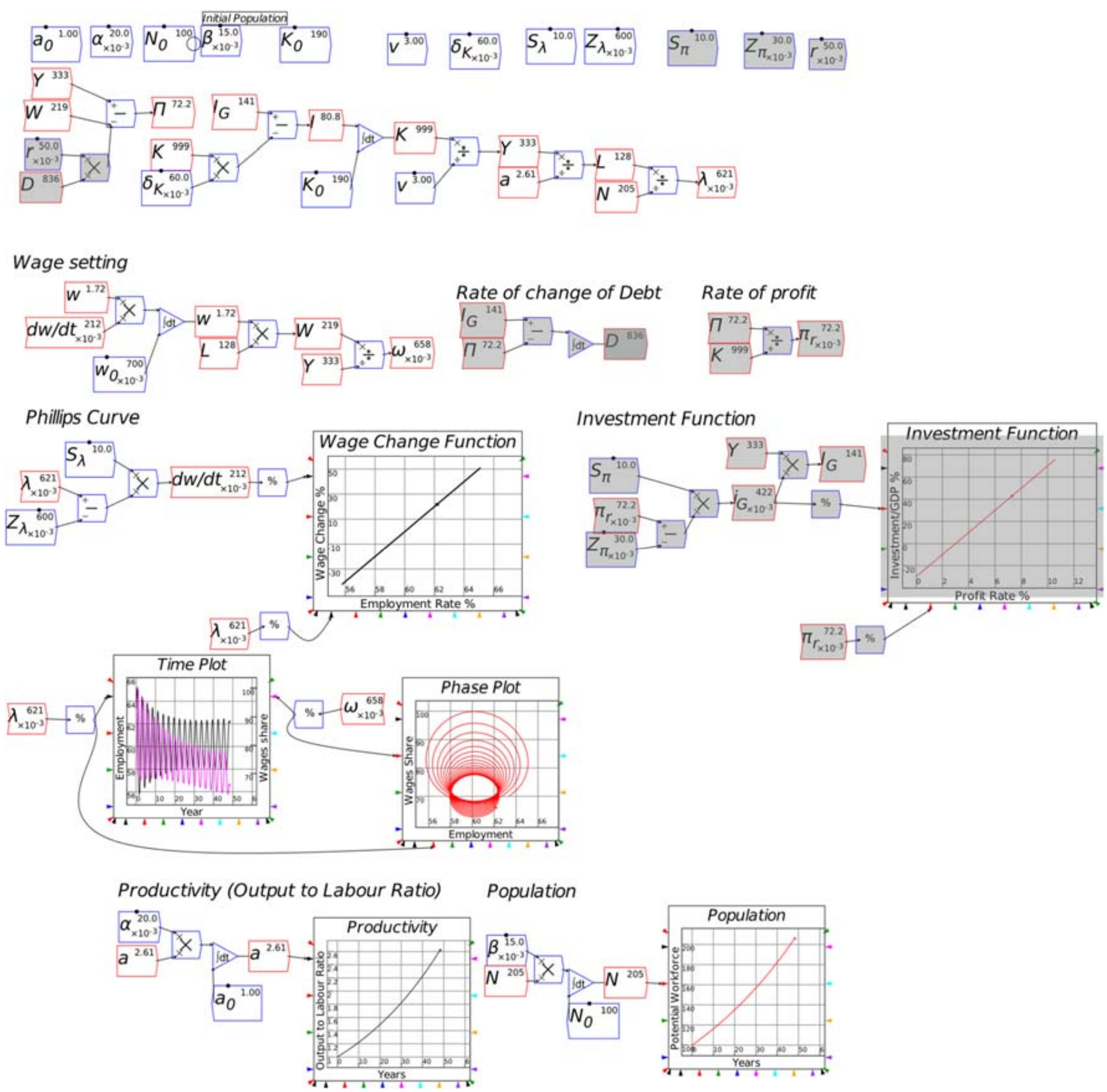
- Add Debt D as a system state, following equation (1.52);
- Delete the link from profit Π to gross investment I_G, since we are relaxing Goodwin's assumption that all profits are invested;
- Redefine profit to be net of wages and interest payments, as in equation (1.53); and

- Add an investment function relating the level of gross investment to the rate of profit. Here, I use the same linear form for the investment function as for the Phillips Curve, where the investment function determines how much of output is used as investment:[39]

$$I_G = f\left(\frac{\Pi}{K}\right)\cdot Y = S_\pi \cdot \left(\frac{Y - W - r\cdot D}{v\cdot Y} - Z_\pi\right) \tag{1.54}$$

Figure 177 shows the result of adding these features to the Goodwin model, with the new components of the model highlighted in grey. It should be obvious from the two plots that we're in a totally new ballgame compared to the Goodwin model: the fixed cycles of the Goodwin model have given way to something very different.

Figure 177: A system dynamics model of Minsky's Financial Instability Hypothesis



[39] Consumption is the residual in a Goodwin model, since Y and I are both determined, so that $C = Y - I$. Any model that determines all 3 will have unsold stocks as a residual, and so on.

Analyzing what lies behind these peculiar dynamics is more easily done in the reduced form model using the variables ω, λ and d_r, if only because this reduces the number of differential equations in the model from 4 in Figure 177 (K, w, a, N) to 3. I'll derive the equation for the rate of change of the debt ratio $\widehat{d_r}$ using the same system I explained in the Manifesto for $\hat{\lambda}$ and $\hat{\omega}$: using $\hat{x} = \frac{1}{x}\frac{dx}{dt}$ notation:

$$\begin{aligned} \widehat{\frac{x}{y}} &\equiv \hat{x} - \hat{y} \\ \widehat{x \cdot y} &\equiv \hat{x} + \hat{y} \end{aligned} \tag{1.55}$$

Applying this to the debt ratio $\widehat{d_r}$:

$$\begin{aligned} \widehat{d_r} &\equiv \widehat{\left(\frac{D}{Y}\right)} \\ &\equiv \hat{D} - \hat{Y} \end{aligned} \tag{1.56}$$

Working firstly on $\hat{D}$, in "hat" notation, equation (1.52) is:

$$\hat{D} \equiv \frac{1}{D}\frac{d}{dt}D = \frac{I_G - \Pi}{D} \tag{1.57}$$

From equation (1.57) we can substitute for $\hat{D}$:

$$\widehat{d_r} = \frac{I_G - \Pi}{D} - \hat{Y} \tag{1.58}$$

We can use the simple trick of multiplying the first term by Y/Y to remove D from the denominator:

$$\begin{aligned} \widehat{d_r} &= \frac{Y}{Y}\frac{I_G - \Pi}{D} - \hat{Y} \\ &= \frac{Y}{D}\frac{I_G - \Pi}{Y} - \hat{Y} \\ &= \frac{1}{d_r}\left(\frac{I_G}{Y} - \frac{\Pi}{Y}\right) - \hat{Y} \\ \widehat{d_r} &= \frac{1}{d_r}\left(\frac{I_G}{Y} - \left(1 - \omega - r \cdot d_r\right)\right) - \hat{Y} \end{aligned} \tag{1.59}$$

This leaves just I_G/Y and $\hat{Y}$ to be defined—the previous definition of $\hat{Y}$ in Equation (1.26) relied upon the previous assumption that gross investment was identical to profit. Here, I use the same linear form for the investment function as for the Phillips Curve, where the investment function determines how much of output is used as investment:[40]

[40] Consumption is the residual in a Goodwin model, since Y and I are both determined, so that $C = Y - I$. Any model that determines all 3 will have unsold stocks as a residual, and so on.

$$\begin{aligned}
\frac{I_G}{Y} &= f\left(\frac{\Pi}{K}\right) \\
&= S_\pi \cdot \left(\frac{Y - W - r\cdot D}{v\cdot Y} - Z_\pi\right) \\
&= S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right)
\end{aligned} \tag{1.60}$$

Since we are still treating v as a constant, we can still use Equation (1.25) to equate $\widehat{Y}$ to $\widehat{K}$:

$$\begin{aligned}
\frac{d}{dt}K &= I_G - \delta_K \cdot K \\
\widehat{K} &= \frac{I_G}{K} - \delta_K \\
&= \frac{I_G}{v\cdot Y} - \delta_K \\
\widehat{K} &= \frac{1}{v} S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right) - \delta_K \\
\widehat{Y} &= \frac{S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right)}{v} - \delta_K
\end{aligned} \tag{1.61}$$

Substituting (1.60) and (1.61) into (1.59) yields:

$$\widehat{d_r} = \frac{1}{d_r}\left(S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right) - \left(1-\omega - r\cdot d_r\right)\right) - \left(\frac{S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right)}{v} - \delta_K\right) \tag{1.62}$$

Lastly, because one term in the equation for λ is the rate of economic growth $\widehat{Y}$, this equation needs to be altered, since the growth rate now depends on the investment function, rather than just the level of profit:

$$\frac{d}{dt}\lambda = \lambda \cdot \left(\left(\frac{S_\pi \cdot \left(\frac{1-\omega - r\cdot d_r}{v} - Z_\pi\right)}{v} - \delta_K\right) - \alpha - \beta\right) \tag{1.63}$$

This gives us a 3-dimensional model which, as I explain in *Manifesto*, reproduces the essence of Minsky's Financial Instability Hypothesis:

$$\frac{d}{dt}\lambda = \lambda \cdot \left(\left(\frac{S_\pi \cdot \left(\frac{1-\omega-r\cdot d_r}{v} - Z_\pi \right)}{v} - \delta_K \right) - \alpha - \beta \right)$$

$$\frac{d}{dt}\omega = \omega \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) - \alpha \right) \tag{1.64}$$

$$\frac{d}{dt}d_r = S_\pi \cdot \left(\frac{1-\omega-r\cdot d_r}{v} - Z_\pi \right) - (1-\omega-r\cdot d_r) - \cdot d_r \cdot \left(\frac{S_\pi \cdot \left(\frac{1-\omega-r\cdot d_r}{v} - Z_\pi \right)}{v} - \delta_K \right)$$

This is a mouthful of equations, so to implement it in Minsky, I'll take advantage of Minsky's capacity to define functions and use them in multiple parts of a model. The functions that need to be defined are the profit share of output, gross investment, the growth rate, and the "Phillips Curve". Respectively, these are:

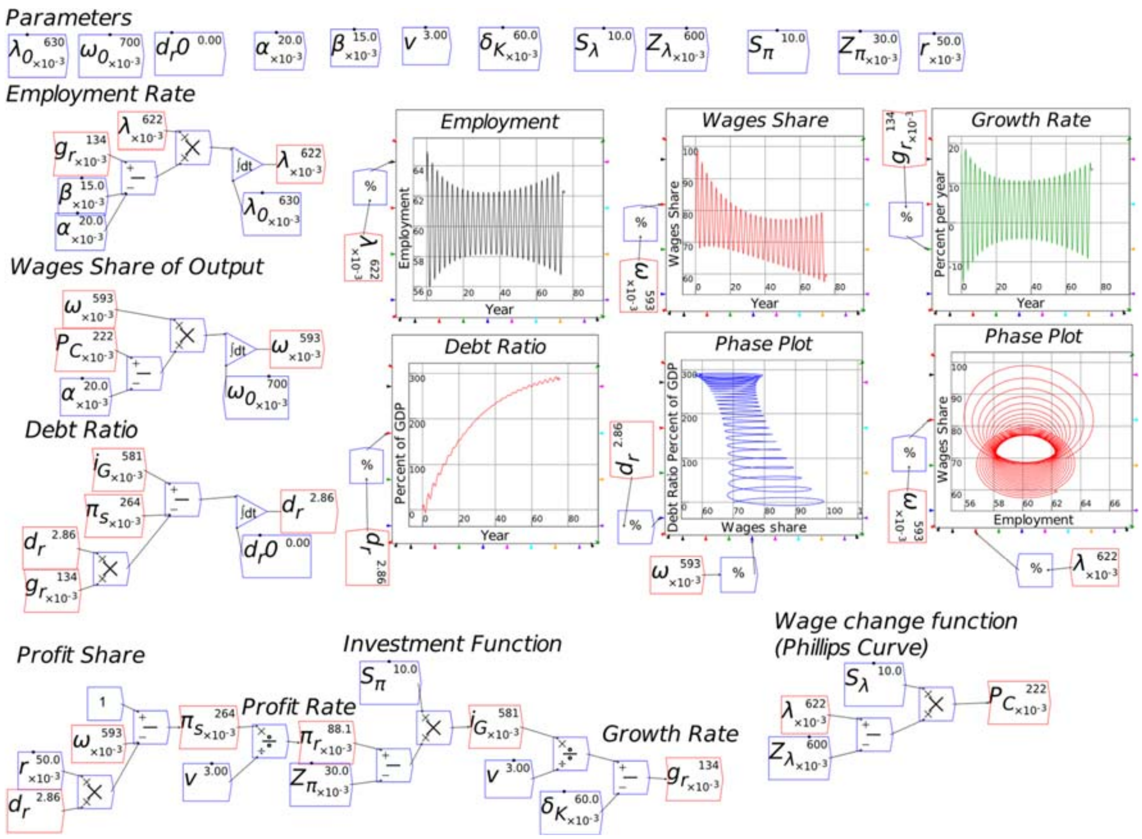
$$\begin{aligned} \pi_s &= 1-\omega-r\cdot d_r \\ i_G &= S_\pi \cdot \left(\frac{\pi_s}{v} - Z_\pi \right) \\ g_r &= \frac{i_G}{v} - \delta_K \\ P_C &= S_\lambda \cdot (\lambda - Z_\lambda) \end{aligned} \tag{1.65}$$

That makes equation (1.64) much more compact:

$$\begin{aligned} \frac{d}{dt}\lambda &= \lambda \cdot (g_r - \alpha - \beta) \\ \frac{d}{dt}\omega &= \omega \cdot (P_C - \alpha) \\ \frac{d}{dt}d_r &= i_G - \pi_s - d_r \cdot g_r \end{aligned} \tag{1.66}$$

These equations are easily entered into Minsky (to speed things up, you can copy and paste the parameters from the model in Figure 177)—see Figure 178.

Figure 178: Reduced form version of Minsky's Financial Instability model



The outcome of the model is identical to that in Figure 177 (given the same initial conditions), and the equations behind it can be analyzed, as shown in Chapter 11, to explain the peculiar dynamics behind the initially falling and then rising cycles in the model. It turns out that these are a particular subset of the dynamics of the Lorenz model, known as the “Intermittent Route to Chaos” (Pomeau and Manneville 1980). But more on that later. Let’s now turn to the essential missing ingredient in economic models of production: energy.

10 Energy

The work in this chapter is the most technically demanding in this book, and also the area most needing follow-up work … by people like you! As I note in *Manifesto*, the fact that economics has persisted for almost a century (Cobb and Douglas 1928; Leontief 1944, 1946a; Leontief 1946b; Leontief 1936) with models of production in which energy plays no role is, arguably, the Original Sin of Economics that has resulted in it being the misleading miasma that it is today. But escaping from that miasma is difficult, as I found as I worked with Matheus Grasselli and Tim Garrett to derive the models outlined here.

The starting point, though, was simple enough. Both Neoclassical and Post Keynesian mathematical models of production functions treated output as a function of inputs of Labor and Capital:

$$Y = F\left(L,K\right) \tag{1.67}$$

However, nothing can be produced without energy and matter inputs as well. The input-output approach to modelling production, pioneered by Leontief (Leontief 1936), did explicitly include inputs of both energy and raw materials (as well as other commodities) to produce output, but in practice, this method was generally implemented in an equilibrium framework in "Computable General Equilibrium" (CGE) models, when the equilibrium of an input-output matrix is unstable.[41] After the "Rational Expectations Revolution", Neoclassicals largely abandoned CGE modeling in favour of single commodity modeling, based on the Ramsey growth model (Ramsey 1928). The Cobb-Douglas Production Function (CDPF) ruled supreme in these models, and portrayed output as being produced by combining technology A, labour L and capital K:

$$Y = A \cdot L^{1-\alpha} \cdot K^{\alpha} \tag{1.68}$$

Post-Keynesian aggregate production form of the Leontief input-output model, which Goodwin used in his cyclical growth model (Goodwin 1967), and I used in my model of Minsky's Financial Instability Hypothesis, is:

$$Y = \min\left(\frac{K}{v}, a \cdot L\right) \tag{1.69}$$

Here v is normally described as the "Capital to Output ratio" (see Figure 57 on page 36), while a is called "Labor Productivity"—though I challenge both these labels later.

Neither aggregate production function explicitly include either matter or energy, something which mainstream economists largely ignored until the publication of the *Limits to Growth* (Meadows et al. 1972). Then, faced with a rival technology—system dynamics—they tried to develop a Neoclassical riposte. Stiglitz (Stiglitz 1974a; Stiglitz 1974b) and Solow (Solow 1974a) both proposed modified Cobb-Douglas functions of the form (Solow 1974a, p. 35, Equation 6):

$$Q = e^{mg \cdot t} \cdot L^{g} \cdot R^{h} \cdot K^{1-g-h} \tag{1.70}$$

Here R stood for "Resources", which include energy.[42] It is treated as a third input on equal footing with Labor and Capital.

[41] See (Blatt 1983) for an excellent explanation of this.

[42] Though the word appeared only once in the text of these three papers: "The proposition that limited natural resources provide a limit to growth and to the sustainable size of population is an old one. The natural resource

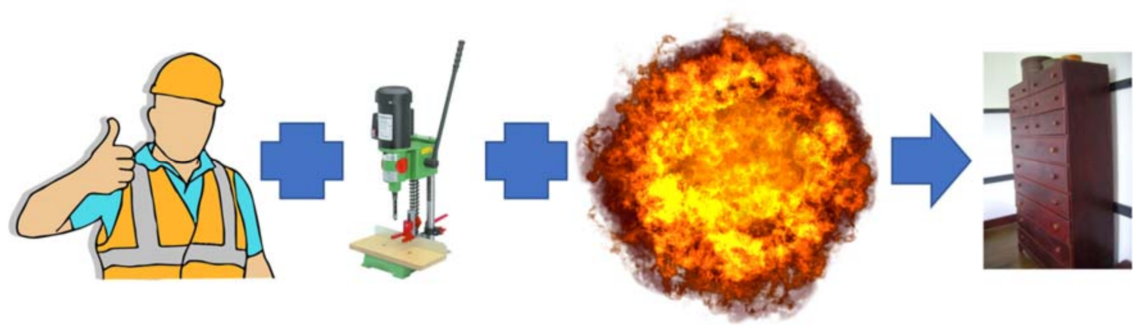
This didn't make sense to me, for two reasons. Firstly, it implied that energy could be added to a production process independently of labor and capital—say, by hitting a factory with a bolt of lightning—and thus producing output. But this was more likely to turn the factory into a smouldering ruin. Secondly, it implied that Labor and Capital could both function without energy—which of course they can't. Figure 179 both portrays and satirizes this approach.

Even far superior attempts to engage with the role of energy in production, like the work of Kümmel and Ayres (Kümmel et al. 2010; Kummel 2011; Lindenberger and Kümmel 2011; Kümmel et al. 2015), used a similar formulation where Capital, Labor and Energy were put on an equal footing. One step in the derivation of their *LinEx* production function was the introduction of a dimensionless specification of a production function, which again put Labor, Capital and Energy on an equal footing. Equation (1.71) shows equations 39 and 50 from (Kümmel et al. 2010, pp. 162,166)

$$\begin{gathered} y[k,l,e;t] \equiv Y\left(k \cdot K_o, l \cdot L_o, e \cdot E_o; t\right) / Y_0 \\ y_{CDE} = y_0 \cdot k^{\alpha_0} \cdot l^{\beta_0} \cdot e^{1-\alpha_0-\beta_0} \end{gathered} \tag{1.71}$$

What was needed was a formulation which made energy absolutely essential to the production process, and didn't pretend that it could be added independently of both labour and machinery.

Figure 179: Treating energy as an equivalent independent input to labour and capital



I was cogitating over this dilemma one evening while walking through Bob Ayres's apartment in Paris—which was full of statues—when the quip "Capital without energy is a sculpture; labor without energy is a corpse" flashed into my mind. This insight revealed that the correct form for incorporating energy in production wasn't equation **Error! Reference source not found.** and Figure 179, but equation (1.72) and Figure 180. Energy is an input to both machinery and labour, without which they can't do useful work:

$$Q=F\big(K(E),L(E)\big) \tag{1.72}$$

In doing useful work, waste is also necessarily generated—an application, in a very limited sense, of the Second Law of Thermodynamics. So the inputs to Labour and Capital are (different forms of) energy, and the outputs are materials transformed from non-usable inputs to usable commodities, plus waste.

---

that was the centre of the discussion in Malthus' day was land; more recently, some concern has been expressed over the limitations imposed by the supplies of oil, or more generally, energy sources, of phosphorus, and of other materials required for production." (Stiglitz 1974a, p. 123), and unlike *Limits to Growth*, no attempt was made to quantify either resources in general or energy in particular.

Figure 180: Labour and capital both need energy inputs to produce output (which inevitably produces waste)

The easiest way to develop a mathematical model of production out of this insight was to treat both $K(E)$ and $L(E)$ as being equal to the product of the units of each (K and L), times the annual energy consumption of each (E_K and E_L), times how efficiently those inputs were turned into useful work (e_K and e_L):

$$\begin{aligned} K(E) &= K \cdot E_K \cdot e_K \\ L(E) &= L \cdot E_L \cdot e_L \end{aligned} \tag{1.73}$$

I fed this into the Capital and Labour components of the Cobb-Douglas Production Function (minus what soon transpired to be the superfluous A):

$$\begin{aligned} Y(t) &= K(t)^{\alpha} \cdot L(t)^{1-\alpha} \\ &= (K \cdot E_K \cdot e_K)^{\alpha} \cdot (L \cdot E_L \cdot e_L)^{1-\alpha} \end{aligned} \tag{1.74}$$

Rearranging this led to the expression in Equation (1.75), where the last two components are the standard expressions for capital and labour in the CDPF:

$$Y(t) = (E_L \cdot e_L)^{1-\alpha} \cdot (E_K \cdot e_K)^{\alpha} \cdot K^{\alpha} \cdot L^{1-\alpha} \tag{1.75}$$

The first component—the energy consumption of the typical worker, times how much of that energy is turned into useful work in production—can be treated as a constant: the capacity for a worker to put energy into useful work hasn't varied since humans evolved, and is roughly 100 Watts. The second is the energy input to the "representative machine" at a given time, multiplied by how much of that energy is turned into useful work. The energy consumption of the "representative machine" has risen from the tonnes of fuel per day that powered James Watt's steam engine to the tonnes per second that fuel Elon Musk's rockets. The efficiency with which machines turn energy into useful work is an unknown scalar bounded by $(0,1)$. Treat the product $(E_L \cdot e_L)^{1-\alpha} \cdot e_K{}^{\alpha}$ as a constant C_L and reserve the exponents for factors that actually change over time: E_K, K, L . Then our energy-modified Cobb-Douglas Production Function is equation (1.76):

$$
\begin{aligned}
Y(t) &= C_L \cdot (E_K \cdot K)^{\alpha} \cdot L^{1-\alpha}; or \\
Y(t) &= C_L \cdot E_K{}^{\alpha} \cdot K^{\alpha} \cdot L^{1-\alpha}
\end{aligned}
\tag{1.76}
$$

Derived this way, the “total factor productivity” term A is actually a constant times the energy input to the “representative machine” of a given time.

This is the form in which I published this work (with Bob Ayres and Russell Standish) in “A Note on the Role of Energy in Production” (Keen et al. 2019, p. 44). But we only used the Cobb-Douglas Production Function in the probably forlorn hope that some Neoclassicals might therefore read the paper. The real basis for modelling the role of energy in production properly is the “Leontief Production Function” used by Post Keynesians (equation (1.77)):

$$
Y(t) = min\left(\frac{K}{v}, a \cdot L\right) = u \cdot \frac{K}{v} = a \cdot L \tag{1.77}
$$

On the other hand, the Cobb-Douglas Production Function belongs in the dustbin of the history of economic thought.

10.1 Forget the “Cobb-Douglas Production Function” (an optional read)

Neoclassicals take great solace in the fact that their preferred aggregate production function fits the national data so well:

> I have always found the high R^2 reassuring when I teach the Solow growth model. Surely, a low R^2 in this regression would have shaken my faith. (Mankiw 1997, p. 104)

This is doubly so, because the model also encapsulates the Neoclassical belief that the real wage is the marginal productivity of labor, and the rate of profit is the marginal productivity of capital. The fact that the empirically measured Cobb-Douglas exponents are very close to the national income shares of labour and capital played a major role in the acceptance of the Cobb Douglas by Neoclassical economists:

> aggregate production functions apparently work nevertheless and do so in a way which is prima facie not easy to explain. It is easy enough to understand why, in economies in which things move more or less together, a relationship giving an aggregate measure of output as dependent on aggregate measures of capital and labor should give a good fit when applied to the data. What is not so easy to explain is the fact that the marginal product of labor in such an estimated relationship appears to give a reasonably good explanation of wages as well. *In its simplest form, this puzzle is set by a remark which Solow once made to me that, had Douglas found labor's share to be 25 per cent and capital's 75 per cent instead of the other way around, we would not now be discussing aggregate production functions*. (Fisher 1971, p. 305)

Tragically, in one of the most insightful and witty papers in the history of economics,[43] “The Humbug Production Function”, Anwar Shaikh (Shaikh 1974) gave the explanation that Fisher craved—and it wasn’t one that Fisher would have enjoyed. *The Cobb-Douglas Production Function is just a tautology*.

[43] Given the ignorant and humourless state of economics in general, this isn’t a high bar: it’s more of a hurdle for sausage dogs. But Anwar cleared that bar by a large margin. Read the paper!

It simply restates, in exponential rather than additive form, the identity that "Income equals Wages plus Profits" under conditions of relatively constant income shares:

> Shaikh demonstrated that the Cobb–Douglas is simply an (anti-)logarithmic transformation of the income identity under the assumption that relative income shares are constant. (Carter 2011, p. 259)

Therefore, regressing the Cobb-Douglas Production Function against national income data is like regressing Y against Y: of course you'll get a high correlation. That correlation falls below 100% only to the extent to which its assumptions—such as a uniform wage rate and constancy of income shares—deviate from actual conditions.

I'll repeat Shaikh's proof here to explain why the Cobb-Douglas function should be rejected as a basis for economic modelling. Start with the identity that income Y equals wages W plus profits Π:

$$Y = W + \Pi \tag{1.78}$$

Assume a uniform real wage rate w and a uniform rate of profit r, applied respectively to a labour force L and stock of capital K:

$$Y = w \cdot L + r \cdot K \tag{1.79}$$

Differentiate both sides and divide by Y:

$$\begin{aligned}\frac{1}{Y}\cdot\frac{d}{dt}Y &= \frac{1}{Y}\cdot\frac{d}{dt}\left(w\cdot L + r\cdot K\right)\\ &= \frac{1}{Y}\cdot\left(w\frac{d}{dt}L + L\cdot\frac{d}{dt}w + r\cdot\frac{d}{dt}K + K\cdot\frac{d}{dt}r\right)\\ &= \frac{w}{Y}\cdot\frac{d}{dt}L + \frac{L}{Y}\cdot\frac{d}{dt}w + \frac{r}{Y}\cdot\frac{d}{dt}K + \frac{K}{Y}\cdot\frac{d}{dt}r\end{aligned} \tag{1.80}$$

Bring in income shares—the proportion of income going to workers and capitalists respectively—by multiplying each fraction by the "missing ingredient": multiply the first term by L/L, the second by w/w and so on:

$$\frac{1}{Y}\cdot\frac{d}{dt}Y = \frac{L}{L}\frac{w}{Y}\cdot\frac{d}{dt}L + \frac{w}{w}\frac{L}{Y}\cdot\frac{d}{dt}w + \frac{K}{K}\frac{r}{Y}\cdot\frac{d}{dt}K + \frac{r}{r}\frac{K}{Y}\cdot\frac{d}{dt}r \tag{1.81}$$

Group the terms so that income shares multiply each differential:

$$\frac{1}{Y}\cdot\frac{d}{dt}Y = \frac{1}{L}\left(\frac{L\cdot w}{Y}\right)\cdot\frac{d}{dt}L + \frac{1}{w}\left(\frac{w\cdot L}{Y}\right)\cdot\frac{d}{dt}w + \frac{1}{K}\left(\frac{K\cdot r}{Y}\right)\cdot\frac{d}{dt}K + \frac{1}{r}\left(\frac{r\cdot K}{Y}\right)\cdot\frac{d}{dt}r \tag{1.82}$$

Call the profit share of GDP $\alpha = \frac{K\cdot r}{Y}$, and the wages share $1-\alpha = \frac{w\cdot L}{Y}$

$$\frac{1}{Y}\cdot\frac{d}{dt}Y = (1-\alpha)\cdot\frac{1}{L}\cdot\frac{d}{dt}L + (1-\alpha)\cdot\frac{1}{w}\cdot\frac{d}{dt}w + \alpha\cdot\frac{1}{K}\frac{d}{dt}K + \alpha\cdot\frac{1}{r}\cdot\frac{d}{dt}r \tag{1.83}$$

"Percentage" rates of change can be expressed as the differential of the logs, so that

$$\frac{1}{Y}\cdot\frac{d}{dt}Y = \frac{d}{dt}\ln(Y) \tag{1.84}$$

And likewise for the other differentials in (1.83):

$$\frac{d}{dt}\ln(Y)=(1-\alpha)\cdot\frac{d}{dt}\ln(w)+(1-\alpha)\cdot\frac{d}{dt}\ln(L)+\alpha\cdot\frac{d}{dt}\ln(K)+\alpha\cdot\frac{d}{dt}\ln(r) \tag{1.85}$$

At this point, we assume that income shares α and $1-\alpha$ are constant. They do change over time—that was the basis of the Goodwin model[44]—but relatively slowly compared to employment, wages, capital and the rate of return on capital, as codified in Kaldor's stylized facts:

> the share of wages and the share of profits in the national income has shown a remarkable constancy in " developed " capitalist economies of the United States and the United Kingdom since the second half of the nineteenth century. (Kaldor 1957, pp. 591-92)

Neoclassical modelers also treat α as a constant in their models. So we can do the same, and then integrate both sides, with integration being the inverse of differentiation:

$$\begin{aligned}\int\frac{d}{dt}\ln(Y)&=\int\left((1-\alpha)\cdot\frac{d}{dt}\ln(w)+(1-\alpha)\cdot\frac{d}{dt}\ln(L)+\alpha\cdot\frac{d}{dt}\ln(K)+\alpha\cdot\frac{d}{dt}\ln(r)\right)\\ \ln(Y)&=(1-\alpha)\cdot\ln(w)+(1-\alpha)\cdot\ln(L)+\alpha\cdot\ln(K)+\alpha\cdot\ln(r)\end{aligned} \tag{1.86}$$

A constant multiplying the logarithm of a variable is the same as the logarithm of the variable raised to the power of that constant: $\alpha\cdot ln(w)=ln(w^{\alpha})$ and so on, so that

$$\ln(Y)=\ln\left(w^{1-\alpha}\right)+\ln\left(L^{1-\alpha}\right)+\ln\left(K^{\alpha}\right)+\ln\left(r^{\alpha}\right) \tag{1.87}$$

Take exponentials of both sides:

$$\begin{aligned}Y&=e^{\ln\left(w^{1-\alpha}\right)+\ln\left(L^{1-\alpha}\right)+\ln\left(K^{\alpha}\right)+\ln\left(r^{\alpha}\right)}\\ &=w^{1-\alpha}\cdot r^{\alpha}\cdot L^{1-\alpha}\cdot K^{\alpha}\end{aligned} \tag{1.88}$$

This is almost the "Cobb-Douglas Production Function": the only difference is that Cobb and Douglas began with a constant in the place of $w^{1-\alpha}\cdot r^{\alpha}$, while later Neoclassicals use a time-varying $A(t)$, which they call "total factor productivity"—and which, as explained previously, is actually the energy consumption level of the "representative machine":

$$Y=A\cdot L^{1-\alpha}\cdot K^{\alpha} \tag{1.89}$$

It's no wonder, therefore, that the "Cobb Douglas Production Function" fits the empirical data on output and income distribution, since it can be derived from that data, under the not entirely false assumption that income shares are relatively constant.

Neoclassicals estimate A as a residual from the time series for Labour and Capital—since the vast majority of them are not aware of Shaikh's proof, and in typical Neoclassical fashion, those that are think that Solow's rejoinder to Shaikh (Solow 1974b) settled the dispute in their favour. But it didn't (Shaikh 1980, 2005; Labini 1995).

[44] The Goodwin model's empirically exaggerated variation in wages share is dramatically reduced when nonlinear behavioural functions, and monetary and price dynamics, are included in the model.

There is a further weakness, pointed out by Mankiw and noted in *Manifesto*, that while the CDPF fits national data well with its exponent conforming to national income distribution data, this value for α also results in predictions of relative economic performance that are disastrously bad:

> Because poor countries have about one-tenth the income of rich countries, they should have returns to capital that are about one hundred times as large. In particular, since the profit rate is about 10 percent per year in rich countries, it should be about 1,000 percent per year in poor countries. (Mankiw et al. 1995, p. 287)

Another good reason to reject the CDPF is its assumed easy substitution of one input for another. This in itself is a dubious assumption—you can't easily vary the labour and capital inputs into a production process—but in the context of energy, it is simply false. Energy can be used more or less efficiently, but there is no substituting for it. If you don't have energy, you don't have output, period. On this basis, the fixed coefficient formulation of the Leontief is more sensible. And, as the next section shows, it is easy to interpret the capital output ratio in the Leontief function as the efficiency with which energy is turned into useful work. The Leontief function has therefore implicitly contained the role of energy all along.

10.2 Generalizing the Leontief Production Function

Superficially, the Leontief Production Function has the same weakness as the Cobb-Douglas when it comes to the role of energy of energy in production—there isn't one. Stating the Leontief in terms of a utilization of capital rate u, a capital to output ratio v, and an output to labour ratio a, it is:

$$Y\left(t\right)=u\cdot\frac{K}{v}=a\cdot L \tag{1.90}$$

In fact, it's relatively easy to show that the capital to output ratio v, which has been treated simply as an empirical regularity with a fairly constant value of between 2 and 4 for most economies, is actually the inverse of e_K: the efficiency with which machines turn energy into useful work.

We have to start by defining what aggregate output Y actually is, mathematically, in a macroeconomic model. Economists treat it as just a number—a scalar—but the real question is "a number of what?". It is ***not*** a pure number, but a dimensioned number: it is a number of identical "things". These "things" are stylized universal commodities, which in the models can be consumed by workers as consumption items C, or used as investment items I, which are inputs to make machines, or "Capital" K. The term for this (highly unrealistic) universal commodity is a "widget". So Y in an aggregate macroeconomic model is the number of widgets produced per year.

In this same sense, we—my collaborators Matheus Grasselli (Grasselli and Costa Lima 2012; Grasselli and Maheshwari 2017; Grasselli and Nguyen-Huu 2018; Giraud and Grasselli 2019) and Tim Garrett (Garrett 2011, 2012a, b, 2014, 2015) and I—introduced Q as the energy equivalent of Y: it was the amount of energy (measured in joules) E_Y contained in a widget, multiplied by the number of widgets produced per year Y.

$$Q=E_Y\cdot Y \tag{1.91}$$

We then equated Q to the energy converted into useful work by machinery, using equation (1.73):

$$Q=u\cdot K\cdot E_K\cdot e_K \tag{1.92}$$

We can now show the relationship between Q and Y, using equation (1.90):

$$\begin{aligned} u\cdot K\cdot E_K\cdot e_K &= E_Y\cdot u\cdot\frac{K}{v} \\ E_K\cdot e_K &= \frac{E_Y}{v} \end{aligned} \tag{1.93}$$

If we now equate terms with the same dimensions—energy per year in the cases of E_Y, E_K and scalars in the cases of e_K, v, we get, firstly, that e_K is the inverse of v:

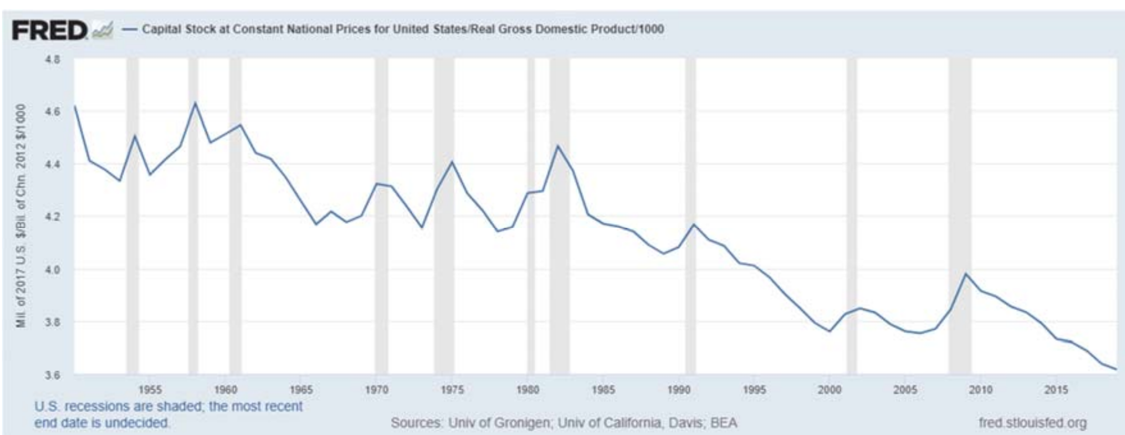
$$e_K = \frac{1}{v} \tag{1.94}$$

Secondly, the conversion factor between output in widgets and output in terms of energy (useful work) at any given time in this single-commodity world is the energy consumption level of the typical machine of that time:

$$E_K = E_Y \tag{1.95}$$

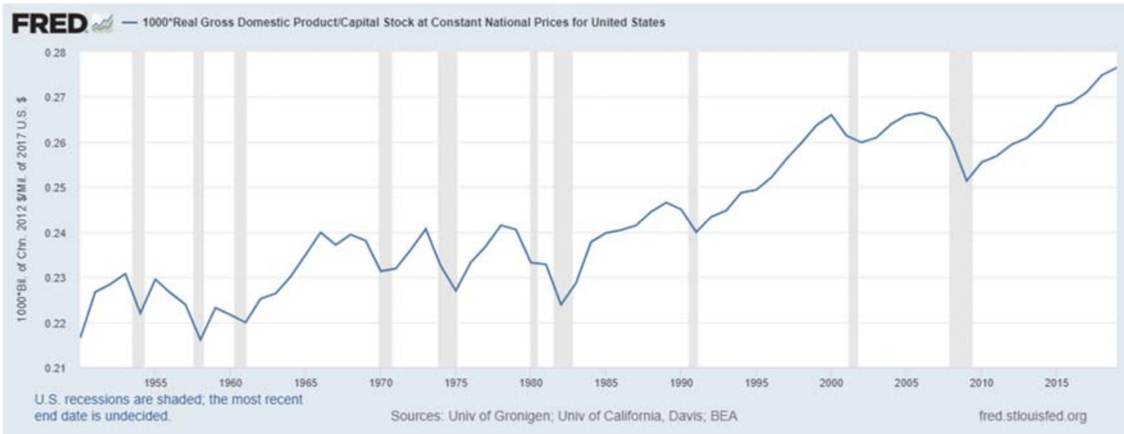
The first finding was a surprise, but one that made intuitive sense once we realized it: the empirically-observed rough proportionality between output Y and capital stock K, which is an essential aspect of the Leontief model, actually represents the efficiency with which machines turn their energy inputs into useful work. In this sense, the Leontief model has always included a role for energy—it just wasn't explicit. This then turns on its head the standard rendition of the capital to output ratio. This has been declining over time, somewhat inexplicably—see Figure 181:

Figure 181: Capital output ratio from https://fred.stlouisfed.org/series/RKNANPUSA666NRUG#0



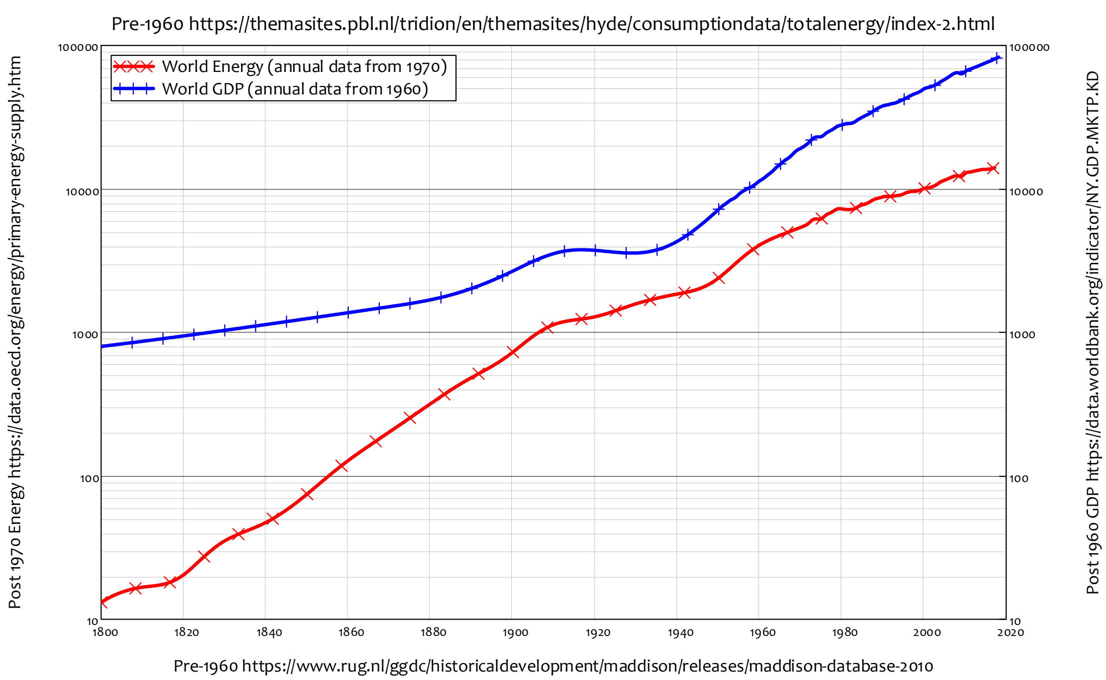
However, from this energy-based perspective, what this actually shows is a rise over time in the efficiency with which machinery turns energy into useful work—or, also quite feasibly, an increase in the amount of GDP which is virtual or non-physical (neither commodities nor directly consumed energy, though of course virtual products—such as video games—require physical resources, including file servers and electricity). Though there is an increasing trend right from the start of the data, it becomes much stronger and more pronounced in the early 1980s, which coincides with the development of the computer and the "virtual" economy it allows, the financialization of capitalism and the rise in what Marx would call "fictitious output" from "fictitious capital",[45] and the start of US capital outsourcing production to China.

Figure 182: The efficiency with which energy is turned into useful work (GDP, or Y)



The rise in the ratio also supports to some degree the "decoupling" argument, that over time less and less of GDP is dependent on physical and energetic output—though it's also important to put this in context: the dependence *at the global level* of output on energy remains extremely high (the data in Figure 182 comes solely from the USA). When one looks at the long-run global data (Figure 183), and especially data for the last half-century (Figure 184), the correlation between GDP and energy is extremely tight.

Figure 183: Global GDP and energy consumption since 1800[46]



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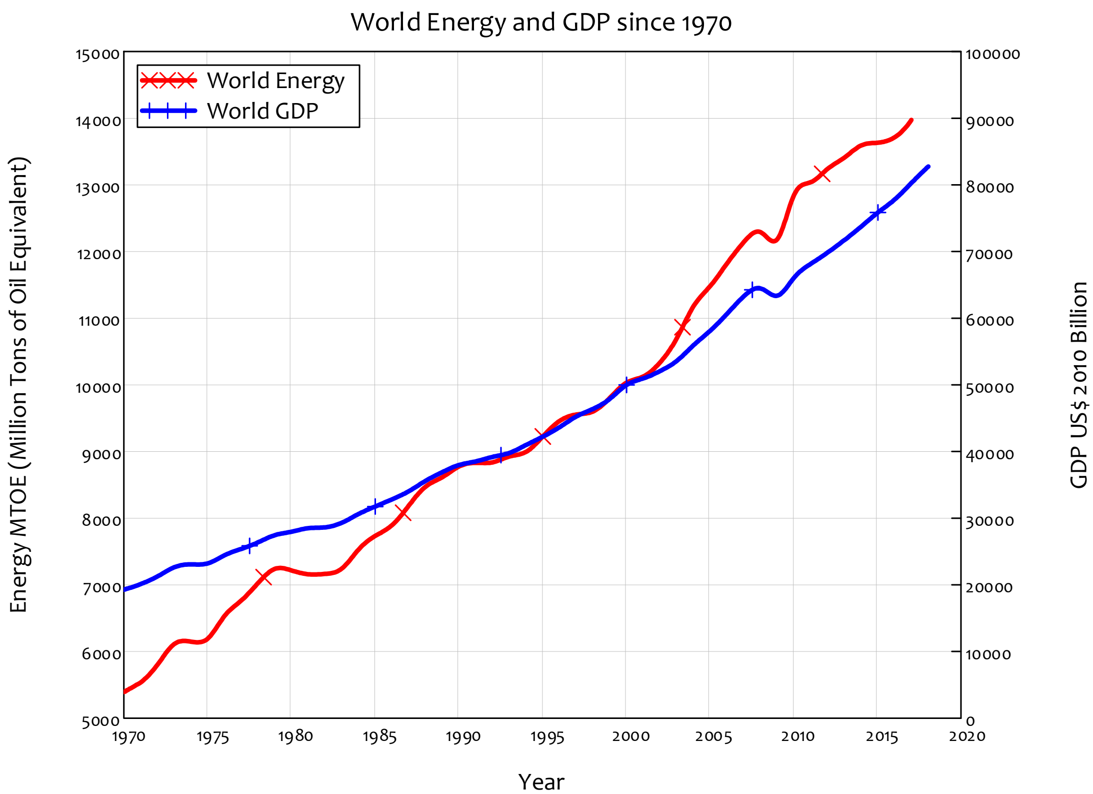
[45] If the latter explanation for the rising ratio is more valid, then we should expect to see this ratio fall in the future if the dominance of the finance sector ever comes to an end.

[46] Data sources Pre-1960:

Figure 184: Global GDP and Energy data since 1970[47]

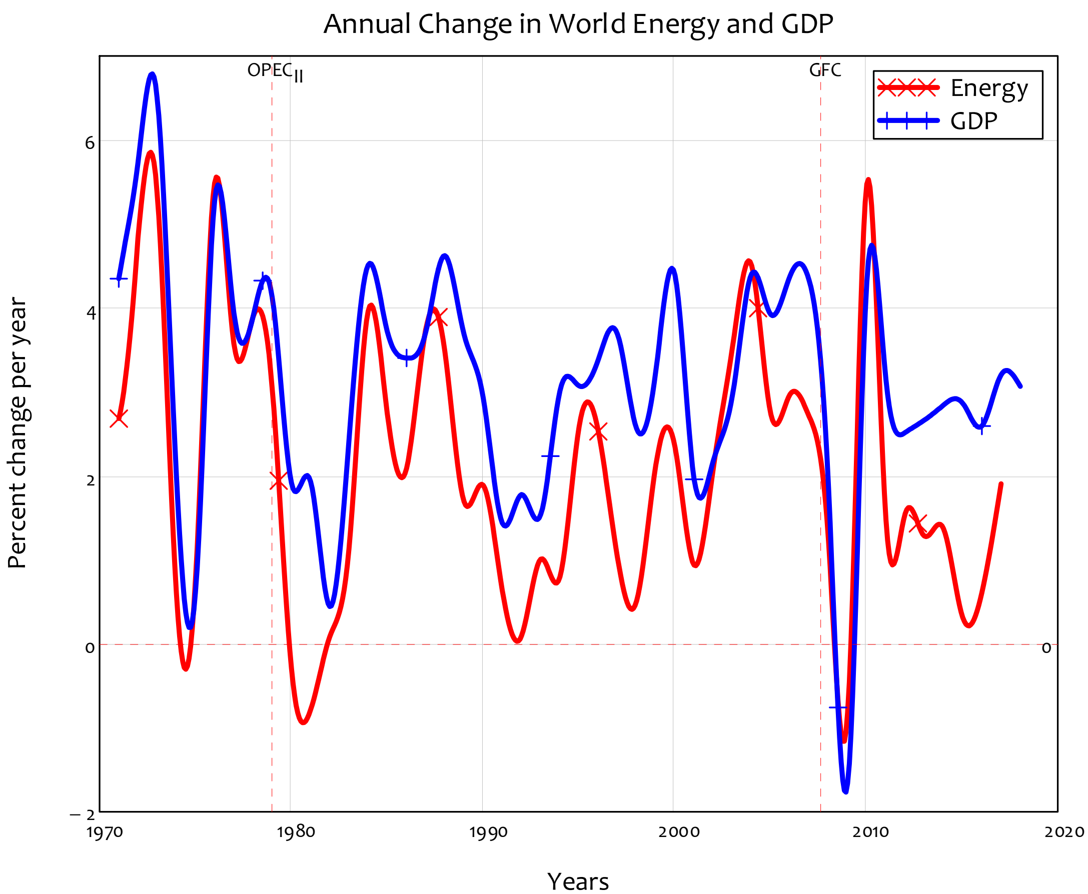


These are both rising trends which generates a spurious correlation of course, but the annual change data is also extremely tightly correlated, and almost 1 for 1—see Figure 185.

Energy https://themasites.pbl.nl/tridion/en/themasites/hyde/consumptiondata/totalenergy/index-2.html
GDP https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-database-2010

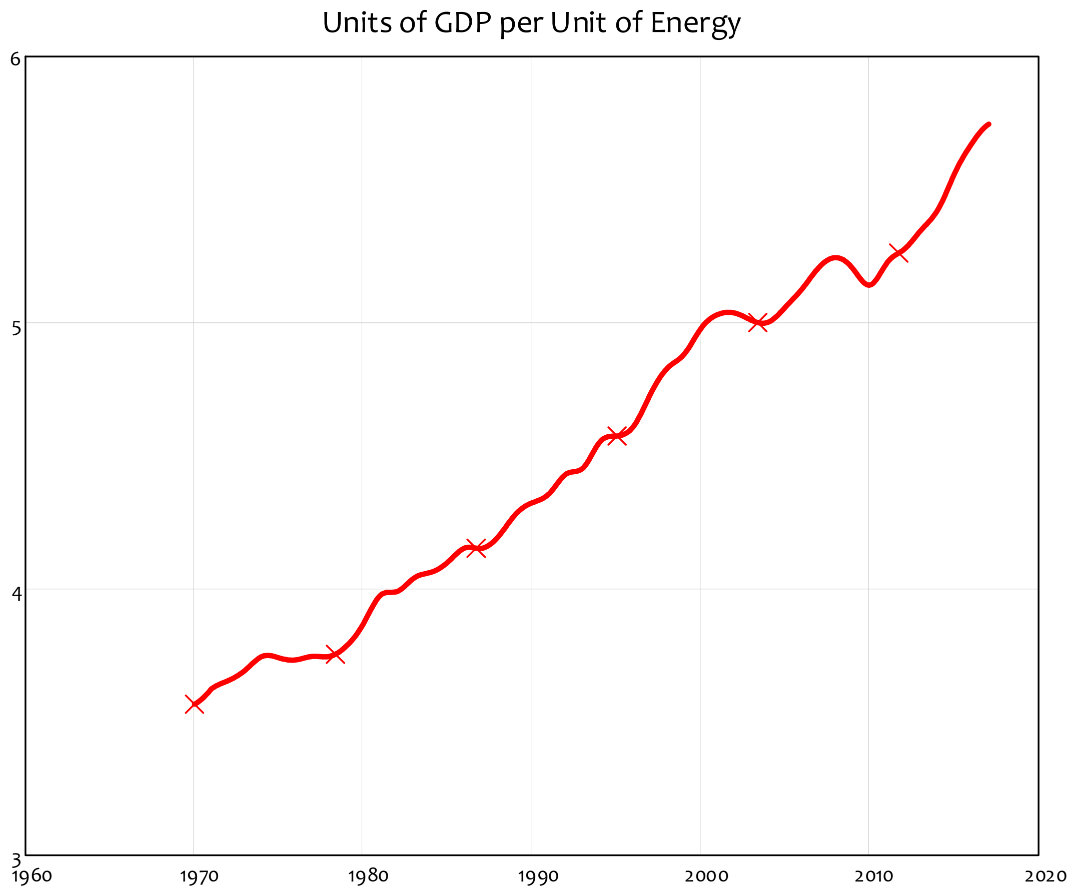
[47] Data sources Post 1970:
Energy https://data.oecd.org/energy/primary-energy-supply.htm
GDP https://data.worldbank.org/indicator/NY.GDP.MKTP.KD

Figure 185: Annual change in GDP against change in energy (Correlation 0.83)



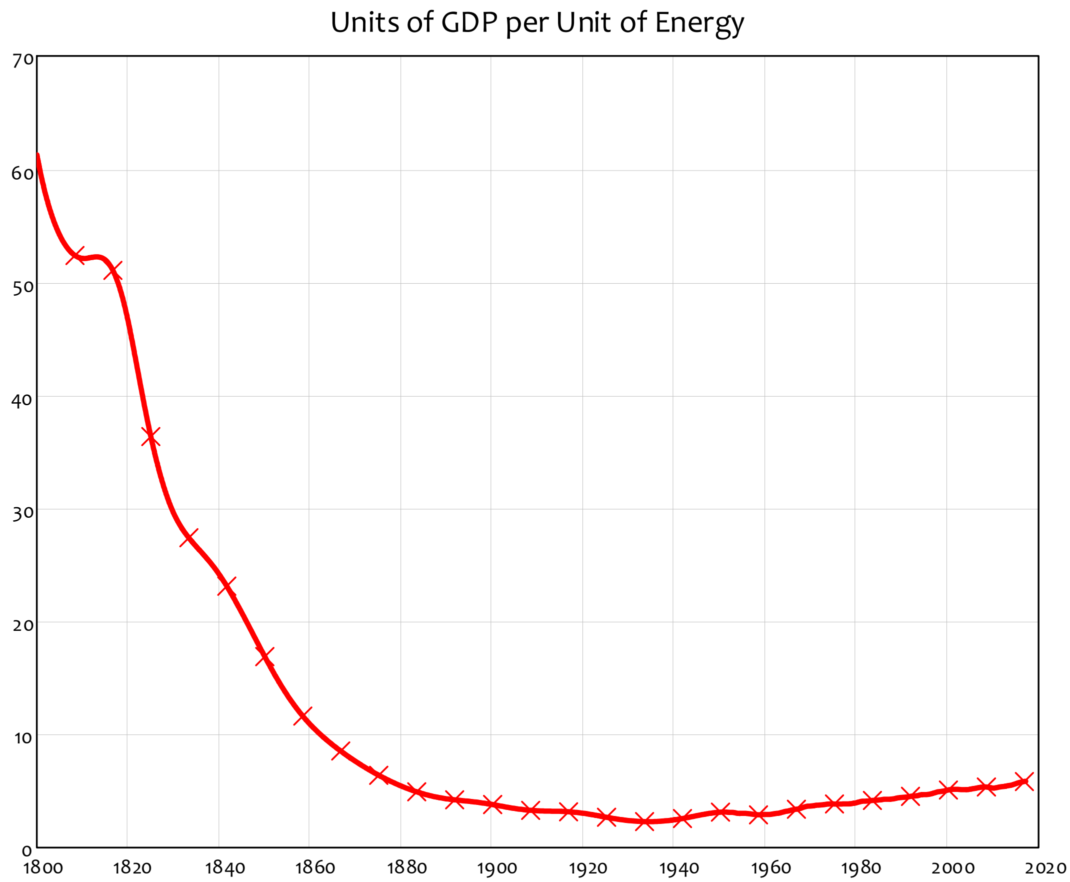
However, the rise in the GDP to energy ratio is also apparent at the global level since the 1970s—see Figure 186.

Figure 186: GDP in constant US$ divided by Energy in BTU



However, the long-term data shows that this is a reversal of the trend since the start of the industrial age—see Figure 187.

Figure 187: GDP to Energy since 1800



Interpretation of this long term trend in GDP to Energy is an open question—it quite possibly represents the change from non-fossil to fossil-fuel driven industry over the course of the 19th century. That said, the very tight fit between energy and GDP from the 20th century on, and especially for the period from 1970 till 2017, provides another strong argument for the Leontief Production Function as the proper tool to model the close to linear relationship between energy consumption and GDP.

10.3 A Goodwin model with Energy

One key element in the previous section was using dimensional analysis to unravel an equation—equation (1.93), where the first term on the left was dimensioned in units of energy per year, and the next term was a scalar. It therefore made sense to equate components in the equation with the same dimensions:

$$\begin{aligned} E_K \cdot e_K &= \frac{E_Y}{v} \\ E_K &= E_Y \\ e_K &= \frac{1}{v} \end{aligned} \tag{1.96}$$

Dimensional analysis is an important technique in science and engineering to check the validity of a model, and it should be in economics too:

> The consistent and correct use of dimensions is essential to scientific work involving mathematics. Their very existence creates the potential for errors: omitting them when they should be included, misusing them when they are included, and others. However, their existence also makes possible dimensional analysis, which can be a significant factor in avoiding error. In the equation $y = f(.)$, if y should have dimensions then so also should $f(.)$, and they should be identical to those of y. If y should not have them then neither should $f(.)$ have them… An error revealed by a correctly performed dimensional analysis indicates a fundamental problem. Therefore, the importance of dimensions for science can hardly be overstated. (Barnett II 2004, p. 95).

Economics has ignored dimensional analysis, as is obvious enough in the Cobb Douglas Production function itself. As Barnett points out, the dimensions of the function can only be made reasonable by ascribing a ludicrous set of dimensions to the $A(t)$ term:

> A typical CD function is given by $Q = A \cdot K^{\alpha} \cdot L^{\beta}$, in which: Q is the output variable; K and L are the capital and labor input variables, respectively; A, may be a constant or a variable; and, α and β are the elasticity of output with respect to capital and with respect to labor, respectively…
>
> If dimensions are used correctly, output, capital, and labor each must have both magnitude and dimension(s), while α and β are pure numbers. Assume, for example, that:
>
> (1) Q is measured in widgets/elapsed time (wid/yr);
>
> (2) K is measured in units of machine-hours/elapsed time (caphr/yr); and,
>
> (3) L is measured in man-hours/elapsed time (manhr/yr). (Barnett II 2004, p. 96)

The only way to balance this equation in dimensional terms is for the A term to have crazy dimensions for something that Neoclassicals, not knowing of Shaikh's critique, describe as "Total Factor Productivity":

$$\begin{aligned}
Q &= A \cdot K^{\alpha} \cdot L^{1-\alpha} \\
\frac{Widgets}{Year} &= A_{Dimensions} \cdot \left(\frac{K_{Hours}}{Year}\right)^{\alpha} \cdot \left(\frac{L_{Hours}}{Year}\right)^{1-\alpha} \\
\frac{Widgets}{Year} &= A_{Dimensions} \cdot K_{Hours}{}^{\alpha} \cdot L_{Hours}{}^{1-\alpha} \cdot \frac{1}{Year} \\
Widgets &= A_{Dimensions} \cdot K_{Hours}{}^{\alpha} \cdot L_{Hours}{}^{1-\alpha} \\
A_{Dimensions} &= \frac{Widgets}{K_{Hours}{}^{\alpha} \cdot L_{Hours}{}^{1-\alpha}}
\end{aligned} \tag{1.97}$$

The "Cobb Douglas Production Function", as well as being based on a tautology, is also dimensionally weird. What we need instead is a model of the biophysical processes by which inputs of energy, raw materials and intermediate products are turned into usable physical products.[48] This chapter will take the first tentative steps towards this, in models in which energy plays a fundamental role. Our first pass was a model in which the inputs are energy, and the outputs are energy: the production process turns energy in a form that can't be consumed by humans—say, coal—into one in which it can—say, electricity.

We started from the points established earlier about the Leontief Production Function (LPF), that by using the redefinition of K and L as means by which energy is used to perform useful work:

$$\begin{aligned} K(E) &= K \cdot E_K \cdot e_K \\ L(E) &= L \cdot E_L \cdot e_L \end{aligned} \tag{1.98}$$

We can recast the standard LPF:

$$Y = u \cdot \frac{K}{v} = a \cdot L \tag{1.99}$$

In terms of energy, so that output Q, in terms of useful energy per year, equals capacity utilization u (a scalar) times the number of machines K, times energy per machine per year E_K, times the efficiency with which that energy input is turned into useful work e_K:

$$Q = u \cdot K \cdot E_K \cdot e_K \tag{1.100}$$

This is dimensionally consistent:

$$\begin{aligned} \frac{Energy}{Year} &= Scalar \cdot Machine \cdot \frac{\frac{Energy}{Year}}{Machine} \cdot Scalar \\ \frac{Energy}{Year} &= \frac{Energy}{Year} \end{aligned} \tag{1.101}$$

The Leontief Production Function in terms of energy per year is mapped across to the standard measure of GDP in Widgets per year by dividing Q by E_K, where $E_K = E_Y$, the energy content of a widget:

$$Y = \frac{Q}{E_Y} = \frac{Q}{E_K} = u \cdot K \cdot e_K \tag{1.102}$$

For simplicity, I'll work with $u = 1$ as in Goodwin's original model.[49]

We start from the definition of Q in terms of K:

$$Q = K \cdot E_K \cdot e_K \tag{1.103}$$

[48] This did exist, to some degree, in the "Computable General Equilibrium" models, but in mainstream economics these have largely been supplanted by Ramsey growth models, most of which use a Cobb-Douglas "production function".

[49] A worthwhile and highly publishable task for a motivated reader is to generalize this and make capacity utilization an endogenous variable of the model. This will create an (at least) 3-dimensional model, whose behaviour will be far more complex than that shown here.

Labour's input also has to be converted into energy terms, where we treat the energy output of the representative worker as a constant:[50]

$$\begin{aligned} L \cdot E_L \cdot e_L &= L \cdot E_l \\ E_l &= constant \end{aligned} \tag{1.104}$$

Labour is a derived demand in the Goodwin model: it is equal to the number of workers needed to operate the machines used to produce output. We therefore need to define a machine to worker ratio:

$$k_L \equiv \frac{K}{L} \tag{1.105}$$

In the original Goodwin model, Goodwin used an output to labour ratio a, which he assumed rose over time at a constant rate α, and this was the same as the rate of growth of the capital to labour ratio (since there was a linear relationship between output and capital). k_L is therefore equivalent to a in (Goodwin 1967). As with Goodwin, we assume that this ratio rises exogenously over time, but as well as giving it a less androgynous term (k_L rather than a), we use a less androgynous Greek letter kappa (κ_L) for its rate of growth:

$$\widehat{k_L} = \kappa_L \tag{1.106}$$

The output to labour ratio in this model is more complicated, since it relates the useful energy from production to the energy input from labour. It therefore includes the dynamics of energy as well as of those of the capital to labour ratio:[51]

$$\begin{aligned} a_E &= \frac{Q}{L \cdot E_l} \\ &= \frac{K \cdot E_K \cdot e_K}{L \cdot E_l} \\ &= k_L \cdot \frac{E_K}{E_l} \cdot e_K \end{aligned} \tag{1.107}$$

E_K, the energy input to the representative machine at time t, is assumed to grow at an exogenously given rate of κ_E:

$$\widehat{E_K} = \kappa_E \tag{1.108}$$

So that the rate of growth of a_E is:

$$\widehat{a_E} = \kappa_L + \kappa_E \tag{1.109}$$

We can now derive L by rearranging the first line of equation (1.107):

[50] The capacity for work for the average human is of the order of 100 Watts.

[51] From this point on we omit capacity utilisation u, as discussed in footnote XX. See XX for the US data.

$$L = \frac{Q}{a_E \cdot E_l} \tag{1.110}$$

Once L is defined, the rest of the model follows logically.

The employment rate λ is employment L divided by population N, which is assumed to grow at an exogenously given rate. Goodwin used β for this rate; in keeping with our eponymous renaming of the capital to labour ratio, we use ν (the Greek equivalent of n) instead:

$$\begin{aligned} \lambda &= \frac{L}{N} \\ \widehat{N} &= \nu \end{aligned} \tag{1.111}$$

The employment rate determines the rate of change of wages:

$$\widehat{w} = S_\lambda \cdot (\lambda - Z_\lambda) \tag{1.112}$$

The wage times Labour determines the wage bill, which determines profit:

$$\Pi = Q - w \cdot L \tag{1.113}$$

Investment is profit minus depreciation:

$$I = \frac{dK}{dt} = \Pi - \delta_K \cdot K \tag{1.114}$$

Capital times the energy output of capital determines output Q in units of energy per year, closing the causal chain of the model:

$$Q = K \cdot E_K \cdot e_K \tag{1.115}$$

10.3.1 Derivation

We start from the same system states as in the original Goodwin model, and then expand them out with the new definitions from equations (1.103) to (1.115).

Firstly the derivation of $\hat{\lambda}$:

$$\begin{aligned} \lambda &= \frac{L}{N} \\ &= \frac{1}{N} \frac{Q}{a_E \cdot E_l} \end{aligned} \tag{1.116}$$

Therefore the "percentage rate of change of λ" is:

$$\begin{aligned} \hat{\lambda} &= \widehat{Q} - \widehat{N} - \widehat{a_E} \\ &= \widehat{Q} - (\nu + (\kappa_L + \kappa_E)) \end{aligned} \tag{1.117}$$

The derivation of $\hat{Q}$ starts from the definition of Q:

$$Q = K \cdot E_K \cdot e_K \tag{1.118}$$

Therefore

$$\begin{aligned}\widehat{Q} &= \widehat{K} + \widehat{E_K} \\ &= \frac{I_G - \delta_K \cdot K}{K} + \kappa_E \\ &= \frac{I_G}{K} + \kappa_E - \delta_K\end{aligned} \tag{1.119}$$

The derivation of $\frac{I_G}{K}$:

$$\begin{aligned}\frac{I_G}{K} &= \frac{\Pi}{K} \\ &= \frac{Y - w \cdot L}{K} \\ &= \frac{Y \cdot \left(1 - \frac{w \cdot L}{Y}\right)}{K} \\ &= \frac{\frac{Q}{E_K} \cdot (1-\omega)}{K} \\ &= \frac{K \cdot E_K \cdot e_K \cdot (1-\omega)}{K \cdot E_K} \\ &= e_K \cdot (1-\omega)\end{aligned} \tag{1.120}$$

Therefore:

$$\hat{\lambda} = e_K \cdot (1-\omega) - (\nu + \kappa_L + \delta_K) \tag{1.121}$$

This is identical to the original Goodwin model (with depreciation) with e_K, the efficiency with which machinery turns its input energy into useful work, taking the place of the capital to output ratio.

Secondly, the derivation of $\widehat{\omega}$:

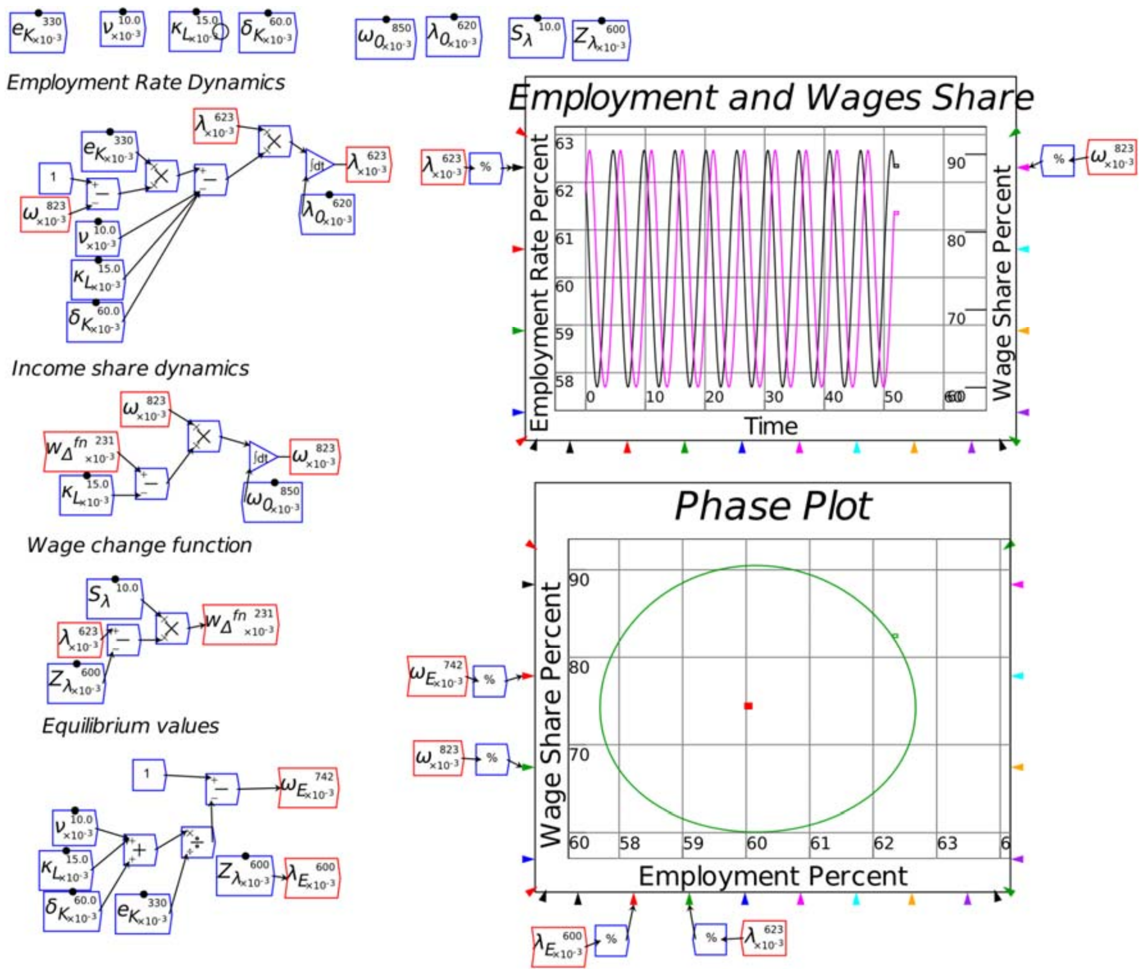
$$\begin{aligned}\widehat{\omega} &= \widehat{\frac{w \cdot L}{Y}} \\ &= \widehat{\frac{w \cdot \frac{Q}{a_E \cdot E_l}}{\frac{Q}{E_K}}} \\ &= \widehat{\frac{E_K \frac{w}{a_E}}{E_l}} \\ &= \widehat{E_K} + \widehat{w} - \widehat{a_E} \\ &= \kappa_E + \widehat{w} - (\kappa_L + \kappa_E) \\ &= S_\lambda \cdot (\lambda - Z_\lambda) - \kappa_L\end{aligned} \tag{1.122}$$

This is strictly identical to the original Goodwin model form:

$$\begin{aligned}\hat{\lambda} &= e_K \cdot (1-\omega) - (\nu + \kappa_L + \delta_K) \\ \widehat{\omega} &= S_\lambda \cdot (\lambda - Z_\lambda) - \kappa_L\end{aligned} \tag{1.123}$$

At this stage the inclusion of energy might look like "much ado about nothing"—see Figure 188.

Figure 188: Goodwin with energy in system state form



However, there are three ways in which this is an advance:

1. The previous empirical regularity of a reasonably constant capital to output ratio is now explained as the efficiency with which energy is converted into useful work;
2. The fact that no quantitative change occurs by introducing energy into the Leontief production function, whereas a significant change occurs when doing the same with the Cobb-Douglas production function, indicates that the Leontief form was effectively correct, though based on a statistical regularity (the relatively constant capital to output ratio) rather than on energy; and
3. The explicit use of energy in the derivation allows both waste production (consistent with the 2nd Law of Thermodynamics) and resource depletion to be added to the basic Goodwin model.

Point 3 above is covered by firstly defining waste energy as the complement to useful energy in Equation (1.115):

$$W_E = K \cdot E_K \cdot (1 - e_K) \tag{1.124}$$

Secondly, to simulate resource depletion, we revise Equation (1.115) to include a factor based on the fraction of remaining fossil fuel reserves:

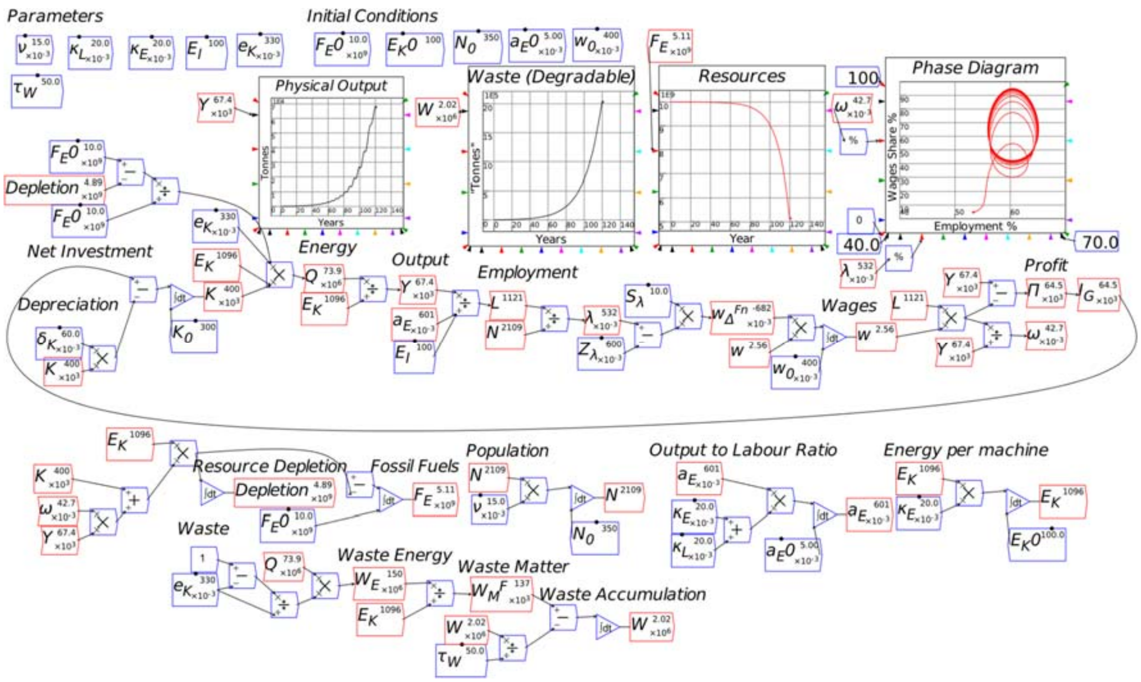
$$\begin{aligned} Q &= K \cdot E_K \cdot e_K \cdot \frac{F_0 - Depletion}{F_0} \\ Depletion &= \int \left((K + \omega \cdot Y) \cdot E_K \right) \end{aligned} \tag{1.125}$$

Depletion includes the use of energy in production, and the energy consumed by workers.

This extension is best shown in an absolute values model of Goodwin with energy. This model is shown in Equation (1.126) and simulated in *Minsky* in Figure 189 (the Minsky model includes a conversion of waste energy into waste matter, which can be degraded over time—we're thinking of CO_2 here obviously).

$$\begin{aligned} \frac{d}{dt} N &= \nu \cdot N \\ \frac{d}{dt} a_E &= (\kappa_L + \kappa_E) \cdot a_E \\ \frac{d}{dt} E_K &= \kappa_E \cdot E_K \\ Q &= K \cdot E_K \cdot e_K \cdot \frac{F_0 - Depletion}{F_0} \\ Y &= \frac{Q}{E_K} \\ W_E &= Q \cdot \frac{1 - e_K}{e_k} \\ \frac{d}{dt} K &= I_G - \delta_K \cdot K \\ I_G &= \Pi \\ \Pi &= Y - w \cdot L \\ L &= \frac{Y}{a_E \cdot E_l} \\ \omega &= \frac{w \cdot L}{Y} \\ \frac{d}{dt} w &= w \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) \right) \\ \frac{d}{dt} F_E &= -E_K \cdot (K + \omega \cdot Y) \\ \frac{d}{dt} Depletion &= (K + \omega \cdot Y) \cdot E_K \end{aligned} \tag{1.126}$$

Figure 189: A Minsky system dynamics model of energy in production and resource depletion



This explains the final figure in *Manifesto*, but it only scratches the surface of properly incorporating inputs from Nature into economic modelling. Though the previous model does introduce energy into the production function, its treatment of matter is too simplistic, with all the "heavy lifting" between matter and energy done by the conversion factor E_K. A model of production entirely in terms of energy is also an extreme simplification. More realistically, energy is used in production to transform matter from less useful forms (raw materials) to more useful (finished products). The next section develops a model with both energy and matter inputs used to produce useful matter output. This model was derived in collaboration with my friends and research colleagues Tim Garrett, an atmospheric physicist, and Matheus Grasselli, a financial mathematician.

10.4 A Goodwin model with matter and energy

Our inspiration here was Hicks's noble but unsuccessful attempt to build a dynamic model of a production economy in the paper "Wages and Interest: The Dynamic Problem" (Hicks 1935), where the output was bread—a consumer good. Though poorly known today, this paper was in fact the real origin of the IS-LM model, as Hicks admitted in 1981 in "IS-LM: An Explanation" (Hicks 1981).[52]

[52] "I must begin with the old story. "Mr. Keynes and the Classics" was actually the fourth of the relevant papers which I wrote during those years. The third was the review of The General Theory that I wrote for the Economic Journal, a first impression which had to be written under pressure of time, almost at once on first reading of the book. *But there were two others that I had written before I saw The General Theory*. One is well known, my "Suggestion for Simplifying the Theory of Money" (1935a), which was written before the end of 1934. The other, much less well known, is even more relevant. "Wages and Interest: the Dynamic Problem"' was a first sketch of what was to become the "dynamic" model of Value and Capital (1939). It is important here, because it shows (I think quite conclusively) that *that model [IS-LM] was already in my mind before I wrote even the first of my papers on Keynes*." (Hicks 1981, p. 140. Emphasis added).
The key evidence to which Hicks alluded was the section of the 1935 paper that used equilibrium in two markets to mean that equilibrium in a third could be assumed—and therefore the analysis could be simplified by omitting that market entirely: "An obvious result, so it would appear! But it conveys the less obvious message, *that in order to determine the rate of interest, we need not examine that elusive thing, the "capital market"; for if the*

In this paper, Hicks attempted to develop a dynamic theory of economics by reconciling the treatment of capital as a "factor of production" by J.B. Clark with its treatment as a produced means of production by Wicksell:

> Most modern theories of capital fall into one or two classes. On the one hand, there is the "timeless" type of theory, which treats capital as a factor of production like any other. Such a theory is that of J. B. Clark. In practice, it assimilates capital to land, treating it as the inexhaustible provider of a regular stream of resources. On the other hand, there is the "period of production" theory of Bohm-Bawerk and Wicksell. This treats capital as "stored-up labour"—labour stored up *in the past*. (Hicks 1935, p. 456)

Hicks characterised both theories as "stationary", and "quite satisfactory under that hypothesis, but incapable of extension to meet other hypotheses, and consequently incapable of application" (Hicks 1935, p. 456), because both theories assumed equalities that applied in a stationary state but could not be assumed in a changing one. Hicks warned that assuming such equalities where they did not exist was dangerous:

> To found a theory upon an assumed equality, which is not a real equality, is a most dangerous thing to do; for the more complex the theory becomes, the more specialised it becomes. The blinkers grow, until they shut out nearly all the landscape. One distinction blurred over breeds another, until we have in the end only a special case of a special case of a special case. (Hicks 1935, p. 457)

Hicks therefore attempted to abandon the assumption of stationarity and develop a dynamic model. After advocating period analysis over the use of continuous time, Hicks set out his simplifying assumptions, which commenced with:

> (1) We shall assume a community which is wholly engaged in the production of a single homogeneous good, which we shall call Bread.
>
> (2) Bread is made by the co-operation of labour (assumed homogeneous) with capital goods (not homogeneous) which we shall call Equipment. Equipment may include land, buildings, machinery, raw materials, and half-finished goods. (Hicks 1935, p. 458)

These assumptions, in the context of a dynamic theory, require a model in which both bread and Equipment are produced—and in which raw materials, including energy, are exploited, as we model here. Had Hicks actually built this model, it would have been a true *tour de force*. Unfortunately, he did not. Instead, at a later stage in the paper, he reduced Equipment to dated bread:

> A production plan can be regarded, on the basis of our simplifying assumptions, as a series of outputs of bread in successive weeks, together with the series of inputs of labour necessary to obtain those outputs. For the entrepreneur has actually to determine, not only how much labour he will employ in the first week, but how he will employ that labour, whether in the production of bread for the next market day, or in the production of bread for the more distant future (activity which, a

---

market for labour is in equilibrium, and if the market for bread is in equilibrium, the market for loans must be in equilibrium too." (Hicks 1935, pp. 465-67. Emphasis added)

> week after, will only have resulted in the production of equipment). (Hicks 1935, p. 460)

Hicks's conceptual apparatus thus reduced to a model in which bread is produced using bread and labour alone, and in which bread functions as a consumer good if used this week, and a capital good if not used this week.[53]

When we first attempted to build a model which did achieve what Hicks set out to do, we felt genuine sympathy for his plight, since our attempt to build a model with the same conceptual foundation—an economy producing a single commodity, which functions as both a consumer and an investment good (which is a common feature of the vast majority of economic models, both Neoclassical and Post Keynesian)—led to a similar intellectual impasse. It is very easy to imagine a world in which consumers consume bread, but very difficult to imagine a world in which bread functions as machinery. In the end, Hicks's sketch of a model described a passingly realistic scenario of consumption, but a trivial and unrealistic scenario for investment.

Our solution was to reverse this dilemma, and to consider a world with a far-fetched model of consumption, but a passingly realistic scenario for investment. What commodity can take the place of bread, and enable a reasonably realistic model of production—including the use of raw materials and

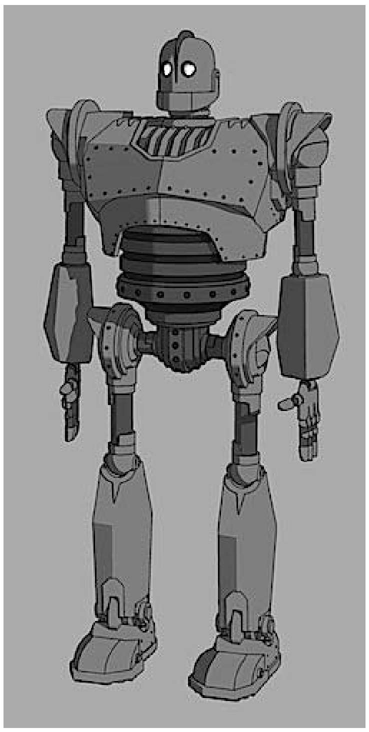
[53] Hicks's time period in "Wages and Interest: The Dynamic Problem" was a week, something which he later admitted made the Walrasian assumptions he made in 1935 inappropriate for the macroeconomic analysis of Keynes's 1936 *General Theory*, which in 1937 he purported to capture with the IS-LM model. While it was appropriate in a week to consider expectations to be constant, it was not appropriate to consider the same when the time period is a year, because it implies constancy of expectations, which means the absence of surprises:

> "Applying these notions to the IS-LM construction, it is only the point of intersection of the curves which makes any claim to representing what actually happened (in our "1975"). Other points on either of the curves—say, the IS curve—surely do not represent, make no claim to represent, what actually happened. They are theoretical constructions, which are supposed to indicate what would have happened if the rate of interest had been different. It does not seem farfetched to suppose that these positions are equilibrium positions, representing the equilibrium which corresponds to a different rate of interest. If we cannot take them to be equilibrium positions, we cannot say much about them. But, as the diagram is drawn, the IS curve passes through the point of intersection; so the point of intersection appears to be a point on the curve; thus it also is an equilibrium position. *That, surely, is quite hard to take. We know that in 1975 the system was not in equilibrium. There were plans which failed to be carried through as intended; there were surprises*. We have to suppose that, for the purpose of the analysis on which we are engaged, these things do not matter. It is sufficient to treat the economy, as it actually was in the year in question, as if it were in equilibrium. Or, what is perhaps equivalent, it is permissible to regard the departures from equilibrium, which we admit to have existed, as being random. There are plenty of instances in applied economics, not only in the application of IS-LM analysis, where we are accustomed to permitting ourselves this way out. But it is dangerous. Though there may well have been some periods of history, some "years," for which it is quite acceptable, it is just at the turning points, at the most interesting "years," where it is hardest to accept it." (Hicks 1981, pp. 149-50)

energy, and the production of machinery using that commodity as an input—at the probable expense of a rather unrealistic consumption good?

Fiction provided an answer with the cult animated movie The Iron Giant. The deuteragonist of that movie was made of iron—see Figure 190. We therefore imagined a "Planet of the Iron Giants", in which Iron Giants were the consumers and workers (and capitalists), iron was used to make the capital goods (blast furnace/rolling mill, iron ore and coal mining machines), energy was essential to all three processes, iron was consumed by the workers as their real wage, and physical waste (slag) was necessarily generated by production, as well as waste energy as in our previous model.

Figure 190: The "Iron Giant"



10.5 Derivation: constant technology and population

This was more easily said than done. To produce a model in which one commodity—iron—was the final output, we needed to model three sectors: the energy-mining sector (most easily thought of as coal mining, since coal—as coke—is also an input to iron manufacturing, and not solely an energy source); the iron-ore-mining sector; and a factory sector which took inputs of coal and iron ore to produce iron and slag. Each sector needed labour, and specialized capital—made of iron. Our mental framework was that everything was made of sheet iron, which could be shaped in the factory sector into shapes specific for each sector, and also as consumption for our Iron Giant workforce.

We needed three production relations, each with a different type of output, but each of which required energy (and capital and labour) as inputs. The outputs were respectively energy (best thought of as coal), iron ore, and iron plus slag.

In keeping with Keen Ayres & Standish 2019, we treat machinery ("Capital") as the means to channel energy to perform useful work. The output of an industry per year is the product of the number of machines K, times the energy per machine per year E, times the efficiency of conversion of energy into useful work ε*, times the yield of product per unit of energy input y*—this is the key extension over the previous model, where all internal processes in the model were in terms of energy only.

$$\begin{aligned} y_E &= \frac{Energy_{Output}}{Energy_{Input}} \\ y_M &= \frac{Ore_{Output}}{Energy_{Input}} \\ y_F &= \frac{Matter_{Output}}{Energy_{Input}}; Matter_{Output} = Iron + Slag \end{aligned} \tag{1.127}$$

With three sectors (E for mining energy, M for mining iron ore, and F for factory), we have five equations: one for the output of each sector in units of energy per year (Joules/Year), units of iron ore per year (Kilograms/Year), units of physical output consisting of both iron and slag (Kilograms/Year), iron itself Y (our single commodity GDP), and slag Y_W (physical waste):

$$\begin{array}{cc} \textit{Equation} & \text{Units} \\ E = K_E \cdot E_E \cdot \varepsilon_E \cdot y_E & \text{Energy/Year} \\ M = K_M \cdot E_M \cdot \varepsilon_M \cdot y_M & \text{Mass/Year (Iron ore)} \\ F = K_F \cdot E_F \cdot \varepsilon_F \cdot y_F & \text{Mass/Year (Iron plus Slag)} \\ Y = \mu \cdot F & \text{Mass/Year (Iron)} \\ Y_W = (1-\mu) \cdot F & \text{Mass/Year (Slag)} \end{array} \tag{1.128}$$

Output Y that is used for investment adds to the stock of machines K, which is denominated in terms of mass: kilograms of iron. This gives us a novel solution to the measurement of capital problem: rather than ignoring the issue entirely as in standard Neoclassical models—despite Samuelson's concession of defeat in the Cambridge Controversies (Sraffa 1960; Samuelson 1966; Pasinetti et al. 2003; Harcourt 1972)—or measuring capital in terms of dated labour as did Sraffa (Sraffa 1960), *we measure capital in terms of kilograms of iron*.[54] Using K to signify the number of machines, and k to signify the weight of each machine, we have:

$$K = K_E \cdot k_E + K_M \cdot k_M + K_F \cdot k_F \tag{1.129}$$

We therefore define κ as the proportion of capital invested in each industry:

$$\begin{aligned} \frac{K}{K} &= \frac{K_E \cdot k_E}{K} + \frac{K_M \cdot k_M}{K} + \frac{K_F \cdot k_F}{K} \\ 1 &= \kappa_E + \kappa_M + \kappa_F \\ K_E &= \kappa_E \cdot \frac{K}{k_E}, K_M = \kappa_M \cdot \frac{K}{k_M}, K_F = \kappa_F \cdot \frac{K}{k_F} \end{aligned} \tag{1.130}$$

Employment L is proportional to the number of machines in each sector. There is a workers per machine ratio λ such that employment in each industry equals this ratio times K:

$$\begin{aligned} L_E &= \lambda_E \cdot K_E \\ &= \lambda_E \cdot \frac{\kappa_E \cdot K}{k_E} \end{aligned} \tag{1.131}$$

[54] Conceptually, the machines are rolled iron sheets molded into different shapes.

Matching equations apply for employment in mining M and fabrication F.

We then define λ_K as the aggregate ratio of labour to capital:

$$\lambda_K = \left(\lambda_E \cdot \frac{\kappa_E}{k_E} + \lambda_M \cdot \frac{\kappa_M}{k_M} + \lambda_F \cdot \frac{\kappa_F}{k_F} \right) \tag{1.132}$$

This enables us to define aggregate employment L and the employment rate λ. For simplicity in the first pass, we worked with a constant population N_0, and a constant labour to capital ratio λ_K:[55]

$$\lambda = \frac{L}{N_0} = \lambda_K \cdot \frac{K}{N_0} \tag{1.133}$$

This in turn enabled us to use the same wage change relation as in the previous models, based on the aggregate level of employment:

$$\frac{1}{w} \cdot \frac{d}{dt} w = S_\lambda \cdot (\lambda - Z_\lambda) \tag{1.134}$$

Three output equations are now needed, in contrast to earlier models with just one. A full, multi-commodity-model would require price relations for each of the energy, iron mining and fabrication sectors, as well as stocks of unsold units of output. To generate a less complex single commodity model, we instead assumed proportionality between each sector, with excess capacity in energy and iron ore mining so that their yields adjust to meet the energy needs of the entire economy.[56] This means that the output of the energy sector equals to energy input needs of all three sectors: itself, mining, and fabrication:

$$K_E \cdot E_E \cdot \varepsilon_E \cdot y_E = K_E \cdot E_E + K_M \cdot E_M + K_F \cdot E_F \tag{1.135}$$

This requires that the yield of the energy sector adjusts to the needs of all three sectors:

$$y_E = \frac{K_E \cdot E_E + K_M \cdot E_M + K_F \cdot E_F}{K_E \cdot E_E \cdot \varepsilon_E} \tag{1.136}$$

Solving for y_E yields:

$$y_E = \frac{1}{\varepsilon_E} \cdot \left(1 + \frac{k_E}{\kappa_E} \cdot \frac{1}{E_E} \cdot \left(\frac{E_M \cdot \kappa_M}{k_M} + \frac{E_F \cdot (1 - \kappa_E - \kappa_M)}{k_F} \right) \right) \tag{1.137}$$

The same assumption for mining, that the yield adjusts to fit the needs of the fabrication sector for material (iron ore) inputs, enables us to link the total output of the two sectors. Since the factory sector converts iron ore to rolled iron sheeting plus slag, then by the conservation of matter, the gross output of the factory F in kilograms of iron plus slag equals the input M in kilograms of iron ore. Therefore:

$$K_M \cdot E_M \cdot \varepsilon_M \cdot y_M = K_F \cdot E_F \cdot \varepsilon_F \cdot y_F \tag{1.138}$$

[55] When the derivation of this model succeeded, we added a growing population N and a falling labour to capital ratio in the final version, which is detailed in the next section.

[56] This assumption is not a bad approximation to reality during a pre-ecological crisis period. It will be relaxed in later extensions to allow analysis of falling EROEI or fossil to renewable energy switching.

From this we can derive the yield (in kilograms per joule) of the factory sector:

$$y_F = \frac{\kappa_M \cdot k_F \cdot E_M \cdot \varepsilon_M}{\kappa_F \cdot k_M \cdot E_F \cdot \varepsilon_F} \cdot y_M \tag{1.139}$$

Output from the factory sector can now be defined:

$$\begin{aligned} F &= K_F \cdot E_F \cdot \varepsilon_F \cdot \frac{\kappa_M \cdot k_F \cdot E_M \cdot \varepsilon_M}{\kappa_F \cdot k_M \cdot E_F \cdot \varepsilon_F} \cdot y_M \\ &= \frac{\kappa_M \cdot E_M \cdot \varepsilon_M \cdot y_M}{k_M} \cdot K \end{aligned} \tag{1.140}$$

Define ϕ_K:

$$\phi_K = \frac{\kappa_M \cdot E_M \cdot \varepsilon_M \cdot y_M}{k_M} \tag{1.141}$$

Then output Y and waste Y_W are:

$$\begin{aligned} Y &= \mu \cdot \phi_K \cdot K \\ Y_W &= (1-\mu) \cdot \phi_K \cdot K \end{aligned} \tag{1.142}$$

With output, labour and wages defined, it is now possible to derive the model in terms of the wages share of GDP and the employment rate. The rate of change of the wages share of GDP is a linear transformation of the rate of change of wages in this model without technical change or growth in population:

$$\phi_K = \frac{\kappa_M \cdot E_M \cdot \varepsilon_M \cdot y_M}{k_M} \tag{1.143}$$

Therefore, the rate of change of wages share is a linear transformation of the wage change function:

$$\begin{aligned} \frac{d}{dt}\omega &= \frac{\lambda_K}{\psi} \cdot \frac{d}{dt} w \\ &= \frac{\lambda_K}{\psi} \cdot w \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) \right) \\ &= \frac{\lambda_K}{\psi} \cdot \frac{\psi}{\lambda_K} \cdot \omega \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) \right) \\ &= \omega \cdot \left(S_\lambda \cdot (\lambda - Z_\lambda) \right) \end{aligned} \tag{1.144}$$

The employment rate is a linear transformation of capital stock:

$$\begin{aligned} \lambda &= \frac{L}{N_0} \\ &= \frac{\lambda_K}{N_0} \cdot K \end{aligned} \tag{1.145}$$

Hence

$$\begin{aligned}\frac{d}{dt}\lambda &= \frac{\lambda_K}{N_0}\cdot\frac{d}{dt}K \\ &= \frac{\lambda_K}{N_0}\cdot\left(\psi_K\cdot(1-\omega)\cdot K-\delta_K\cdot K\right) \\ &= \frac{\lambda_K}{N_0}\cdot\left(\psi_K\cdot(1-\omega)-\delta_K\right)\cdot\frac{N_0}{\lambda_K}\cdot\lambda \\ &= \lambda\cdot\left(\psi_K\cdot(1-\omega)-\delta_K\right)\end{aligned} \tag{1.146}$$

This results in the classic Goodwin model, with ψ_K taking the place of $1/v$:

$$\begin{aligned}\frac{d}{dt}\omega &= \omega\cdot\left(S_\lambda\cdot(\lambda-Z_\lambda)\right) \\ \frac{d}{dt}\lambda &= \lambda\cdot\left(\psi_K\cdot(1-\omega)-\delta_K\right)\end{aligned} \tag{1.147}$$

The final step in this process was to introduce a growing population and changing technology, manifest as a falling ratio of workers to machines. This in turn provides the scaffolding on which to add the accumulation of waste in the biosphere.

10.6 Growth and pollution

We replace a constant population with a growing one, and a constant labour to capital ratio with a falling one:

$$\begin{aligned}\frac{d}{dt}N &= \nu\cdot N, \nu>0 \\ \frac{d}{dt}\lambda_K &= \lambda_\kappa\cdot\lambda_K, \lambda_\kappa<0\end{aligned} \tag{1.148}$$

A variable N thus replaces N_0in (1.145) while λ_K becomes a variable in (1.143). The state space equation for ω thus becomes:

$$\begin{aligned}\frac{d}{dt}\omega &= \frac{1}{\psi}\frac{d}{dt}(w\cdot\lambda_K) \\ &= \omega\cdot\left(S_\lambda\cdot(\lambda-Z_\lambda)+\lambda_\kappa\right)\end{aligned} \tag{1.149}$$

That for λ becomes:

$$\begin{aligned}\frac{d}{dt}\lambda &= \frac{d}{dt}\frac{\lambda_K\cdot K}{N} \\ &= \lambda\cdot\left(\psi_K\cdot(1-\omega)-\delta_K+\lambda_\kappa-\nu\right)\end{aligned} \tag{1.150}$$

This is once more the classic Goodwin model:

$$\begin{aligned}\frac{d}{dt}\omega &= \omega\cdot\left(S_\lambda\cdot(\lambda-Z_\lambda)+\lambda_\kappa\right) \\ \frac{d}{dt}\lambda &= \lambda\cdot\left(\psi_K\cdot(1-\omega)-\delta_K+\lambda_\kappa-\nu\right)\end{aligned} \tag{1.151}$$

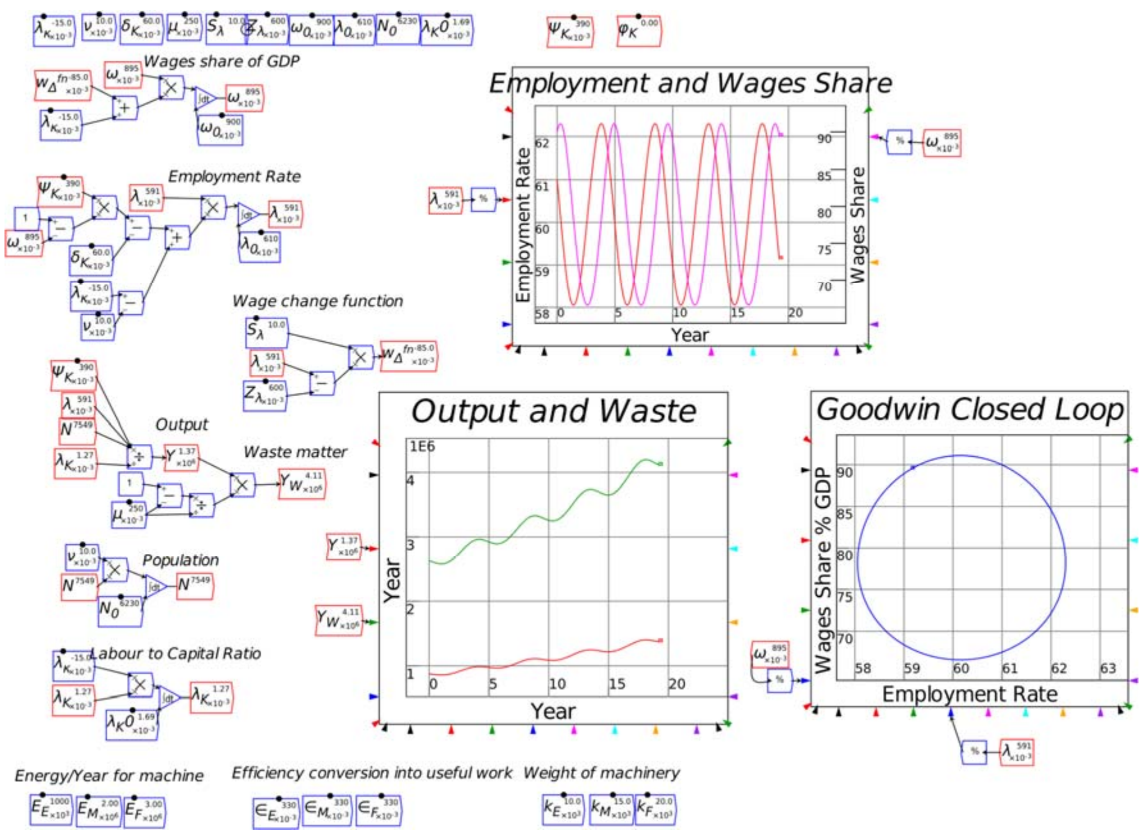
The strengths of this model over the previous versions are:

- The introduction of the concept of an “energy return on energy invested” in the yield from the energy mining sector, y_E, since the input and the output are both energy. This is a critical concept in biophysical economics (Hall 2011), but, to our knowledge, has not previously been incorporated in a macroeconomic model.
    - In this initial model, this is a constant derived from the requirement of the other two sectors. Our ambition is to make this an empirically derived quantity in future extensions, and to consider the extent to which this determines and constrains economic performance.
- It is now possible to link this model directly to the generation of waste matter Y_W using equation (1.142):

$$\begin{aligned} Y(t) &= \psi_K \cdot \frac{\lambda(t) \cdot N_0 \cdot e^{\nu \cdot t}}{\lambda_K(t)} \\ Y_W(t) &= \frac{1-\mu}{\mu} \cdot Y(t) \end{aligned} \tag{1.152}$$

Figure 191 shows a simulation of this model in Minsky, including both output and waste derived from Equation (1.152).

Figure 191



This model thus achieves what Hicks attempted in "Wages and Interest: The Dynamic Problem" (Hicks 1935) in the context of a single commodity model of production. Future extensions will address the unrealism of this foundational model by introducing multiple commodities, and multiple forms of waste as well.

11 Analyzing a Model

To come: basic mathematics of stability analysis.

12 Estimating a model

To come: exploring a multi-dimensional parameter space.

13 Planned improvements in forthcoming releases of *Minsky*

To come.

14 Ravel

Ravel is a commercial extension to Minsky for analyzing multidimensional data. I'll give more details in a subsequent version of this manual.

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